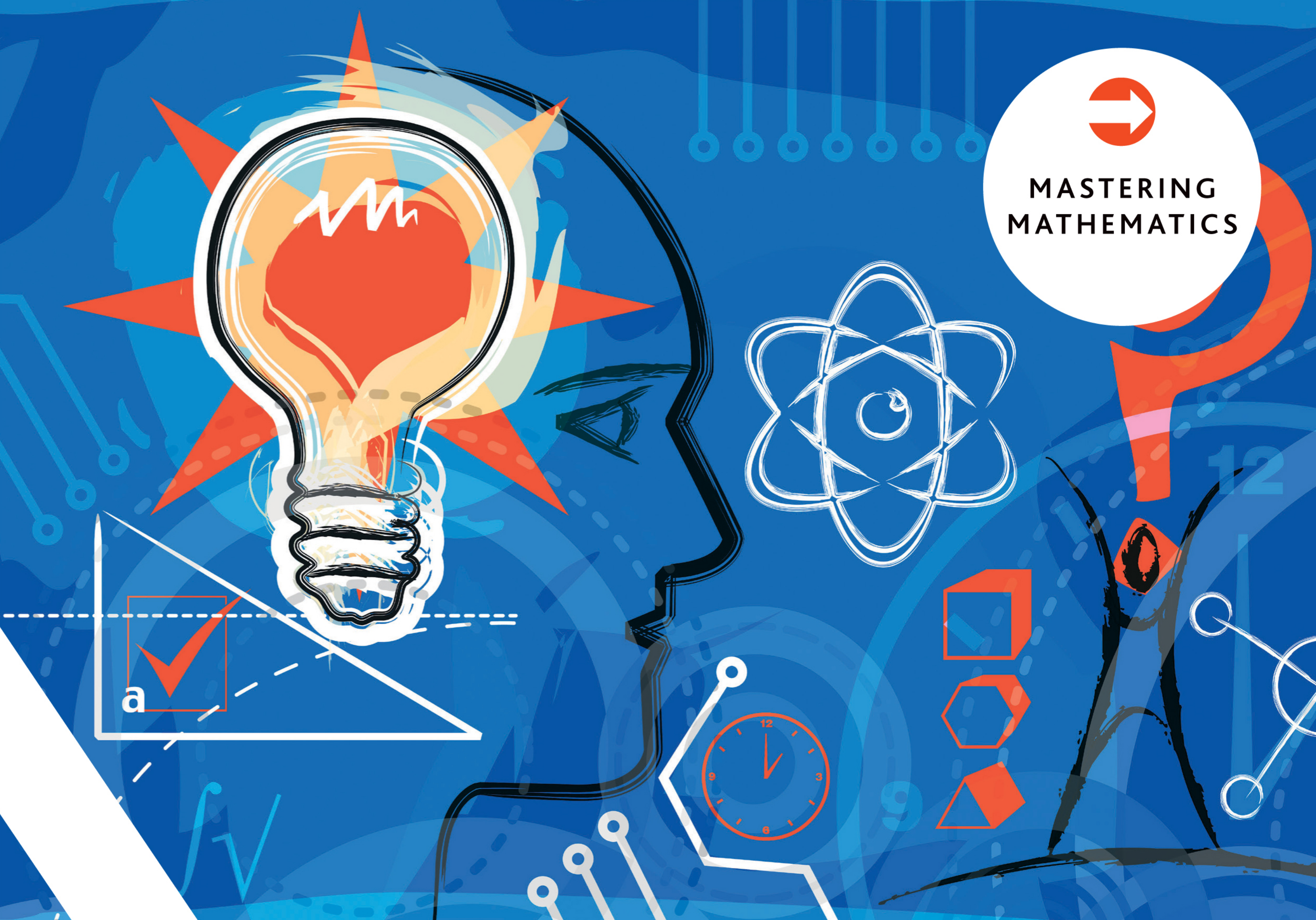

EDEXCEL GCSE (9 – 1)
MATHEMATICS
Success in a Year

Heather Davis

Series Editor
Elaine Lambert

DYNAMIC LEARNING

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HODDER EDUCATION
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Contents

Each chapter number corresponds to a topic. Most topics appear in both the Essential Topics and Next Steps sections, but some only appear in one section. Therefore, Essential Topics has no Chapter 12 and no Chapter 19 as these are only included in Next Steps. Likewise there are no Chapters 1, 4, 5,16, 20 or 21 in Next Steps as these topics are only in Essential Topics.

STATISTICS AND PROBABILITY

MINI TEST PAPERS

Problem Solving

Next Steps

NUMBER

ALGEBRA

GEOMETRY AND MEASURES

STATISTICS AND PROBABILITY

MINI TEST PAPERS

Answers to all questions in the book are provided online at www.hoddereducation.co.uk/EdexcelGCSESuccessInAYear

CHAPTER 1

ESSENTIAL TOPICS – NUMBER

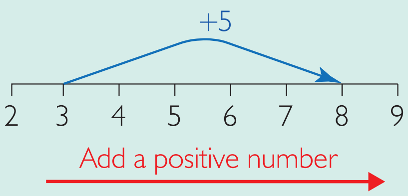
Calculating

JUST IN CASE

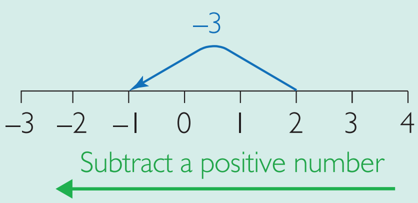
Adding and subtracting

You can use a number line to help you work out these.

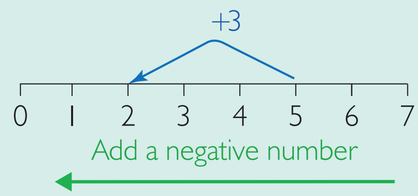
$3 + (+5) = +8$



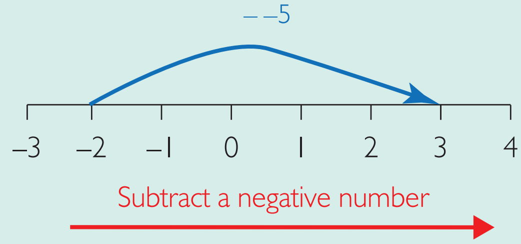
$2 - 3 = -1$



$5 + (-3) = +2$

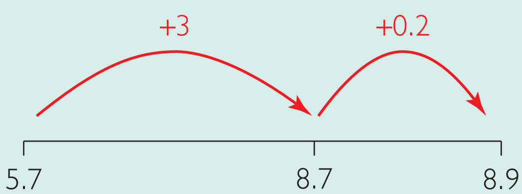


$-2 - (-5) = +3$



Number lines can also be used to add and subtract decimals.

$5.7 + 3.2 = 5.7 + 3 + 0.2$



So $5.7 + 3.2 = 8.9$

Multiplying and dividing whole numbers

Use the **grid method** to multiply two numbers.

Calculate 123×26

Solution

×	100	20	3
20	2000	400	60
6	600	120	18
	2600	520	78

$2600 + 520 + 78 = 3198$

Use **short division** to divide two numbers.

Calculate 276 ÷ 6

Solution

6 into 2 doesn't go.

6 into 27 goes 4 times with 3 left over.

6 into 36 goes 6 times.

$$\begin{array}{r} 46 \\ 6\overline{)27^{3}6} \end{array}$$

Multiplying and dividing negative numbers

When multiplying and dividing negative numbers, multiply or divide as usual, then the sign of the answer follows these patterns.

- When you multiply or divide two numbers with the same sign, the answer is positive.

  $7 \times 3 = 21$

  $(-7) \times (-3) = 21$

  $12 \div 4 = 3$

  $(-12) \div (-4) = 3$

- When you multiply or divide two numbers with different signs, the answer is negative.

  $(-3) \times 2 = -6$

  $3 \times (-2) = -6$

  $15 \div (-3) = -5$

  $(-15) \div 3 = -5$

SKILLS CHECK

→ Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Answer the questions in this section *without* using a calculator.

→ Adding and subtracting whole numbers

The chart shows the distances in miles between some UK airports.

	Aberdeen	London	Birmingham	Edinburgh
London	551			
Birmingham	437	139		
Edinburgh	130	390	306	
Leeds	333	203	118	206

a How far would you travel if you flew from Aberdeen to Edinburgh, then from Edinburgh to Leeds?

b How far is it to travel from Aberdeen to Leeds directly?

c What is the difference between your answers to part **a** and part **b**?

Multiplying whole numbers

What is the product of 217 and 12?

Adding and subtracting decimals

Work out these.

a 8.1 + 2.59 **b** 4.76 + 8.9 **c** 9.75 − 2.19 **d** 4.31 − 2.9

Dividing whole numbers

Work out these.

a 150 ÷ 6 **b** 985 ÷ 15

Adding and subtracting negative numbers

Work out each of these.

a $(-7) + (-10)$ **b** $(-3) - (+8)$ **c** $(-4) + (-9)$ **d** $(+10) - (-15)$

e $(+100) + (-20)$ **f** $(-70) - (-30)$ **g** $(+0.5) + (-0.5)$ **h** $(+1.5) - (+0.5)$

Multiplying and dividing negative numbers

Calculate these.

a $(+3) \times (-2)$ **b** $(-5) \times (-4)$ **c** $(+24) \div (-8)$ **d** $\frac{(-20)}{(-5)}$

Applying the knowledge

① Waqas is going to tile the walls in his kitchen.
The tiles are sold in boxes.
Each box contains 24 tiles and costs £15.
Waqas has £275 to spend on tiles.
He estimates that he will need 18 boxes of tiles.

a Does Waqas have enough money to buy 18 boxes of tiles?

In total, Waqas will use 425 tiles to tile the walls in his kitchen.

b Work out the number of tiles that Waqas will have left over.

② Mrs Green has £150 in her bank account. She receives a cheque for £50 from an insurance company. She has a phone bill of £95 to pay.

Mrs Green records her finances on this account sheet.

Credits (+)	Debits (−)	Balance (£)
		150
50		200
	95	?

a Copy the account sheet and fill in the unknown balance.

b Extend your copy of the account sheet and write these items in the correct columns.

Payment for dress £85
Competition win £100
Payment for food £25
Payment to hairdresser £40
Payment for car repairs £150
Sale of old car £400

c What is the final balance?

1.1 Order of operations

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Graham and Susan each work out this sum.

$3 \times 2 + 9 - 3 \div 3$

Graham says the answer is 4.
Susan says the answer is 8.

a They are both wrong. Work out the correct answer.

b Rewrite the sum with brackets to show how they each got their answers.

If you can do the question above, try this one on problem solving.

② Here are some numbers and some symbols.

1 9 + − × 2 5) (

Arrange them to form a statement that gives the answer 3.
Use each symbol and number exactly once.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 6 (Problem solving exercise 1.1 Order of operations).

What you need to know

Did you know?

Electricity bills come in two parts – a standing charge and a price per unit of electricity used. You need to do a calculation like the one below to work out your bill.

$7 + 0.1 \times 50$

Addition and **subtraction** are the most basic operations.

Multiplication can be thought of as repeated addition and **division** as repeated subtraction.

Powers are used to show repeated multiplication or division.

The order of operations is:
- Powers (indices)
- Multiplication and division
- Addition and subtraction

Brackets are used to show that a different operation needs to be done first.

Follow the order of operations to work out these.

a $(8 + 5) \times 4$

b $50 - 2 \times (9 + 6)$

a $(8 + 5) \times 4$ Brackets

$= 13 \times 4$ Multiplication

$= 52$

b $50 - 2 \times (9 + 6)$ Brackets

$= 50 - 2 \times 15$ Multiplication

$= 50 - 30$ Subtraction

$= 20$

How to do it

Using powers

Work out the answers to these calculations.

a $(20 + 7) \div 3^2 + 4$

b $(10 - 3) \times 2^3 - 4$

Solution

a $(20 + 7) \div 3^2 + 4$ Brackets

$= 27 \div 3^2 + 4$ Powers

$= 27 \div 9 + 4$ Division

$= 3 + 4$ Addition

$= 7$

b $(10 - 3) \times 2^3 - 4$ Brackets

$= 7 \times 2^3 - 4$ Powers

$= 7 \times 8 - 4$ Multiplication

$= 56 - 4$ Subtraction

$= 52$

Using fractions (division)

Work out $\frac{6 \times 8}{14 - 2}$.

Solution

The fraction bar acts in the same way as a bracket.

Brackets

So this is the same as $(6 \times 8) \div (14 - 2) = 48 \div 12$

$= 4$ Division

Unless otherwise stated, answer the questions in the following exercises *without* the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

Work these out.

1. **a** $(5 + 2) \times 3$ **b** $5 + 2 \times 3$
2. **a** $7 \times (10 - 4)$ **b** $7 \times 10 - 4$
3. **a** $(20 + 15) \div 5$ **b** $20 + 15 \div 5$
4. **a** $20 - 12 \div 2$ **b** $(20 - 12) \div 2$
5. **a** $(21 + 12) \times 3$ **b** $21 + 12 \times 3$
6. **a** $(13 + 5) \times (7 - 3)$ **b** $13 + 5 \times 7 - 3$

7 **a** $(8 + 12) \div (4 + 1)$ **b** $8 + 12 \div 4 + 1$

8 **a** $13 + 5 - 7 - 3$ **b** $(13 + 5) - (7 - 3)$

9 **a** 4×3^2 **b** $(4 \times 3)^2$

10 **a** $8^2 - 5^2 + 1^2$ **b** $(8 - 5 + 1)^2$

11 **a** $5^2 - (4 - 3)^2$ **b** $(5 - 4 - 3)^2$

12 **a** $5 + 2 \times 2 - 2$ **b** $(5 + 2) \times (2 - 2)$

13 **a** $4 - 6 - 8 \div 2$ **b** $4 - (6 - 8) \div 2$

14 **a** $7 \times 7 - 7$ **b** $7 \times (7 - 7)$

15 Find the value of $(a + b) \times (c - d)$ when $a = 1$, $b = -1$, $c = 4$, $d = 1$.

16 Find the value of $\sqrt{x \times (y - z)}$ when $x = 8$, $y = 12$, $z = 4$.

17 Insert brackets in each calculation to make these correct.

a $6 + 3 \times 2 = 18$ **b** $5 + 9 - 2 \times 2 = 19$

c $8 + 3 \times 2 + 1 = 23$ **d** $4 + 3 \times 3 + 2 = 35$

e $6 + 8 - 2 + 1 = 11$ **f** $13 - 5 + 4 - 2 = 6$

18 **a** $\dfrac{5+1}{2+1}$ **b** $5 + 1 \div 2 + 1$

19 **a** $\sqrt{(3^2 + 4^2 + 12^2)}$ **b** $\sqrt{(3 + 4 + 12)^2}$

20 **a** $\dfrac{(3 + 4) \times 2}{\sqrt{36} + \sqrt{64}}$ **b** $\dfrac{3^2 + 4^2}{\sqrt{36 + 64}}$

Problem solving exercise

1 Rewrite each statement with brackets to make it true.

a $10 - 1 \div 3 + 6 = 9$

b $3 + 7 \times 2 - 5 = -30$

c $2 \times 1 + 3^2 = 32$

2 Michelle worked out the value of $6x + 3x^2$ when $x = 2$.
Hannah worked out the value of $4y^2 - 52$ when $y = 5$.
Finlay worked out the value of $4z^2 \div 3$ when $z = 3$.
They all got an answer of 48.

a Whose answer of 48 is correct?

b Use brackets to show how the other two got their answers.

3 Here are four different numbers.

2	3	4	11

a Put one number in each box to make a correct statement.
You may only use each number once.

$\square + \square \times \square = \square$

b Using just three of these numbers, what is the greatest number that could appear in the answer box?

④ Here are four different symbols.

) + × (

a Copy the statement below and use each symbol once only to make the statement correct.

6 1 7 = 48

Here are four more symbols.

) ÷ − (

b Copy the statement below and use each symbol once only to make the statement correct.

9 7 4 = 3

⑤ **a** Use your calculator to work out $\frac{5.36 + 62.87}{19.86 - 6.52}$

b Peter's calculator gave an answer of 2.005659617.
Write, in order, the buttons Peter could have pressed on his calculator to get this answer.

Do I know it now?

① Work these out.

a $(7 - 2) \times 3$ **b** $7 - 2 \times 3$ **c** $7 - (2 \times 3)$

② Work these out.

a $100 \div (10 - 6)$ **b** $100 \div 10 - 6$ **c** $\sqrt{100} \div 10 - 6$

③ Work these out.

a $5^4 \div (5^3 - 5^2)$ **b** $5^4 \div 5^3 - 5^2$ **c** $(5^4 \div 5^3 - 5)^2$

④ Work these out.

a $\frac{8 + 4}{2 + 1}$ **b** $(8 + 4) \div (2 + 1)$ **c** $8 + 4 \div 2 + 1$

⑤ Find the value of $(a - b) \div c$ when

a $a = 10, b = 5, c = 2$ **b** $a = 5, b = 5, c = 2$

c $a = 5, b = 10, c = 2$.

Can I apply it now?

① Rewrite each statement with brackets to make it true.

a $12 + 4 \div 5 - 1 = 4$

b $7 - 3 \times 4 - 2 = 1$

1.2 Multiplying decimals

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

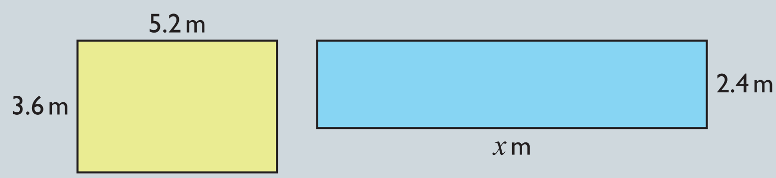
① Work out each of these.

a 0.4 × 0.1 **b** 0.32 × 0.4 **c** 2.2 × 0.03

d 2.4 × 0.8 **e** 0.04 × 0.04 **f** 1.6 × 0.05

If you can do the question above, try this one on problem solving.

② Here are two rectangles.
The areas of the two rectangles are the same.
Work out the value of x.



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 11 (Problem solving exercise 1.2 Multiplying decimals).

What you need to know

Think about these two things when you multiply decimals.

① Getting the correct digits
② Getting the decimal point in the right place

Here are some methods you can use.

- **Rewrite the decimals** so you are working with whole numbers.
  For example
  0.03 = 3 ÷ 100
  8 × 0.03 = 8 × 3 ÷ 100 = 24 ÷ 100 = 0.24
- **The grid method** (*see below*)

How to do it

Rewriting the decimals

Work out these calculations.

a 6×0.3

b 12×0.005

Solution

a $6 \times 0.3 = 6 \times 3 \div 10$ ← 0.3 is rewritten as $3 \div 10$

$= 18 \div 10$

$= 1.8$

b $12 \times 0.005 = 12 \times 5 \div 1000$ ← 0.005 is rewritten as $5 \div 1000$

$= 60 \div 1000$

$= 6 \div 100$

$= 0.06$

The grid method

Calculate 4.36×23.

Solution

×	4	0.3	0.06	
20	80	6	1.2	87.2 ← Total each row.
3	12	0.9	0.18	13.08
				100.28 ← Add the totals.

Answer the questions in the following exercises *without* the use of a calculator.

Learning exercise

1. You know that $6 \times 4 = 24$. Use it to find these.

   a 6×0.4 **b** 6×0.004 **c** 0.6×0.4

2. You know that $5 \times 8 = 40$. Use it to find these.

   a 5×0.8 **b** 5×0.08 **c** 0.5×0.8

3. You know that $15 \times 8 = 120$. Use it to find these.

   a 15×0.08 **b** 1.5×0.8 **c** 0.15×0.8

4. Work out these.

   a 25×0.4 **b** 25×0.004 **c** 0.25×0.04

5. You are given that $19 \times 53 = 1007$. Find these.

   a 1.9×5.3 **b** 190×5.3 **c** 0.19×0.53

6 Work out these.

a 0.6×0.7 **b** 0.5×0.4

7 Work out these.

a $12 \times 0.5 \times 0.5$ **b** $0.1 \times 0.2 \times 0.3$

8 Find these.

a 0.3^2 **b** 0.03^2

9 Calculate the cost of these.

a 5 books at £2.58 each

b 12 pens at £1.09 each

10 Work out these.

a $3 \times (-0.4)$ **b** $0.3 \times (-0.4)$ **c** $(-0.3) \times 0.4$

11 Work out these.

a $9 \times (-1.2)$ **b** $(-9) \times (-1.2)$ **c** $(-900) \times (-0.0012)$

12 Petrol costs £1.42 per litre. Calculate the cost of 55 litres.

13 Turkey costs £8.10 per kilogram and chicken costs £6.45 per kilogram.
Alana buys 0.6 kg of turkey and 1.2 kg of chicken.
What is the total cost?

14 An electricity bill shows the following information.

Electricity bill	
Previous reading	11 988 units
Present reading	12 768 units
Cost per unit	£0.152
Standing charge	£19.50

Work out the total charge.

15 The table shows the calories used up when doing some types of exercise.

Exercise	Calories per minute
Aerobics	10.5
Running	12.4
Swimming	6.8

a Jana has a 40 minute workout. She spends 15 minutes doing aerobics and the rest swimming. How many calories does she burn?

b Jack goes running for 45 minutes. How many more calories does he burn than Jana?

16 Zack's pay is £9.28 per hour, Monday to Friday. On Saturdays he is paid time and a half.
Last week he worked 6 hours on Monday, 7.2 hours on Tuesday and 8.5 hours on Saturday.
Calculate his pay for the week.

Problem solving exercise

① David and Elaine both work at the sports centre.
David works from 6 p.m. to 9.30 p.m. from Monday to Friday each week.
He earns £7.80 per hour.
Elaine works on Saturdays and Sundays.
She works from 8.30 a.m. to 12.30 p.m. and from 1.30 p.m. to 4.30 p.m.
She earns £9.50 per hour.
Who earns more money each week?
Show clearly how you got your answer.

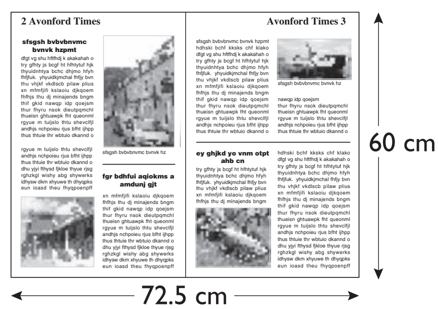
② It costs £12.50 for a rail ticket lasting one day from Ashton to Manchester.
It costs £2700 for a rail ticket lasting one year from Ashton to Manchester.
Becky travels from Ashton to Manchester on 220 days in every year.
Is it cheaper for Becky to buy a £12.50 ticket each day or to buy a £2700 ticket for the year?

③ Caroline works in a factory.
The table shows the number of hours she works each day one week.

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of hours worked	8.5 hours	8.4 hours	8.5 hours	10.2 hours	7 hours

Caroline is paid for the total time she works in a week.
Her pay is £9 per hour for the first 40 hours she works in the week.
She is paid £12 per hour for any extra hours she works in the week.
Work out Caroline's pay for the week.

④ A double sheet of a newspaper measures 72.5 cm by 60 cm.
The newspaper contains six double sheets.
Work out the area of paper in one copy of the newspaper.
Give your answer in square metres.



Do I know it now?

① Work out these.

a 8×0.2 **b** 5×0.09 **c** 0.3×70
d 1.6×10 **e** 1.3×3 **f** 0.8×0.6
g 0.6^2 **h** 0.04^2

② Use the fact that $186 \times 9.4 = 1748.4$ to write down the answers to the following.

a 18.6×9.4 **b** 1.86×9.4 **c** 186×0.94
d 0.186×9.4 **e** 0.0186×0.94 **f** 18.6×94

Can I apply it now?

① Find these costs.

a Sylvia buys 0.8 kg of cherries and 0.2 kg of strawberries.

b Kyle buys 3.5 kg of damsons and 0.5 kg of blackberries.

c Ramona buys 400 g of each type of fruit.

Strawberries......... £5.40 per kilo
Damsons............... £4.20 per kilo
Cherries................ £8.60 per kilo
Blackberries.......... £4.80 per kilo

1.3 Dividing decimals

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

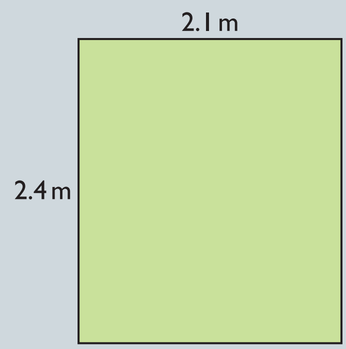
① Work out these.

a 22.72 ÷ 0.04 **b** 51.6 ÷ 0.06 **c** 0.65 ÷ 0.001

d 11.2 ÷ 0.2 **e** 2 ÷ 0.005 **f** 7.2 ÷ 0.4

If you can do the question above, try this one on problem solving.

② Richard is going to cover a bathroom wall with tiles.
The wall is in the shape of a rectangle.
The wall is 2.1 m long and 2.4 m high.
The tiles are squares with sides of 0.3 m.
There are 12 tiles in a box. Richard buys 4 boxes of tiles.
Will he have enough tiles to cover the wall in his bathroom?



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 15 (Problem solving exercise 1.3 Dividing decimals).

What you need to know

Here is a method you can use for working out a division problem.

Using equivalent fractions

- Write the division as a fraction.
- Multiply the top and bottom numbers by a power of 10 so that both become whole numbers.
- Work with the simpler equivalent fraction.

Here is how to find 0.6 ÷ 0.05:

$\times 100$

$$\frac{0.6}{0.05} = \frac{60}{5} = 12$$

$\times 100$

$\frac{0.6}{0.05}$ and $\frac{60}{5}$ are equivalent fractions.

How to do it

Using equivalent fractions

Work out these calculations.

a 3 ÷ 0.2 **b** 1.2 ÷ 0.03

Solution

a Write as a fraction. $\frac{3}{0.2}$

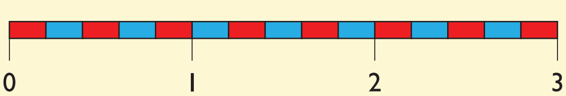
Then multiply the top and bottom numbers by 10 to make them both whole numbers.

$\times 10$

$$\frac{3}{0.2} = \frac{30}{2} = 15$$

$\times 10$

You can see this result on the number line from 0 to 3. It is divided into 15 pieces each 0.2 long.



b $1.2 \div 0.03 = \frac{1.2}{0.03}$

$\times 100$

$$\frac{1.2}{0.03} = \frac{120}{3} = 40$$

$\times 100$

Using short division

Work out 3 ÷ 8.

Solution

$$8\overline{)3.0^{6}0^{4}00}\quad 0.375$$

You can write extra zeros at the end of the number, after the decimal point.

This extra zero wasn't needed.

Answer 0.375

Unless otherwise stated, answer the questions in the following exercises *without* the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

① Work these out.

a 6.48 ÷ 2 **b** 9.70 ÷ 2 **c** 6.5 ÷ 2

d 0.054 ÷ 6 **e** 14.028 ÷ 7 **f** 29.613 ÷ 4

② You know that 45 ÷ 3 = 15. Work these out.

a 4.5 ÷ 3 **b** 45 ÷ 0.3 **c** 0.45 ÷ 3

③ Work these out.

a 3.5 ÷ 0.5 **b** 2.4 ÷ 0.4 **c** 1.9 ÷ 0.1

d 0.1 ÷ 0.4 **e** 0.315 ÷ 0.9 **f** 2.4934 ÷ 0.07

④ Work these out.

a 74 ÷ 0.2 **b** 25.32 ÷ 0.3 **c** 16.92 ÷ 1.2

d 0.009 ÷ 0.01 **e** 2.07 ÷ 0.002 **f** 32.04 ÷ 0.009

⑤ Which division is the odd one out?

16 ÷ 0.2 160 ÷ 20 1.6 ÷ 0.02 0.16 ÷ 0.002

⑥ Match each division to its correct answer.

72 ÷ 9	0.008
0.72 ÷ 9	800
7.2 ÷ 9	8
720 ÷ 0.9	0.8
7.2 ÷ 0.09	0.08
0.72 ÷ 90	80

⑦ Work these out.

a $6.9 \div (-2)$ **b** $(-7.8) \div 5$ **c** $(-0.01) \div 4$

d $(-5) \div (-0.8)$ **e** $(-0.01) \div 20$ **f** $0.8 \div 5$

⑧ Work these out.

a $0.1 \times 0.4 \div 0.2$ **b** $\dfrac{0.1 \times 0.4}{0.2}$ **c** $\dfrac{0.06}{0.2 \times 0.003}$

d $\dfrac{(0.6)^2}{0.4 \times 0.9}$ **e** $(11 + 2.2) \div 0.12$ **f** $\dfrac{(0.2)^5}{64}$

⑨ Beef costs £7.80 per kilogram. Ruth buys some beef and pays £9.75.
How much beef does she buy?

Problem solving exercise

① Angus bought a new television.
The television cost £628.40.
Angus paid a deposit of £50 when he bought the television.
He paid the rest of the money in 12 equal monthly payments.
Work out how much Angus paid each month.

② There is 400 g of hot chocolate in this jar.

a What is 400 g in kilograms?

A factory produces 10 000 kg of hot chocolate in one day.
The jars are packed in boxes of 50 jars.

b How many full boxes of these jars are produced each day?

③ A book is 30 cm by 20 cm and 1.4 cm thick.
Work out the greatest number of books of this size that will fit into a box with dimensions 0.9 m by 0.4 m by 0.35 m.

④ Tea bags are sold in two sizes of box.

Small

Large

a Compare the number of tea bags per £ that can be bought from each size of box.

An extra-large box is produced selling 450 tea bags for £16.

b Show that this offers better value for money than both the small and large boxes.

Do I know it now?

① Work these out.

a $7.2 \div 0.9$ **b** $4.8 \div 0.2$ **c** $51 \div 0.03$ **d** $3.14 \div 0.002$

② Work these out.

a $0.8 \div 0.01$ **b** $0.27 \div 0.03$ **c** $0.009 \div 0.06$ **d** $0.05 \div 4$

③ Work these out.

a $0.3 \times 0.4 \div 0.12$ **b** $\frac{(0.4)^2}{0.008}$ **c** $\frac{1}{0.2 \times 0.4}$ **d** $\frac{0.5 \times 0.022}{11}$

④ Work these out.

a $(-0.2) \div 0.01$ **b** $(-8) \div 0.025$ **c** $0.33 \div (-1.1)$ **d** $(-2.64) \div (-0.24)$

⑤ How many 60p stamps can you buy for £300?

Can I apply it now?

① Kay has two bottles of cola. She shares the 1.5 litre bottle equally among six boys, and the 2.5 litre bottle equally among eight girls.

a How much more does each girl get than each boy?

b Suppose instead Kay shares the cola equally between all the children.
How much would each of them get?

CHAPTER 2

ESSENTIAL TOPICS – NUMBER

Using our number system

JUST IN CASE

Notation

You need to know these signs.

$<$ means 'less than'

$\leqslant$ means 'less than or equal to'

$>$ means 'greater than'

$\geqslant$ means 'greater than or equal to'

$=$ means 'is equal to or worth the same as'

$\neq$ means 'is not equal to'

Understanding place value in whole numbers and decimals

Use a place-value table to help you think about place value.

Th	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
3	0	4	7	.	5	2	1

As you move from left to right, the value of the digit decreases.

Ordering decimals

Place these decimals in order, from smallest to largest.

0.3 0.31 0.7 0.13 0.71 0.07

Solution

U	.	$\frac{1}{10}$	$\frac{1}{100}$
0	.	3	
0	.	3	1
0	.	7	
0	.	1	3
0	.	7	1
0	.	0	7

A place-value table can help to show which digits represent the greatest value.

Looking at the place-value table, there is only one number that has no tenths, 0.07, so this is the smallest number.

The next smallest value in the tenths column is 1, so 0.13 is next.

There are two numbers that have 3 tenths, 0.3 and 0.31.

0.3 has no hundredths and 0.31 has 1 hundredth, so 0.31 is very slightly larger than 0.3.

Using the same reasoning $0.71 > 0.7$.

So the order is

0.07 0.13 0.3 0.31 0.7 0.71

The place value decreases by 10 times for each step to the right.

The place value increases by 10 times for each step to the left.

We use this idea to multiply and divide by 10 and other powers of 10.

0.32×1000

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
		0	.	3	2	
3	2	0	.			

Each digit moves 3 places to the left.

$32 \div 10$

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	3	2				
		3	.	2		

Each digit moves 1 place to the right.

Negative numbers

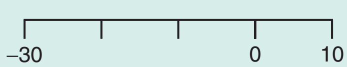
A number line shows how negative numbers work.

On a number line, negative numbers are to the left of zero and positive numbers are to the right.

Zero itself is neither positive nor negative.

Words such as 'big' and 'small' can be confusing when thinking about negative numbers.

Think about how the number –30 could be considered to be 'bigger' than 10 because it is further from zero on the number line.



It is better to use 'higher' and 'lower'. –30 is definitely lower than 10!

SKILLS CHECK

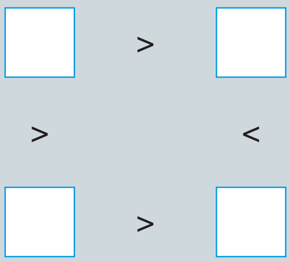
Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Using whole numbers

Copy the diagram. Write these numbers in the boxes so that all four inequalities are true.

940 409 490 904



Understanding decimals

What is the value of the red digit in each of these numbers?

a 35.71 **b** 215.67 **c** 1920.04 **d** 0.009416

Multiplying and dividing by powers of 10

Copy and complete this table.

× 1000					4 200 000
× 100	2300		46 100		
Number	23	3000		2750	
÷ 10	2.3				

Negative numbers

Write these numbers in order of size, from lowest value to highest value.

−1 0 −6 +1 +5 −3

Applying the knowledge

1. Four timekeepers at a race work out the difference in seconds between the first and second place competitors. Here are their results.

Timekeeper A	Timekeeper B	Timekeeper C	Timekeeper D
2 and 3 hundredths	23 tenths	2330 thousandths	2.003

The correct difference in time was 2.03 seconds.

a Which timekeeper worked out the time correctly?

b Which timekeeper worked out the time difference furthest from 2.03 seconds?

2. **a** Hannah is thinking of two numbers.
She says that when she puts a negative sign in front of each number, they stay in the same order on a number line.
Is Hannah correct? Explain your answer.

b Jeremy is thinking of two numbers.
He multiplies the two numbers together and adds 5 to the result.
His final answer is -3.
Suggest one possibility for the two numbers that Jeremy thought of.

2.1 Using the number system effectively

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

1. Work these out.

a $4.2 \div 0.1$ **b** $4.2 \div 0.01$ **c** $4.2 \div 0.001$

d $0.57 \div 0.1$ **e** $0.63 \div 0.01$ **f** $0.08 \div 0.001$

g $0.46 \div 0.01$ **h** $3.9 \div 0.1$

What you need to know

You can think of dividing as 'how many are there in…?'

How many 2s are there in 8?

$2 + 2 + 2 + 2 = 8$

There are four 2s in 8 so $8 \div 2 = 4$.

How many 0.1s are there in 1.3?

$0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 = 1.3$

There are thirteen 0.1s in 1.3 so $1.3 \div 0.1 = 13$.
This is the same as multiplying 1.3 by 10.

This place-value table shows 4.67 divided by 0.01.

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
		4	.	6	7
4	6	7	.		

It also shows that dividing by 0.01 has exactly the same effect as multiplying by 100.

In the same way

- multiplying by 0.1 is the same as dividing by 10
- multiplying by 0.01 is the same as dividing by 100
- multiplying by 0.001 is the same as dividing by 1000 and so on.

How to do it

Multiplying and dividing by 0.1 and 0.01

Work out the answers to these calculations.

a 32×0.1 **b** 320×0.01 **c** $32 \div 0.1$ **d** $32 \div 0.01$

Solution

a Using a place-value table to multiply by 0.1, think of 30 lots of 0.1 which makes 3, and 2 lots of 0.1 which makes 0.2.

H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
	3	2	.			
		3	.	2		

So 32 lots of $0.1 = 3.2$

Using the same idea for the other calculations:

b $320 \times 0.01 = 3.2$

c $32 \div 0.1 = 320$

d $32 \div 0.01 = 3200$

Learning exercise

1. Work these out.

   a **i** 2468×1000 **ii** 2468×100
   iii 2468×10 **iv** 2468×1
   v 2468×0.1 **vi** 2468×0.01
   vii 2468×0.001

   b Copy and complete these sentences. Use either 'less' or 'more' in the gaps.

   7000×10 gives an answer which is ____ than 7000.

   7000×0.1 gives an answer which is ____ than 7000.

2. Work these out.

   a **i** $6500 \div 1000$ **ii** $6500 \div 100$
   iii $6500 \div 10$ **iv** $6500 \div 1$
   v $6500 \div 0.1$ **vi** $6500 \div 0.01$
   vii $6500 \div 0.001$

   b Copy and complete these sentences. Use either 'less' or 'more' in the gaps.

   $8000 \div 10$ gives an answer which is ____ than 8000.

   $8000 \div 0.1$ gives an answer which is ____ than 8000.

3. Work these out.

   a 40×0.1 **b** 600×0.1
   c 9×0.1 **d** 8.4×0.1
   e $125 \div 0.1$ **f** $993 \div 0.1$
   g $6.2 \div 0.1$ **h** $5.17 \div 0.1$

4. Work these out.

   a 0.6×0.01 **b** 500×0.01
   c 8000×0.01 **d** 145×0.01
   e $246.9 \div 0.01$ **f** $61.3 \div 0.01$
   g $32 \div 0.01$ **h** $2000 \div 0.01$

5. Work these out.

   a 225.9×0.001 **b** 638×0.001
   c 8×0.001 **d** 0.4×0.001
   e $5.84 \div 0.001$ **f** $0.7 \div 0.001$
   g $24.9 \div 0.001$ **h** $0.0815 \div 0.001$

6. Work these out.

   a $6 \div 0.1$ **b** 13×0.1
   c $4.7 \div 0.001$ **d** $52.9 \div 0.01$
   e 0.8×0.1 **f** $7.65 \div 0.001$
   g $5 \div 0.01$ **h** 46×0.01

⑦ Work these out.

a 180 ÷ 0.01 **b** 2.3 × 0.01 **c** 6.91 ÷ 0.01

d 0.072 ÷ 0.01 **e** 50 × 0.01 **f** 3.2 ÷ 0.001

g 1.64 × 0.001 **h** 5.899 ÷ 0.0001

⑧ Beef costs £8.60 per kilo.
Copy and complete each sentence.

a 1 kg of beef costs 8.60 × 1 = £____

b 10 kg of beef costs 8.60 × ____ = £____

c 0.1 kg of beef costs 8.60 × ____ = £____

⑨ Here is Kevin's work on decimals.
Mark his work and correct any answers that are wrong.

1 86 × 0.1 = 8.6
2 20 × 0.1 = 200
3 2.5 ÷ 0.1 = 0.25
4 3 ÷ 0.1 = 30
5 18 × 0.01 = 0.18
6 1121 ÷ 0.01 = 11210
7 60 × 0.001 = 0.6
8 2.04 ÷ 0.1 = 2.004

⑩ Work out the missing numbers.

a 180 × ____ = 1800 **b** 65 × ____ = 6.5

c 900 × ____ = 9 **d** 2.1 × ____ = 210

e 6.7 × ____ = 0.67 **f** 0.9 × ____ = 0.009

⑪ Write down the missing numbers.

a 4500 ÷ ____ = 45 **b** 2 ÷ ____ = 20

c 7.8 ÷ ____ = 7800 **d** 68 ÷ ____ = 680

e 40 ÷ ____ = 0.4 **f** 0.06 ÷ ____ = 0.6

Do I know it now?

① Work these out.

a 8.2 × 0.1 **b** 130 × 0.1 **c** 4 × 0.01 **d** 8 × 0.001

e 63 × 0.001 **f** 0.9 × 0.01 **g** 2.01 × 0.1 **h** 0.7 × 0.001

② Work these out.

a 2.8 ÷ 0.1 **b** 30 ÷ 0.01 **c** 0.08 ÷ 0.001 **d** 0.2 ÷ 0.001

e 600 ÷ 0.1 **f** 0.004 ÷ 0.01 **g** 100 ÷ 0.0001 **h** 0.001 ÷ 0.0001

③ How many 1p coins make £100?

2.2 Understanding standard form

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the questions below.

① Write each set of numbers in order of size, starting with the smallest.

a 2.3×10^4 32 000 5.47×10^3 1.36×10^3 40 thousand

b 4×10^{-5} 3.7×10^{-4} 1.8×10^{-4} 0.000 65   0.000 03

② Which of the following numbers are in standard form? Convert those that are not into standard form.

a 23.4   **b** 23 400 000   **c** 0.0234   **d** 0.0234×10^4

e 0.0234×10^{-4} **f** 2.34×10^4 **g** 23.4×10^{-4} **h** 2.34

What you need to know

Standard form is used to write large numbers and small numbers.

For example, the speed of light is 2.998×10^8 metres per second and the mass of an electron is 9.110×10^{-31} kg.

A number in standard form is

(a number between 1 and 10) × (a power of 10).

So 3.9×10^{-4} is standard form, but 39×10^{-5} isn't.

When thinking about numbers in standard form, it can be helpful to have a place-value table in mind. The table below shows four million six hundred and seventy thousand (4 670 000), which is sometimes said as 4.67 million.

	M	HTh	TTh	Th	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$
							4	.	6	7
4.67×10^6	4	6	7	0	0	0	0	.		

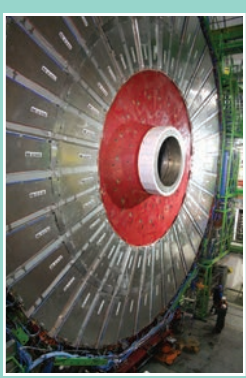
4.67 million is $4.67 \times 1\,000\,000$ or 4.67×10^6. The table shows you that the $\times 10^6$ moves all of the digits in 4.67 six places to the left; any blanks are filled with zeros.

In 6.8×10^{-3} all of the digits of 6.8 are moved three places to the right and the gaps are filled with zeros.

	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$	$\frac{1}{10\,000}$
			6	.	8			
6.8×10^{-3}				.	0	0	6	8

Did you know?

Scientists use standard form to write very large numbers such as the speed of light, and very small distances like the size of an atom.



How to do it

Converting small numbers to standard form

A flea weighs around 0.000 087 kg. Write this number in standard form.

Solution

For the number to be in standard form, it must start 8.7.

$0.000\,087 = 0.000\,87 \times 0.1$

$0.000\,087 = 0.0087 \times 0.01$

$0.000\,087 = 0.087 \times 0.001$

$0.000\,087 = 0.87 \times 0.0001$

$0.000\,087 = 8.7 \times 0.000\,01$

So
$0.000\,087 = 8.7 \times 0.000\,01 = 8.7 \times 10^{-5}$

Converting large numbers from standard form

Write 6.387×10^6 as an ordinary number.

Solution

M	HTh	TTh	Th	H	T	U	.	$\frac{1}{10}$	$\frac{1}{100}$	$\frac{1}{1000}$
						6	.	3	8	7
6	3	8	7	0	0	0	.			

Using a place-value table, the digits will move up six columns when multiplied by 10^6.
So $6.387 \times 10^6 = 6\,387\,000$

Answer the questions in the following exercises *without* the use of a calculator.

Learning exercise

1 **a** Write these numbers in standard form.

i 5120 **ii** 512 **iii** 51.2

iv 0.512 **v** 0.005 12 **vi** 0.000 512

b How would you write 5.12 in standard form?

2 These numbers are expressed in standard form. Write them as ordinary numbers.

a 5×10^2 **b** 8×10^4 **c** 2.6×10^3

d 1.9×10^5 **e** 8.17×10^3 **f** 9.05×10^4

g 7.4×10^7 **h** 1.004×10^4

③ Write these numbers in standard form.

a 600 b 70 000 c 8900
d 816 e 133 000 f 4 million
g 95 million h 4 billion

④ These numbers are written in standard form. Write them as ordinary numbers.

a 6.8×10^{-2} b 5×10^{-3} c 2.99×10^{-2}
d 7×10^{-4} e 1.04×10^{-1} f 8.6×10^{-5}
g 5×10^{-6} h 3.227×10^{-2}

⑤ Write these numbers in standard form.

a 0.69 b 0.052 c 0.0114
d 0.0007 e 0.0038 f 0.000 006
g 0.955 h 0.000 09

⑥ Which of these are not written in standard form?

5×10^4 1600 0.8×10^3 6.2×10^5 9×100^3 7.1×10^{-4}

⑦ Rewrite the quantities in these sentences using standard form.

a The total length of veins in the human body is 60 000 miles.
b On average a person's heart beats 108 000 times a day.
c The distance between the Sun and the Moon is about 150 000 000 km.
d A single coffee bean weighs about 0.003 kg.
e The mass of a grain of rice is 0.000 002 6 kg.

⑧ These numbers are in standard form. Write them as ordinary numbers and in words.

a 9×10^3 b 2.1×10^3 c 6.8×10^2
d 9.22×10^2 e 1.08×10^4 f 7×10^1
g 7×10^{-1} h 3×10^{-2}

⑨ Write these numbers in standard form.

a six thousand b seventy-four
c eight hundred and ten d two thousand and fifteen
e four tenths f three hundredths
g 0.000 002 24 h 5 108 000
i 67 800 000 j 23 million
k 6 billion l 0.000 000 007 001

⑩ Write these numbers in order of size, starting with the smallest.

7100 6.8×10^4 9×10^4 7.95×10^2 7.09×10^3

⑪ Write these numbers in order of size, starting with the biggest.

3.82×10^{-2} 0.04 2×10^{-3} 3.9×10^{-2} 2.2×10^{-3}

⑫ Copy and complete these statements. Use $<$, $>$ or $=$.

a 7×10^2 □ 750 **b** 6.2×10^{-3} □ 8×10^{-2}

c 5×10^3 □ 5000 **d** 0.009 □ 9×10^4

e 1.65×10^8 □ 2.4×10^7 **f** 8×10^7 □ 9 million

⑬ The table shows the closest distances of the Sun and seven planets from Earth.

Planet	Distance from Earth (in kilometres)
Jupiter	6.244×10^8
Mars	7.83×10^7
Mercury	9.17×10^7
Neptune	4.35×10^9
Saturn	1.25×10^9
Sun	1.496×10^8
Uranus	2.72×10^9
Venus	4.1×10^7

Finlay lists these planets in order of distance from Earth, starting with the planet that is nearest to Earth.

Finlay writes them in the correct order. Write down Finlay's list.

⑭ In a test, Daniel had to write down the value of the first digit of ten numbers written in standard form. The table shows Daniel's answers.

Question	Number	Value of first digit
1	6.1×10^4	**6000**
2	3.62×10^4	**3 units**
3	2.9×10^7	**20 million**
4	4.5×10^9	**4 billion**
5	1.236×10^9	**1 trillion**

Question	Number	Value of first digit
6	2×10^{-2}	**2 hundredths**
7	1.46×10^{-2}	**1 unit**
8	3×10^{-4}	$\frac{\mathbf{3}}{\mathbf{10\,000}}$
9	6.2×10^{-4}	$\frac{\mathbf{6}}{\mathbf{100\,000}}$
10	3.12×10^{-6}	**3 millionths**

How many correct answers did Daniel get?

Do I know it now?

① These numbers are expressed in standard form. Write them as ordinary numbers.

a 2.008×10^5 **b** 2.45×10^6 **c** 7.803×10^9 **d** 6.45×10^8

e 9×10^{-1} **f** 2.07×10^{-7} **g** 6.145×10^{-3} **h** 1.007×10^{-1}

② Write these numbers in standard form.

a 20 250 **b** 23 million **c** 654.7 **d** 25 624.87

e 3 tenths **f** 7 hundredths **g** 0.002 04 **h** 0.099

③ Write these quantities in standard form.

a The population of the world is approximately 7 billion.

b The diameter of a red blood cell is 0.008 mm.

CHAPTER 3

ESSENTIAL TOPICS – NUMBER

Accuracy

JUST IN CASE

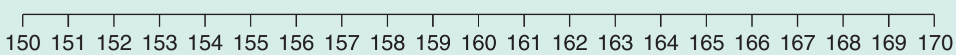
Rounding

Rounding means writing a number less accurately. It can help to make numbers more understandable and manageable.

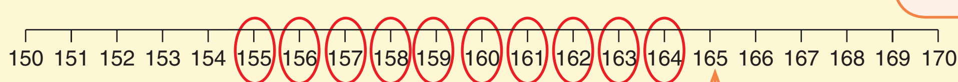
Rounding to the nearest 10 means you write down the nearest multiple of 10 to the number. Numbers that are the same distance from two multiples of 10, ending in 5, round up. This is so we all do the same thing; there is no mathematical reason for it.

Here is a number line.

Circle the whole numbers that round to 160 to the nearest 10.



Solution



Notice that 165 rounds to 170, not 160.

Rounding to one decimal place

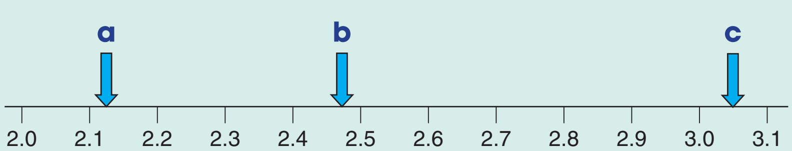
You can round to any number but usually you will be asked to round to a power of 10, such as 1000, 100, 10 or 1 whole, or to a number of decimal places.

Round each of these numbers on this number line to 1 decimal place.

a 2.127

b 2.4736

c 3.0491758



Solution

a rounds down to 2.1 (because of the 2).

b rounds up to 2.5 (because of the 7).

c rounds down to 3.0 (because of the 4).

Look at the second decimal place.

The digit after the place you are rounding to tells you whether it is nearer the lower number or the higher one. So you round down when the next digit is 0 to 4, and up when it is 5 to 9.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Rounding to the nearest 10 or 100

Copy and complete this table.

Number	Nearest 100	Nearest 10
371	400	
402		400
6399		
1045		

Rounding larger numbers

One of the following statements is incorrect. Which one? Explain why.

a 83 860, rounded to the nearest 1000, is 84 000.

b 8386, rounded to the nearest 100, is 8400.

c 8 386 500, rounded to the nearest million, is 8 000 000.

d 8 386 500, rounded to the nearest ten thousand, is 8 380 000.

Rounding decimals to the nearest integer

Round these numbers to the nearest whole number.

a 14.7 **b** 9.17 **c** 49.9 **d** 106.5 **e** 28.3

Rounding to a number of decimal places

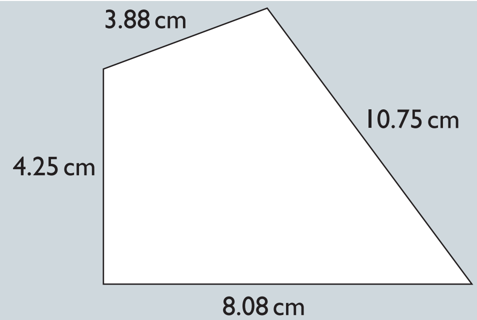
Write these numbers correct to the number of decimal places shown in brackets.

a 34.34 (1) **b** 14.45 (1) **c** 1.646 (1) **d** 1.646 (2)

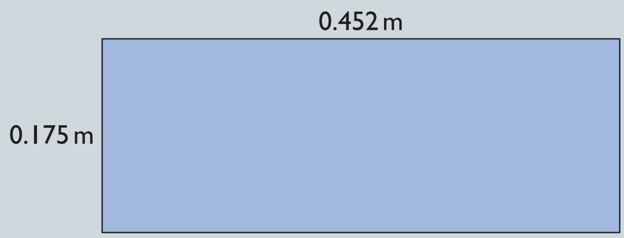
e 0.137 (2) **f** 0.95 (1) **g** 0.4962 (2) **h** 0.998 (2)

→ Applying the knowledge

① Salome wants to work out the perimeter of this quadrilateral.
She uses her calculator to add the lengths of the four sides together.
Her answer is 48.97 cm.
Salome's answer is wrong.
Without working out the exact answer, show why Salome's answer must be wrong.



② Mel needs to work out the area of this rectangle.
Mel's answer is 0.08 m^2 to 2 significant figures.
This answer is wrong. Explain why and give the correct answer to 2 significant figures.



3.1 Significance

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① Write these numbers correct to the number of significant figures shown in brackets.

a 26.45 (2)	**b** 89.149 (3)	**c** 56.257 (3)	**d** 1766 (3)
e 0.0276 (2)	**f** 7121.9 (1)	**g** 0.002 17 (1)	**h** 0.0305 (2)

What you need to know

The length of one year is 365.2422 days. This is 365 when rounded to the nearest whole number.

This is the same as saying it has been rounded to 3 significant figures (s.f.).

365.2422 is

- 400 to 1 significant figure
- 370 to 2 significant figures
- 365.2 to 4 significant figures
- 365.24 to 5 significant figures

The first non-zero digit is always the first significant figure.

After the first significant figure, all digits are significant.

How to do it

Rounding very small numbers

The answer 0.000 253 745 was given on a calculator. Write the number to 2 significant figures.

Solution

These three zeros are place holders.

2 is the first significant digit.

0.000 253 745

3 is smaller than 5 so the number rounds down.

The answer is 0.000 25 (to 2 significant figures).

Learning exercise

1. Here are some measurements. How many significant figures are in each number?

   a 36 cm **b** 1297 km **c** 0.9 kg
   d 0.053 m **e** 208 mm **f** 0.0251 m
   g 30.97 kg **h** 700.49 m

2. Round these numbers to 1 significant figure.

   a 29 **b** 45 **c** 361
   d 852 **e** 7422 **f** 21 652
   g 18.4 **h** 62.9 **i** 0.943
   j 0.652 **k** 0.0194 **l** 0.0248

3. Round these numbers to 2 significant figures.

a 873 **b** 924 **c** 615

d 708 **e** 704 **f** 3261

g 5119 **h** 18 642 **i** 73 281

j 8042 **k** 0.635 **l** 0.6041

4. Write the number 384 027 correct to

a 1 significant figure **b** 2 significant figures **c** 3 significant figures

d 4 significant figures **e** 5 significant figures.

5. Write the number 7.999 999 9 correct to

a 1 significant figure **b** 2 significant figures **c** 3 significant figures

d 4 significant figures **e** 5 significant figures.

6. Write the number 0.008 106 049 9 correct to

a 1 significant figure **b** 2 significant figures **c** 3 significant figures

d 4 significant figures **e** 5 significant figures.

7. Write these numbers correct to the number of significant figures (s.f.) shown in brackets.

a 17.65 (1 s.f.) **b** 0.597 (2 s.f.) **c** 71 046 (3 s.f.)

d 3.74 (1 s.f.) **e** 6.5092 (3 s.f.) **f** 26.9999 (4 s.f.)

8. Use a calculator to work these out. Give each answer to the degree of accuracy shown in brackets.

a $861 \div 45$ (1 s.f.) **b** 2.3^3 (1 s.f.)

c 7.89×6.45 (2 s.f.) **d** $11.6 \div 240$ (2 s.f.)

e 64.8^4 (3 s.f.) **f** 0.89×156.11 (1 s.f.)

g $\sqrt{89956}$ (2 s.f.) **h** $\sqrt[3]{1.0256}$ (3 s.f.)

i $\dfrac{4.3^2 \times 72}{\sqrt{3.864}}$ (3 s.f.)

9. Decide if each statement is true or false.

a 91.684 (to 2 s.f.) > 91.684 (to 1 s.f.)

b 0.3079 (to 3 s.f.) = 0.3079 (to 3 d.p.)

c 16.9949 (to 2 s.f.) ≠ 16.9949 (to 2 d.p.)

d 0.002 713 (to 2 s.f.) > 0.002 713 (to 1 s.f.)

10. Five friends round numbers in this way.
Ada rounds to 1 significant figure.
Ben rounds to 2 significant figures.
Cain rounds to the nearest integer.
Dave rounds to 1 decimal place.
Ella rounds to 2 decimal places.

a They each round the number 9.463. Whose answers are the same?

b They each round the number 59.698. Whose answers are the same?

c They each round the number 109.655. Whose answers are the same?

11. At an international football match at Wembley, the attendance was announced as 80 641.
Four people who were at the match were asked to round this figure to 2 significant figures.
Dan said 80 000.

Milly said 80 600.
Ami said 81 000.
Bob said, 'You are all wrong. 80 641 correct to 2 significant figures is 81.'
Who is right?
Explain your answer, indicating the mistakes that some of the people made.

⑫ The formula to find the circumference, C, of a circle of diameter, d, is $C = \pi d$.
The value of π is 3.141 592 654…
Keith wants to compare the circumference of a circle, diameter 8 cm, for different values of π.
He works out C using π correct to

a 1 significant figure **b** 2 significant figures

c 3 significant figures **d** 4 significant figures.

Work out Keith's results.

Do I know it now?

① Round these numbers to 1 significant figure.

a 0.994 **b** 0.009 74

c 993 **d** 999 943

② Round these numbers to 2 significant figures.

a 6.382 **b** 19.84

c 0.005 19 **d** 0.009 97

③ Use a calculator to work these out. Give each answer to the degree of accuracy shown in brackets.

a $\frac{2.35}{7.66}$ (4 s.f.) **b** $452 \times 60 \times 19$ (4 s.f.) **c** $\frac{2.3 \times 4.7}{9.22 - 3.7}$ (2 s.f.)

3.2 Approximating

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

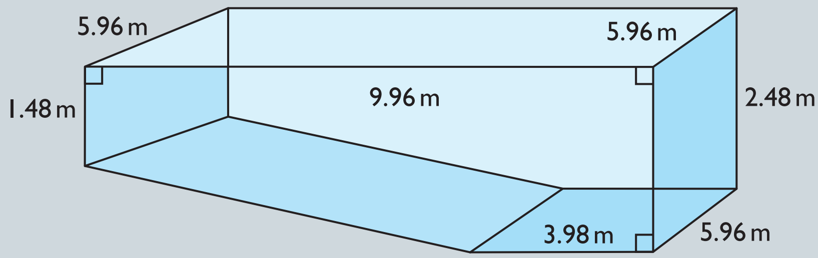
① You are told that $T = \frac{7.8^2 + 5.79^2}{17.67}$.

a Estimate an approximate value for T by rounding all the numbers to 1 significant figure.

b Use your calculator to find the true value of T correct to 3 significant figures.

If you can do the question overleaf, try this one on problem solving.

2. Gareth has just had a swimming pool built in his garden.
   The diagram shows the shape of the pool and its dimensions.
   Gareth fills the pool with water using a hosepipe.
   The hosepipe delivers water at a rate of 980 cm^3 per second.
   Gareth turns on the hosepipe at 10 a.m. on Monday morning.
   Estimate at what time the swimming pool will be full with water.



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 37 (Problem solving exercise 3.2 Approximating).

What you need to know

Approximating is about:

- **rounding numbers** for a calculation so that you can do it in your head
- making a **rough calculation** to anticipate what sort of answer to expect
- recognising when an **error** has been made
- giving a number to the level of **detail** that suits the context.

An approximate answer is often found by rounding numbers to 1 significant figure.

How to do it

➤ Rounding numbers to make a calculation easier

Mo and Sahar are buying a new radiator for their bedroom.
To choose the right radiator, they first need to know the volume of the room.
Mo measures the room in metres. It is 3.8 m long, 3.2 m wide and 2.7 m high.
Use rounded numbers to find the approximate volume of the room.

Solution

Volume $= 3.8 \times 3.2 \times 2.7$

Approximate values $4 \times 3 \times 3$

The approximate volume is $4 \times 3 \times 3 = 36$ m^3.

➤ Rounding to see when an error has been made

Pete's dog is 7 years 10 months old.

Pete says that this is equivalent to a human aged 70 because 1 dog year is like 7 human years.

Is he correct?

Solution

7 years 10 months is nearly 8 years.

$7 \times 8 = 56$ so Pete's dog would be less than 56, not 70.

You could also use an inverse argument to say that, to get a result of 70 when multiplying by 7, you would need to start with 10 (Pete's dog is not yet 10 years old).

Unless otherwise stated, answer the questions in the following exercises *without* the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

1. Estimate the answers to these calculations.

   a $68.79 + 21.96$ **b** $858.74 - 111.79$ **c** 30.8×45.3

   d $28.4 \div 1.99$ **e** 29.7^2 **f** $6371 + 4912$

2. For each of these calculations, there is a choice of answers. Use approximation to help you select the correct answer.

   a 6.4×8.8
   - **i** 15.2
   - **ii** 28.62
   - **iii** 56.32

   b 7.23^2
   - **i** 5227.29
   - **ii** 52.2729
   - **iii** 14.49

   c $836 \div 19$
   - **i** 15884
   - **ii** 855
   - **iii** 44

3. Quick checks by rounding the numbers will tell you that three of these answers are wrong. Which three?

   a $168 \times 94 = 1592$ **b** $18.6 \times 4.5 = 83.7$ **c** $\frac{56}{3.8} = 19.74$

   d $1200 \div 48 = 25$ **e** $8.9^2 = 79.21$ **f** $2.8^3 = 31.92$

4. A t-shirt costs £7.99 and a pair of shorts costs £11.49.
   Amber wants to buy 2 pairs of shorts and 5 t-shirts.
   She has £60.
   Make an estimate and decide if Amber has enough money.

5. Micah has done several calculations.
   He then does rough estimates to check his answers.
   Here are his results. Which calculations should he look at again?

   a calculation 12.64; estimate 80 **b** calculation 611; estimate 600

   c calculation 0.072; estimate 0.08 **d** calculation 19.32; estimate 6.5

   e calculation 341.8; estimate 360 **f** calculation 0.0156; estimate 0.002

6. Estimate the answer to each calculation.

   a $649 + 382$ **b** 7.15×13.06 **c** 62.4^2

   d $\frac{815 \times 6.4}{2.85}$ **e** $\frac{97 \times 94}{8.96}$ **f** $(2.1 + 9.4)^2$

⑦ For each calculation, use approximation to help you select the correct answer from the four possible answers given. Explain your choice.

a 514^2
- **i** 664 196
- **ii** 26 416
- **iii** 264 196
- **iv** 246 196

b 71.4×6.8
- **i** 4015.52
- **ii** 465.52
- **iii** 595.52
- **iv** 485.52

c $4.2^2 + 2.9^3$
- **i** 42.029
- **ii** 430.221 96
- **iii** 73.08
- **iv** 0.723

⑧ Alana saves £18.25 every week for a year.
Approximately how much does she save in a year?

⑨ Walt's mobile phone bill is £27.49 per month.
Approximately how much does he pay for his mobile phone in a year?

⑩ A journey of 104 miles, 528 yards took 2 hours and 3 minutes.
Estimate the average speed in miles per hour.

⑪ Amir's weekly wage is usually between £350 and £400.
4.8% of his wage is deducted for his pension.
Estimate how much is deducted over 2 years.

⑫ A typical adult sleeps about 7 hours every night.
Estimate how many months an adult sleeps in a year.

⑬ Here are some calculations.

Estimate the answers, then decide if the answer given is definitely wrong.

a $\dfrac{6149 \div 28}{3.8 \times 4.7}$ Answer 22.26

b $\dfrac{8.4^2 \times 9.75}{0.68 - 0.4}$ Answer 2457

c $\dfrac{5.4^2 \times 5.8^2}{61.5 \div 2.2}$ Answer 14.4

⑭ The diameter of the Sun is about 1 392 000 km.
The diameter of the Earth is about 12 700 km and the diameter of the Moon is about 3500 km.

a Estimate roughly how many times bigger the diameter of the Sun is than the diameter of the Earth. Give your answer to 1 significant figure.

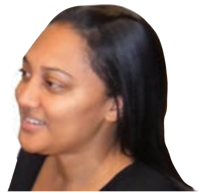
b Estimate how many times bigger the diameter of the Earth is than the diameter of the Moon, to 1 significant figure.

c Now estimate how many times bigger the diameter of the Sun is than the diameter of the Moon. Again, give your answer to 1 significant figure.

Problem solving exercise

① The table shows how long Aimee worked at different rates of pay last week.

	Hours worked	Rate of pay
Normal pay	25 hours 45 mins	£7.90 per hour
Overtime	5 hours 20 mins	£9.95 per hour
Sunday	2 hours 40 mins	£12.75 per hour



Aimee

This week, Aimee earned £305.

a Using approximations, show whether Aimee earned more or less last week than she earned this week.

Aimee is hoping to get a mortgage to buy a house.
She needs to earn more than £13 000 a year to get a mortgage for the house she wants.
In a year, Aimee works for 41 weeks and she earns a similar amount each week.
In the remaining weeks of the year, she is paid holiday pay at a rate of £152 per week.

b Estimate whether Aimee earns enough each year to be able to get a mortgage.

② Country Farm yogurt is sold in pots.
A machine fills the pots in batches of ten at the same time.
The machine can fill 9040 pots in one hour.

a Estimate the number of seconds it takes the machine to fill a batch of pots.

In one week, the machine fills pots for a total of 31 hours.
The pots of yogurt are then packed in cartons, each holding 96 pots.

b Estimate how many cartons are filled that week.

③ Andy is treating himself and five friends to a chip shop supper.
Andy orders the food that everyone wants.
He then realises that he only has a £20 note.
Andy quickly estimates to work out if he will have enough money.

Fryin - 2 - Nite

Chips .... 89p per portion
Peas .... 25p per tub
Fish .... £1.95 each
Pies .... £1.49 each
Sausages .... £1.10 per portion
Drinks .... 90p per can

Andy

a Does Andy have enough money?

His friends say that they each would like a drink. Andy does not want one himself.

b Will Andy have to borrow any money from his friends?

4 This is part of a timetable for trains from London to Stoke-on-Trent.

London Euston	07:20	08:00	08:20	09:00	09:20	10:00
Milton Keynes Central	07:50		08:50		09:50	
Stoke-on-Trent	08:48	09:25	09:48	10:25	10:48	11:15

This is part of a timetable for trains from Stoke-on-Trent to London.

Stoke-on-Trent	16:50	17:12	17:50	18:12	18:50	19:12
Milton Keynes Central	17:46		18:46		19:46	
London Euston	18:24	18:43	19:24	19:43	20:24	20:42

Lucy lives in London.
She has a meeting in Stoke-on-Trent at 11 a.m.
It takes Lucy about 15 minutes to walk from her home to London Euston train station.
The journey from the train station in Stoke-on-Trent to her meeting takes about 35 minutes.
The meeting is due to finish at 5.15 p.m.
Show and explain why Lucy is likely to be away from home for about 12 hours.

Do I know it now?

1 Estimate the answers to these calculations.

a $9728 - 9061$ **b** $814 \div 36.9$ **c** 7.84×194.3

2 For each of these calculations, there is a choice of answers. Use approximation to help you select the correct answer.

a $\dfrac{126 \times 68}{48}$

i 178.5

ii 17.85

iii 1785

b $9398.4 \div 26.4$

i 0.002 809

ii 9372

iii 35.6

iv 356

3 Estimate the answers to these calculations.

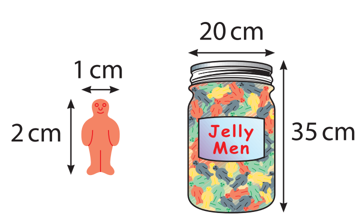
a $\dfrac{1684 - 324}{4.93}$ **b** $\dfrac{6.93 \times 55.4}{0.132}$ **c** $\dfrac{917 + 458}{0.41 - 0.23}$

4 Jake uses about 8.5 litres of petrol every day that he works.
Petrol costs 132.9 p per litre.
Approximately how much does Jake pay for petrol for a 4-day working week?

Can I apply it now?

1 Estimate the number of jelly men in the jar. Show all your working.

A jelly man is 1 cm thick.



CHAPTER 4

ESSENTIAL TOPICS – NUMBER

Fractions

JUST IN CASE

A fraction is a number that has a top (the numerator) and a bottom (the denominator).

For example, $\frac{3}{6}$

The top number tells you the number of parts you have.

The bottom number tells you the total number of equal parts.

To find a fraction of an amount you first divide the amount into a number of equal parts.
The bottom number of the fraction tells you how many parts.
Then you multiply by the number of parts you want.
The top number of the fraction tells you how many parts you want.

So, to find $\frac{2}{5}$ of 60

60				
12	12	12	12	12
12	12	12	12	12

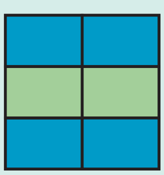
Divide 60 into 5 equal parts; each part represents $\frac{1}{5}$.

Two pieces will give you $\frac{2}{5}$.

So $\frac{2}{5}$ of $60 = 2 \times 12 = 24$

Equivalent fractions

Look at Jack's badge.



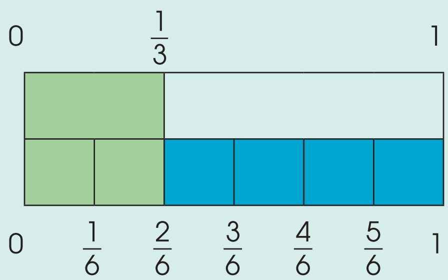
Two out of the six rectangles are green.

$\frac{2}{6}$ of the badge is green.

$\frac{2}{6}$ can be simplified or cancelled down or reduced to give $\frac{1}{3}$.

To simplify a fraction, you divide the top and bottom by the same number.

$$\frac{2}{6} \overset{\div 2}{=} \frac{1}{3}$$



Sometimes there is more than one way to get to the simplest form.

$$\frac{54}{60} \overset{\div 2}{=} \frac{27}{30} \overset{\div 3}{=} \frac{9}{10} \qquad \frac{54}{60} \overset{\div 6}{=} \frac{9}{10}$$

When you cannot cancel any further, the fraction is in its simplest form.

$\frac{54}{60}$, $\frac{27}{30}$ and $\frac{9}{10}$ are called **equivalent fractions**.

Equivalent fractions are found by dividing the top and bottom by the same number *or* by multiplying the top and bottom by the same number.

Multiplying fractions

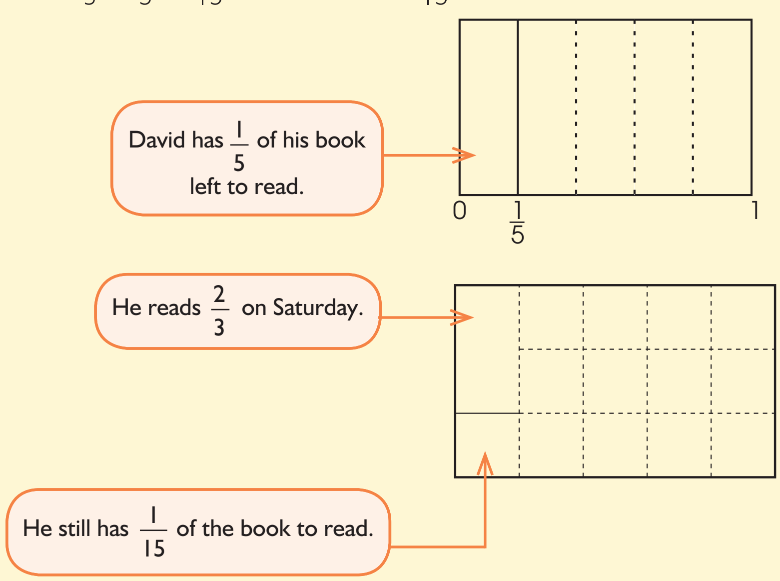
David has $\frac{1}{5}$ of his book left to read over the weekend.

He reads $\frac{2}{3}$ of it on Saturday.

What fraction of the book does he read on Saturday?

Solution

This is $\frac{2}{3} \times \frac{1}{5} = \frac{2}{15}$. So David reads $\frac{2}{15}$ on Saturday.



Adding and subtracting fractions

Jared has $\frac{7}{12}$ of his cake left.

He gives $\frac{1}{2}$ of the cake to his sister.

How much is left now?

Solution

Jared has $\frac{7}{12} - \frac{1}{2}$ left.

$$\frac{7}{12} - \frac{1}{2} = \frac{7}{12} - \frac{6}{12} = \frac{1}{12}$$

$\frac{1}{12}$ of the cake is left.

$$\frac{1}{2} \overset{\times 6}{=} \frac{6}{12}$$

$\frac{1}{2}$ and $\frac{6}{12}$ are equivalent fractions.

$$\frac{7}{12} - \frac{1}{2} = \frac{7}{12} - \frac{6}{12} = \frac{1}{12}$$

Once the denominators are the same, you can subtract the top numbers.

12 is the lowest common denominator of 12 and 2.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Understanding fractions

Work out each of these.

a $\frac{1}{3}$ of £24

b $\frac{3}{4}$ of 84 people

c $\frac{5}{8}$ of 16 ounces

d $\frac{1}{4}$ of £16

➜ Finding equivalent fractions

Write these fractions in their simplest form.

a $\frac{4}{8}$ **b** $\frac{12}{16}$ **c** $\frac{25}{30}$ **d** $\frac{12}{18}$

➜ Multiplying fractions

Work out each of these.

a $\frac{3}{4} \times \frac{5}{7}$ **b** $\frac{2}{5} \times \frac{4}{11}$ **c** $\frac{5}{9} \times \frac{4}{7}$

➜ Adding and subtracting fractions

Work out each of these.

a $\frac{3}{7} + \frac{4}{9}$ **b** $\frac{5}{6} - \frac{2}{3}$ **c** $\frac{1}{4} + \frac{1}{5} + \frac{1}{10}$ **d** $\frac{7}{9} - \frac{5}{18}$

➜ Applying the knowledge

① A manager of a small factory carries out a survey to find out how each employee gets to work. The manager finds:

- $\frac{1}{3}$ of the employees come by car
- $\frac{1}{4}$ of the employees come by bus
- the rest of the employees walk.

What fraction of the employees walk to work?

② Kate is baking flapjacks to sell at a fete.
She has 6 lb of oats, 5 lb of butter, 4 lb of sugar and 7 lb of golden syrup.
Each flapjack requires $\frac{4}{5}$ ounce of oats, $\frac{2}{3}$ ounce of butter, $\frac{1}{3}$ ounce sugar and $\frac{1}{2}$ ounce of golden syrup.
How many flapjacks can she make?
1 lb = 16 ounces

4.1 Working with mixed numbers

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Work out these. Give your answers as mixed numbers.

a $3 \times 2\frac{1}{2}$

b $2\frac{1}{5} - \frac{2}{3}$

c $1\frac{1}{8} + \frac{1}{2}$

d $1\frac{2}{3} \times 1\frac{1}{4}$

If you can do the question above, try this one on problem solving.

② Erica is preparing a party for 30 people. She estimates the amount of food each person will eat. How much of each type of food should Erica buy?

Pizza	$\frac{1}{4}$
Garlic bread	$\frac{1}{3}$ loaf
Lettuce	$\frac{1}{6}$
Tomato	$\frac{3}{4}$
Salad cream	$\frac{1}{12}$ bottle
Coleslaw	$\frac{1}{8}$ tub

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 47 (Problem solving exercise 4.1 Working with mixed numbers).

What you need to know

Did you know?

Measurements in recipes are often given in cups in the United States. This means that they use mixed numbers to describe how much of an ingredient is needed.

To find out if all the sand can be placed in one bucket:

$$\frac{1}{2} + \frac{2}{5} + \frac{1}{3} = \frac{15 + 12 + 10}{30} = \frac{37}{30}$$

$\frac{30}{30}$ is one whole.

$\frac{37}{30}$ is one whole and $\frac{7}{30}$. That is $1\frac{7}{30}$.

So the sand takes up one whole bucket and $\frac{7}{30}$ of a second one.

$\frac{37}{30}$ is called a **top-heavy fraction** or an **improper fraction**.

$1\frac{7}{30}$ is called a **mixed number**.

Every improper fraction can be written as a mixed number.

How to do it

Converting between improper fractions and mixed numbers

a Express $\frac{19}{5}$ as a mixed number.

b Express $2\frac{7}{8}$ as an improper fraction.

Solution

a Change $\frac{19}{5}$ to a mixed number by dividing 19 by 5.

$19 \div 5 = 3$ remainder 4

So $\frac{19}{5} = 3\frac{4}{5}$

b To find how many eighths there are in $2\frac{7}{8}$, multiply 2 by 8 and then add 7.

$2\frac{7}{8} = \frac{16}{8} + \frac{7}{8} = \frac{23}{8}$

So $2\frac{7}{8} = \frac{23}{8}$

Multiplying, adding and subtracting mixed numbers

Work out each of these.

a $2\frac{1}{3} \times 1\frac{4}{5}$ **b** $2\frac{1}{3} + 1\frac{4}{5}$ **c** $2\frac{1}{3} - 1\frac{4}{5}$

Solution

a Start by writing $2\frac{1}{3}$ and $1\frac{4}{5}$ as top-heavy fractions.

$2\frac{1}{3} = \frac{7}{3}, 1\frac{4}{5} = \frac{9}{5}$

So $2\frac{1}{3} \times 1\frac{4}{5} = \frac{7}{\cancel{3}_1} \times \frac{\cancel{9}^3}{5}$ Cancel using a common factor.

$= \frac{21}{5}$

$= 4\frac{1}{5}$

b Add the whole numbers and the fractions separately.

$2\frac{1}{3} + 1\frac{4}{5}$

$= (2 + 1) + \left(\frac{1}{3} + \frac{4}{5}\right)$

$= 3 + \left(\frac{5}{15} + \frac{12}{15}\right)$ 15 is the common denominator.

$= 3 + \frac{17}{15}$ Change this top-heavy fraction into a mixed number.

$= 3 + 1\frac{2}{15}$

$= 4\frac{2}{15}$

c Convert the top-heavy fractions to equivalent fractions with a common denominator.

$$2\frac{1}{3} - 1\frac{4}{5} = \frac{7}{3} - \frac{9}{5}$$

You can also use a method like that in part **b**.

$$\frac{7}{3} \underset{\times 5}{\overset{\times 5}{=}} \frac{35}{15} \qquad \frac{9}{5} \underset{\times 3}{\overset{\times 3}{=}} \frac{27}{15}$$

$$\frac{7}{3} - \frac{9}{5} = \frac{35}{15} - \frac{27}{15} = \frac{8}{15}$$

Answer the questions in the following exercises *without* the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

1. Change these mixed numbers to top-heavy fractions.

   a $1\frac{1}{3}$ **b** $2\frac{1}{4}$ **c** $3\frac{1}{2}$ **d** $1\frac{2}{5}$

   e $2\frac{3}{4}$ **f** $5\frac{1}{6}$ **g** $2\frac{2}{9}$ **h** $6\frac{6}{7}$

2. Change these top-heavy fractions to mixed numbers.

   a $\frac{5}{4}$ **b** $\frac{5}{2}$ **c** $\frac{10}{3}$ **d** $\frac{11}{4}$

   e $\frac{17}{5}$ **f** $\frac{23}{6}$ **g** $\frac{80}{9}$ **h** $\frac{51}{4}$

3. Work out these. Write each answer as a mixed number.

   a $2\frac{1}{4} + 3\frac{1}{2}$ **b** $1\frac{2}{3} + \frac{1}{6}$ **c** $2\frac{4}{5} + 3\frac{1}{5}$

   d $5\frac{3}{4} - 1\frac{1}{4}$ **e** $5\frac{1}{5} - 3\frac{1}{10}$ **f** $3\frac{1}{3} + 1\frac{1}{6}$

4. Work out these. Write each answer as a mixed number.

   a $1\frac{1}{2} + 2\frac{2}{3}$ **b** $1\frac{3}{4} + 2\frac{1}{2}$ **c** $2\frac{3}{5} + 1\frac{1}{3}$

   d $5\frac{3}{5} + 1\frac{7}{10}$ **e** $2\frac{5}{6} + 2\frac{3}{4}$ **f** $3\frac{7}{8} + 2\frac{4}{5}$

5. Work out these. Write each answer as a proper fraction or a mixed number.

   a $1\frac{1}{3} - \frac{7}{8}$ **b** $2\frac{1}{2} - \frac{7}{10}$ **c** $3\frac{1}{4} - 1\frac{2}{5}$

   d $4\frac{2}{3} - 1\frac{7}{8}$ **e** $5\frac{1}{4} - 2\frac{5}{6}$ **f** $3\frac{1}{10} - 1\frac{7}{9}$

6. Work out these. Write each answer as a mixed number.

   a $4\frac{1}{3} \times 2$ **b** $4 \times 2\frac{1}{5}$ **c** $2\frac{1}{4} \times 3$

   d $5 \times 1\frac{2}{7}$ **e** $6 \times 2\frac{1}{3}$ **f** $3\frac{2}{5} \times 9$

⑦ Work out these. Write each answer as a mixed number.

a $2\frac{1}{4} \times 3\frac{1}{2}$ **b** $1\frac{2}{3} \times 4\frac{1}{3}$ **c** $3\frac{1}{6} \times \frac{2}{3}$

d $\frac{5}{6} \times 12\frac{4}{5}$ **e** $1\frac{3}{8} \times 2\frac{1}{3}$ **f** $2\frac{3}{4} \times 4\frac{1}{2}$

⑧ Work out these.

a $2\frac{3}{4} \times 3\frac{2}{5}$ **b** $4\frac{1}{3} - 1\frac{5}{8}$ **c** $5\frac{2}{5} - 2\frac{9}{10}$

d $2\frac{1}{4} \times 1\frac{5}{6}$ **e** $2\frac{3}{4} \times 2\frac{1}{3}$ **f** $7\frac{1}{10} - 2\frac{7}{8}$

⑨ Work out these.

a $1\frac{1}{3} \times 2$ **b** $1\frac{1}{3} \times 3$ **c** $1\frac{1}{3} \times 4$

d $1\frac{1}{3} \times 5$ **e** $1\frac{1}{3} \times 6$ **f** $1\frac{1}{3} \times 7$

⑩ Bob runs $4\frac{1}{2}$ km on Monday, $5\frac{1}{4}$ km on Tuesday, $4\frac{7}{8}$ km on Wednesday.
His plan is to run 20 km.
How far does he need to run on Thursday to complete the 20 km?

⑪ Decide if these are true or false. Explain how you know.

a $\frac{1}{3}$ of $1\frac{1}{2}$ kg $\neq \frac{1}{2}$ kg

b $\frac{1}{2}$ km + $\frac{1}{3}$ km + $\frac{1}{4}$ km + $\frac{1}{5}$ km = $1\frac{7}{60}$ km

c 6 pints − $2\frac{1}{3}$ pints − $1\frac{3}{4}$ pints = $1\frac{5}{12}$ pints

d $3\frac{1}{2}$ m $\times 1\frac{1}{2}$ m = $5\frac{1}{4}$ m^2

⑫ Three friends are going on holiday.

Max's hand luggage is $6\frac{1}{3}$ kg, Ned's hand luggage is $5\frac{7}{8}$ kg and Oran's hand luggage is $3\frac{3}{4}$ kg.

a What is the total mass of their hand luggage?

b How much heavier is Max's hand luggage than Oran's hand luggage?

c On the return flight, Max's hand luggage is $\frac{7}{10}$ kg lighter, Ned's is $1\frac{3}{5}$ kg heavier and Oran's is twice as heavy.

What is the total mass of their hand luggage now?

⑬ Work out these. Write your answers as mixed numbers.

a $\left(1\frac{1}{2}\right)^2$ **b** $\left(2\frac{1}{2}\right)^2$ **c** $\left(\frac{5}{4}\right)^2$

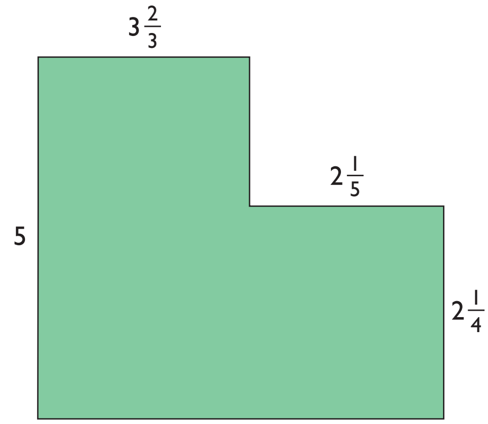
d $\left(1\frac{1}{2}\right)^3$ **e** $\left(2\frac{1}{5}\right)^3$ **f** $\left(1\frac{2}{3}\right)^4$

⑭ Copy and complete these.

a $1\frac{3}{4} \times 2\frac{2}{5} = 2\frac{4}{5} + \frac{\square}{\square}$ **b** $\frac{\square}{\square} - 1\frac{3}{4} = 3\frac{1}{3} \times 1\frac{7}{8}$ **c** $3\frac{1}{2} \times 1\frac{5}{6} = 7\frac{1}{2} - \frac{\square}{\square}$

⑮ The diagram gives some of the dimensions of a room in metres.

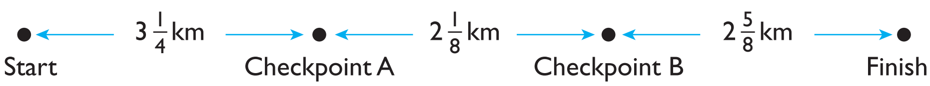
Calculate its perimeter.



Problem solving exercise

① Peter takes part in a charity fun run.

The diagram shows the distance between checkpoints on the run.

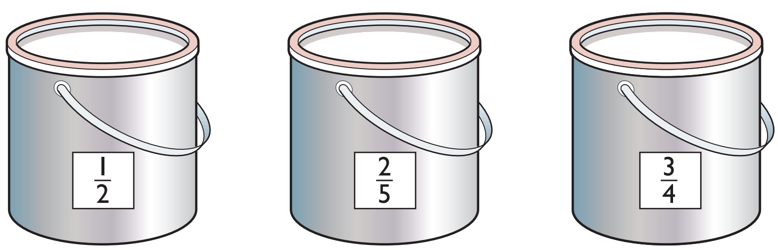


a How long is the race?

b How far is Checkpoint A from the finish?

c Peter is halfway between Checkpoints A and B. How far has he run?

② Dave has three tins of white paint.
Dave has used some of the paint from each tin.
There was 1 litre of paint in each tin.
Now one tin is half full of paint, one tin is $\frac{2}{5}$ full of paint and one tin is $\frac{3}{4}$ full.
Dave needs $1\frac{4}{5}$ litres of white paint.
Does Dave have enough paint?



③ Tom needs $3\frac{1}{2}$ cups of dried fruit for a cake recipe.

He only has $\frac{3}{4}$ cup of sultanas and $1\frac{1}{3}$ cups of raisins. He has plenty of currants.

a How many cups of currants does he need to use?

Tom, Linda and Les share the cake.

Les has $\frac{1}{9}$ of the cake.

Linda has $\frac{1}{10}$ of the cake.

Tom has $\frac{2}{15}$ of the cake.

b Tom says that there is still over half of the cake left. Is he right?

Do I know it now?

1. Change these mixed numbers to top-heavy fractions.

   a $4\frac{3}{5}$ **b** $6\frac{2}{3}$ **c** $9\frac{8}{9}$ **d** $12\frac{3}{4}$

2. Change these top-heavy fractions to mixed numbers.

   a $\frac{37}{8}$ **b** $\frac{41}{7}$ **c** $\frac{68}{5}$ **d** $\frac{137}{20}$

3. Work these out.

   a $2\frac{1}{4}+1\frac{3}{4}$ **b** $4\frac{1}{3}+\frac{2}{3}$ **c** $7\frac{5}{6}-2\frac{1}{2}$ **d** $5\frac{1}{2}-2\frac{3}{4}$

   e $5\frac{1}{4}-3\frac{1}{2}$ **f** $3\frac{1}{3}-1\frac{4}{5}$

4. Work these out.

   a $2\frac{1}{4}\times 2$ **b** $2\frac{1}{4}\times 2\frac{1}{4}$ **c** $2\frac{1}{4}\times 4\frac{1}{2}$ **d** $2\frac{1}{4}\times 5$

   e $4\frac{3}{4}\times 2\frac{2}{5}$ **f** $1\frac{3}{4}\times 2\frac{1}{2}$

Can I apply it now?

1. A bus arrives at a bus stop. It is already $\frac{3}{4}$ full. The number of people standing at the stop could fill $\frac{1}{3}$ of the bus. What fraction of a bus load are left at the bus stop?

4.2 Dividing fractions

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the questions below.

1. Work out these.

   a $7\div\frac{1}{5}$ **b** $\frac{6}{7}\div 3$ **c** $5\div\frac{2}{7}$

   d $\frac{3}{4}\div\frac{1}{5}$ **e** $3\frac{1}{3}\div 2\frac{1}{4}$

2. Write down the reciprocal of each number.

   a $\frac{1}{2}$ **b** $\frac{4}{5}$ **c** $1\frac{2}{3}$

3. A farmer has a field with an area of $7\frac{3}{4}$ acres. Each day he uses $1\frac{1}{3}$ acres to graze his cows. For how many days can he graze his cows in this field?

What you need to know

Reciprocals

$\frac{1}{4}$ is known as the **reciprocal** of 4.

The fraction $\frac{1}{3}$ is the reciprocal of 3.

You can write the whole number 3 as $\frac{3}{1}$.

You turn a fraction upside down to find its reciprocal.

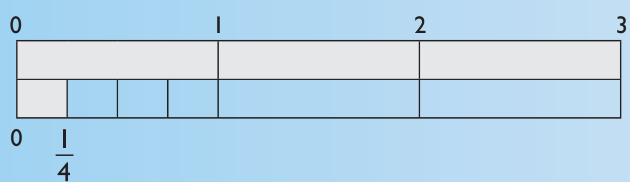
$\frac{5}{2}$ is the reciprocal of $\frac{2}{5}$. $\frac{2}{5}$ is the reciprocal of $\frac{5}{2}$.

Division problems

Each glass holds $\frac{1}{4}$ of a bottle of lemonade.

How many glasses can be filled from three bottles?

The problem can be solved by finding how many quarters in 3.



$3 \div \frac{1}{4} = 12$

It can also be solved as a multiplication.

So **dividing by a fraction is the same as multiplying by its reciprocal**.

$3 \times 4 = 12$

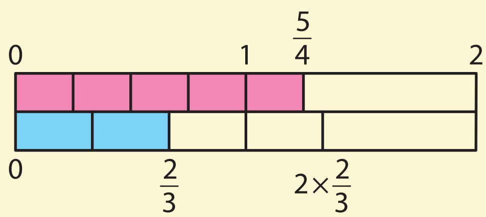
You should change mixed numbers into improper fractions before dividing.

How to do it

➤ Dividing a fraction by a fraction

Work out $\frac{5}{4} \div \frac{2}{3}$.

Solution



The bar diagram shows the answer will be just less than 2.

Find the answer by multiplying by the reciprocal.

The reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$.

$$\frac{5}{4} \div \frac{2}{3} = \frac{5}{4} \times \frac{3}{2}$$
$$= \frac{15}{8}$$

The questions in the following exercise should be answered *without* the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

1. Write down the reciprocal of each of these.

a $\frac{1}{7}$ **b** $\frac{5}{7}$ **c** 20 **d** $1\frac{1}{2}$

e $2\frac{5}{8}$ **f** $4\frac{5}{6}$ **g** $3\frac{7}{8}$ **h** $6\frac{2}{9}$

2. Rewrite each division as an equivalent multiplication, then work out the answer.

a $\frac{1}{5} \div 2$ **b** $\frac{1}{3} \div 4$ **c** $\frac{3}{4} \div 5$ **d** $\frac{3}{5} \div 3$

e $\frac{7}{9} \div 6$ **f** $\frac{9}{10} \div 4$ **g** $\frac{1}{20} \div 2$ **h** $\frac{1}{20} \div 5$

3. Rewrite each division as an equivalent multiplication, then work out the answer.

a $3 \div \frac{1}{2}$ **b** $2 \div \frac{1}{4}$ **c** $3 \div \frac{1}{5}$ **d** $2 \div \frac{2}{3}$

e $5 \div \frac{3}{5}$ **f** $6 \div \frac{3}{4}$ **g** $6 \div \frac{1}{12}$ **h** $12 \div \frac{1}{6}$

4. Rewrite each division as an equivalent multiplication, then work out the answer.

a $\frac{1}{3} \div \frac{3}{4}$ **b** $\frac{1}{6} \div \frac{2}{5}$ **c** $\frac{2}{9} \div \frac{1}{2}$ **d** $\frac{2}{7} \div \frac{3}{4}$

e $\frac{3}{5} \div \frac{9}{10}$ **f** $\frac{5}{8} \div \frac{2}{3}$ **g** $\frac{1}{6} \div \frac{2}{3}$ **h** $\frac{2}{3} \div \frac{1}{6}$

⑤ Work out these.

a $1\frac{1}{4} \div 3$ **b** $2\frac{1}{3} \div \frac{1}{2}$ **c** $3 \div 2\frac{1}{4}$ **d** $2\frac{4}{5} \div 1\frac{1}{2}$

e $2\frac{2}{7} \div 3\frac{1}{5}$ **f** $3\frac{1}{8} \div 1\frac{5}{8}$ **g** $4\frac{1}{3} \div 1\frac{1}{8}$ **h** $1\frac{1}{8} \div 4\frac{1}{5}$

⑥ $\frac{7}{8}$ of a litre of cola is shared between three friends.
What fraction of a litre of cola does each receive?

⑦ Daryl eats $\frac{2}{3}$ of a pizza.
The rest is divided between his two little sisters.
What fraction of the pizza does each sister eat?

⑧ Decide if these are true or false. Explain how you know.

a $\frac{1}{3} \div 4 = 4 \div \frac{1}{3}$ **b** $6 \div \frac{1}{2} = 3 \div \frac{1}{4}$ **c** $\frac{2}{5} \div 3 \neq \frac{3}{5} \div 2$

⑨ Copy and complete this multiplication grid.

$\times$		$\frac{2}{3}$
$\frac{1}{5}$	$\frac{1}{20}$	
		$\frac{5}{9}$

⑩ A room has area $30\,m^2$ and length $6\frac{1}{4}$ m.
Work out

a the width of the room

b the perimeter of the room.

⑪ Work out these.

a $\frac{1}{2} \div \frac{5}{8} + \frac{5}{8} + \frac{1}{6}$ **b** $\frac{7}{8} - \frac{5}{8} \div \frac{5}{6}$

c $\frac{9}{10} + \frac{3}{5} \div \frac{2}{7}$ **d** $\frac{4}{5} \div \frac{3}{4} + \frac{1}{3}$

⑫ It takes Jenny $\frac{3}{4}$ of an hour to drive the $26\frac{1}{4}$ miles to her friend Sandra's house.
What is her average speed?
(Use the formula speed = distance ÷ time.)

⑬ This was part of an article in a local newspaper.
When Liz read this article, she asked 'How many children did Martin Miller have?'
Work out the answer to Liz's question.

Local Millionaire Leaves Fortune

Martin Miller, local millionaire, left $\frac{1}{2}$ of his fortune of £$17\frac{1}{2}$ million to his wife of 32 years.

His children are to share the rest of the money, each receiving £$1\frac{3}{4}$ million.

Do I know it now?

① Write down the reciprocal of each of these.

a 8 **b** $\frac{3}{4}$ **c** $2\frac{1}{3}$ **d** $1\frac{4}{5}$

② Work out these.

a $\frac{7}{8} \div 2$ **b** $\frac{5}{6} \div 10$ **c** $4 \div \frac{1}{6}$ **d** $7 \div \frac{3}{8}$

③ Work out these.

a $\frac{3}{13} \div \frac{1}{3}$ **b** $\frac{2}{9} \div \frac{5}{6}$ **c** $\frac{5}{9} \div 1\frac{2}{3}$ **d** $6\frac{1}{4} \div 2\frac{2}{3}$

④ Meena has a pendant.
She wants to know what it is made of and so she tries to measure its density.
She finds its mass is $34\frac{1}{5}$ grams and its volume is $1\frac{4}{5}$ cubic centimetres.
Work out the density of the material. Use the formula

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

CHAPTER 5

ESSENTIAL TOPICS – NUMBER

Percentages

JUST IN CASE

Percentage means 'out of a hundred' or 'parts per hundred'.
You can think of percentages as hundredths (fractions).
They are also decimals because they are hundredths, just written a little differently.

Calculating percentages of quantities

Find 54% of 450.

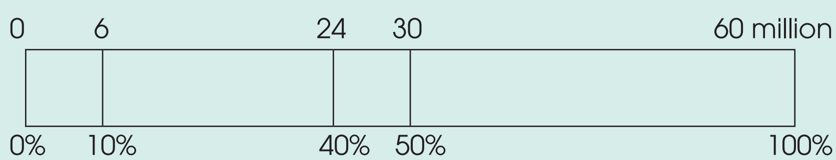
100% is 450

So 1% is $\frac{450}{100} = 4.5$

Remember that 'of' means $\times$.

54% is $54 \times 4.5 = 243$

Calculate 40% of £60 million.



100% is 60 million

10% is 6 million

40% is $4 \times 6 = 24$ million, so £24 million.

Alternatively you could stay with fractions and do this as $\frac{40}{100} \times \frac{60}{1} = 24$

Or

40% of £60 million

$\frac{40}{100} \times 6$ million

0.4×60 million $= 24$ million

Converting fractions and decimals to and from percentages

Convert $\frac{1}{8}$ to a percentage.

$$\frac{1}{8} \overset{\times 50}{=} \frac{50}{400} \overset{\div 4}{=} \frac{12.5}{100} = 12.5\%$$

Convert 0.125 (decimal equivalent of $\frac{1}{8}$) to a percentage.

$0.125 \times \frac{100}{100} = \frac{12.5}{100} = 12.5\%$

Convert 12.5% to a fraction.

$12.5\% = \frac{12.5}{100}$ ← Multiply by 2 to remove the decimal.

$= \frac{25}{100}$ ← Simplify by dividing by 5.

$= \frac{5}{40}$ ← Simplify by dividing by 5.

$= \frac{1}{8}$ ← Using place value.

Convert 12.5% to a decimal.

$12.5\% = \frac{12.5}{100} = 0.125$ ← Using place value.

SKILLS CHECK

→ Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

→ Understanding and using percentages

Mr Green decided that he should recycle his rubbish if possible. In one week his rubbish weighed 25 kg. He was able to recycle 16 kg of it.

a What fraction did he recycle?

b What percentage is this?

Calculating percentages of quantities

Work out each of these.

a 20% of £300

b 3% of £5000

c 10% of 220 km

d 13% of 250 kg

Converting fractions and decimals to and from percentages

Copy and complete this table.

Fraction	Decimal	Percentage
$\frac{1}{2}$		
	0.15	
		75%
$\frac{1}{10}$		

Applying the knowledge

1 Andy has just been promoted.
His new annual salary will be £38 500.
Andy wants to know how much money he is actually going to get each month.
From his salary there will be deductions for income tax, National Insurance and his pension.
7% of his salary will be taken for National Insurance.
His pension contribution will be 5% of his salary.
Andy estimates that 18% of his salary will be taken for income tax.
Work out how much money Andy will get each month.

2 Harry is taking an accountancy course. If he passes he will get a promotion. In his first assignment, Harry gets 36 out of 50. In the next one he gets 56 out of 80.

a Which is his better score?

b The last assignment is out of 80. Across all three assignments, he must have scored an average of 70% to pass. What mark will he need on the last assignment?

5.1 Applying percentage increases and decreases to amounts

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① The government awards a 4% pay increase to all public sector workers.

Work out the new salary for

a Beatrice, who earned £15 000

b Brett, who earned £24 000

c Bert, who earned £17 000.

If you can do the question above, try this one on problem solving.

② Bill is a sheep farmer. He has 350 female sheep on his farm. This year, each female sheep produced, on average, 2 lambs. Bill sold 65% of these lambs for £58 each. The selling price of lambs then increased by 3%. Bill sold the rest of his lambs at this increased price. Work out the total amount Bill received for the lambs that he sold.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 60 (Problem solving exercise 5.1 Applying percentage increases and decreases to amounts).

What you need to know

Did you know?

Online and high street shops use percentage decreases to calculate sale prices.

There are several ways to work out percentage increases and decreases. Look at the example below.

Work out the amount when £120 is

a increased by 15%
b decreased by 15%.

Finding the increase or decrease

100% is £120

1% is $\frac{1}{100} \times £120$

15% is $\frac{15}{100} \times £120 = £18$

a An increase of 15% gives £120 + £18 = £138
b A decrease of 15% gives £120 − £18 = £102

Using a ratio table

15% = 10% + 5%

£120 − £18 = £102

%	100	10	5	15	115	85
Amount	120	12	6	18	138	102

b A decrease of 15% gives £102.

£120 + £18 = £138

a An increase of 15% gives £138.

This is the amount of the increase or decrease. In this case it is £18.

Using a percentage bar chart

15% is £18

85% is 100% − 15% so £120 − £18 = £102

115% is 100% + 15% so £120 + £18 = £138

The chart shows that:

a An increase of 15% gives an amount of £138.
b A decrease of 15% gives an amount of £102.

Using a multiplier

a The original amount is 100% so it is 100 + 15 = 115% when 15% is added.
So the new amount is 115% of £120 $= \frac{115}{100} \times £120$
$= £138$

You can also write this as $1.15 \times £120$.

b In the same way, the amount is 100 − 15 = 85% when 15% is subtracted.

$$\frac{85}{100} \times £120 = 0.85 \times £120$$
$$= £102$$

How to do it

Percentage increase

Anneka earns £21 000 per year. She is given a 3% increase.
Calculate how much she now earns in a year. Use two different methods.

Solution

Using a ratio table

100%	1%	3%	103%
£21 000	£210	£630	£21 630

103% of £21 000 = £21 630

Finding the increase or decrease

100% is £21 000

1% is $\frac{1}{100} \times £21\,000 = £210$

3% is $\frac{3}{100} \times £21\,000 = £630$

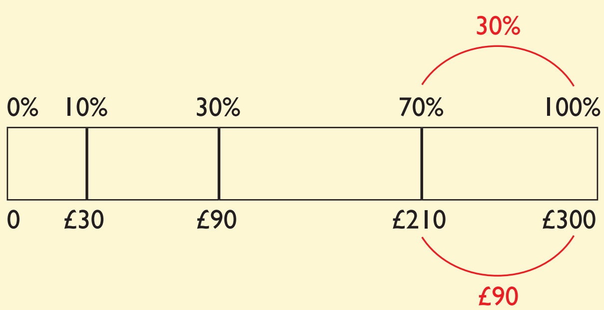
An increase of 3% gives £21 000 + £630 = £21 630

Percentage decrease

William paid £300 for a new bicycle three years ago. It has lost 30% of its value.
What is its value now? Use two different methods.

Solution

Using a percentage bar chart



70% of £300 = £210

Using a multiplier

The new amount is 100% − 30% = 70%

This is $\frac{70}{100} \times £300 = 0.7 \times £300 = £210$

Learning exercise

Answer questions 1 to 12 *without* using your calculator.

1. **a** What is 10% of £70?
   b Increase £70 by 10%. **c** Decrease £70 by 10%.
2. In a sale there is 10% off.
   Find the sale price of each item.
   a jeans £20 **b** shirt £18 **c** tie £6.50
3. The price of each of these items goes up by 30%.
   What are the new prices?
   a coat £40 **b** jumper £32 **c** socks £9.20
4. In a sale there is a reduction of 25%.
   For each item, find the reduction in price and the new price.
   a chair £68 **b** table £244 **c** lamp £19
5. Work out these.
   a Increase 30 by 10%. **b** Decrease 35 by 20%.
   c Decrease 32 by 25%. **d** Increase 80 by 50%.
   e Decrease 60 by 30%. **f** Decrease 170 by 40%.
   g Increase 15 by 100%. **h** Increase 10 by 200%.
   i Increase 200 by 10%.
6. All prices are reduced by 30% in a sale.
   Samantha buys shoes priced at £28, a coat priced at £43 and a shirt priced at £17.50.
   What is her total bill if she buys these items during the sale?
7. Boxes of confectionary are labelled '25% extra free'.
   Work out the new number of contents when the original amount was
   a 48 bars **b** 60 lollipops **c** 160 chews.
8. Steven buys a car for £7200.
   He sells it and makes 15% profit.
   What price does he sell the car for?
9. Sid buys a house for £185 000.
   After one year its value has risen by 3%.
   What is the value of the house after one year?
10. Sally earns £9.50 per hour.
    She gets a pay rise of 6% starting in June.
    She works 32 hours in the first week of June.
    What is her pay for that week?
11. A carton of cream normally contains 230 ml.
    The label says 20% extra free.
    Sandy buys five cartons.
    How many litres of cream does she buy?

⑫ Shaun is at a DIY store.
He buys a drill, a wheelbarrow and a hammer.
What is Shaun's final bill when the VAT is included? (VAT is 20%.)


£38.00 plus VAT


£40.00 plus VAT


£18.00 plus VAT

⑬ Top Rates Bank offers simple interest at a rate of 1.6% per annum.
Work out the total value when each amount is invested for these times.

a £700 for 2 years

b £1340 for 3 years

c £190 for 6 months

d £825 for 9 months

⑭ Sandra invests £410 for 3 years at 2.1% per annum simple interest.
Simon invests £520 for $2\frac{1}{2}$ years at 1.9% per annum simple interest.
Who gains more interest and by how much?

Problem solving exercise

① Natasha joins a small company. Her starting salary is £16 000 a year.

a In her first year, the company does very well. Everyone is given a 25% pay rise for the next year.
What is Natasha's new salary?

b The next year the company does not do at all well. Everyone's salary is reduced by 25% for the following year.
What is Natasha's salary now?

c Natasha says, 'That's not fair! + 25 − 25 should be zero.'
Comment on this statement.

② Cars depreciate in value as they get older.
Mansoor is buying a car.

He finds three cars that he likes. He wants to buy a car that has depreciated by the least amount of money after one year.

	Original cost	Rate of depreciation after one year
Car A	£17 500	9%
Car B	£23 000	7.5%
Car C	£15 650	10%

a Which car should Mansoor buy?

b What is the value of this car after one year?

③ Caroline wants to go on holiday.
She is going to take out a loan of £1500 to help pay for the holiday.
Caroline will have to pay back the £1500 plus 20% interest over 12 months.
She will pay back the same amount of money each month.
How much money will she need to pay back each month?

Do I know it now?

1. **a** What is 20% of £80? **b** Increase £80 by 20%. **c** Decrease £80 by 20%.

2. In a sale there is a reduction of 20%.
   Work out the new price for each item.
   a shirt £30 **b** coat £99 **c** rugby shirt £55.50

3. Work out these.
   a Increase 120 grams by 5%. **b** Increase 160 metres by 35%. **c** Decrease 150 calories by 3%.

4. Sonya's car cost £9600.
   After one year its value has depreciated by 12%.
   By how much has the value of her car fallen?

Can I apply it now?

1. When water is frozen, its volume increases by 4%.
   a 5 litres of water is frozen. How many cubic centimetres of ice will be made?
   b A machine makes identical ice cubes, each with a volume of $10\,cm^3$. 2.5 litres of water are used to make these ice cubes. How many ice cubes are made?
   1 litre $= 1000\,cm^3$

5.2 Finding the percentage change from one amount to another

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

1. **a** A house was bought for £160 000 and sold for £210 000. Calculate the percentage profit.
   b A flat was bought for £90 000 and sold for £83 000. Calculate the percentage loss.

If you can do the question overleaf, try this one on problem solving.

② **a** A colony of bees is 400 strong in April. By the end of June, the number has increased to 528. What is the percentage increase?

b The colony is then struck by a disease and goes down to 132. What is the percentage decrease?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 64 (Problem solving exercise 5.2 Finding the percentage change from one amount to another).

What you need to know

Did you know?

The cost of the same basket of shopping is worked out each month to see how much more it costs now than it did a year ago. The increase tells you the **inflation rate**.

To calculate a **percentage change**, follow the method shown in this example.

The population of an island increases from 500 to 650 people. What is the percentage increase?

The increase is $650 - 500 = 150$ people

As a fraction this is $\frac{150}{500}$ ← the increase / the original population

Either find the equivalent fraction with 100 on the bottom line…

$\frac{150}{500} = \frac{30}{100} = 30\%$

or

write the fraction as a decimal…

$\frac{150}{500} = 0.3$

… then multiply by $\frac{100}{100}$ to make it a percentage.

$0.3 \times \frac{100}{100} = \frac{30}{100} = 30\%$

You may find a percentage bar or a ratio table useful to illustrate this.

How to do it

Percentage increase

Maya deals in musical instruments. She buys a guitar for £75 and restores it. She sells it for £120. Work out her percentage profit.

Solution

Profit $= £120 - £75$ ← Profit = sale price − cost price

$= £45$

As a fraction this is $\frac{45}{75}$ ← Remember to always write the original value on the bottom line.

So Maya's profit is 60%.

➤ Percentage decrease

Zorro was an overweight dog. He was 14.0 kg.
His owner put him on a programme of long daily walks. After six months his weight is 9.8 kg.
What percentage of his weight has Zorro lost?

Solution

Weight lost $= 14.0 - 9.8 = 4.2$ kg

As a fraction this is $\frac{4.2}{14.0}$

Percentage decrease $= \frac{4.2}{14.0} \times \frac{100}{100}$ ← Find an equivalent fraction with 100 on the bottom line.

$= \frac{420}{1400} = \frac{60}{200} = \frac{30}{100}$ ← This is the equivalent fraction.

The fraction lost is $\frac{30}{100}$

So Zorro has lost 30% of his original body weight.

Learning exercise

① Write the first quantity as a percentage of the second quantity.

a £7, £100 **b** €9, €10 **c** 5 cm, 25 cm
d 12 g, 300 g **e** 30p, £2 **f** 14 cm, 1 m
g 90 cm, 3 m **h** 85p, £5 **i** 800 m, 5 km

② Work out the profit and percentage profit for each of these items in a DIY store.
The first one has been done for you.

	Item	Cost price	Selling price	Profit	Percentage profit
	Drill	£30	£33	£3	10%
a	Saw	£12	£21		
b	Hammer	£10	£17		
c	Plane	£20	£32		
d	Spanner set	£35	£56		

③ Work out the loss and percentage loss for each of these items at a car boot sale.

	Item	Cost price	Selling price	Loss	Percentage loss
a	Book	£10	£2		
b	Saucepan	£25	£22		
c	Dinner set	£200	£184		
d	Armchair	£70	£63		
e	Bicycle	£120	£84		
f	Cushion	£2	96p		

④ A full carton of juice used to contain 750 ml.
It now contains 810 ml.

a What is the increase in millilitres?

b What is the percentage increase?

⑤ A packet of biscuits used to contain 30 biscuits.
It now contains 24 biscuits.

a What is the decrease in the number of biscuits?

b What is the percentage decrease?

⑥ A chair cost £60 and is sold for £75.
Work out the percentage profit.

⑦ A TV cost £250 and is sold for £240.
Work out the percentage loss.

⑧ Nick's rent last year was £400 per month.
This year it is £460 per month.
Work out the percentage increase in his rent.

⑨ Ned's mass fell from 90.5 kg to 81.8 kg.
Work out the percentage decrease in his mass.

⑩ A tree grew in height from 75 cm to 1.2 m.
Work out the percentage increase in height.

⑪ The mass of Sam's prize calf increased from 40 kg to 43 kg.
The mass of Sally's prize calf increased from 46 kg to 49 kg.
For each calf, calculate the percentage increase in mass. Which calf had the greater percentage increase?

⑫ The value of Mary's house fell from £160 000 to £147 000.
The value of Martin's house fell from £185 000 to £172 000.
Calculate the percentage fall in the value of both houses. Whose house had the greater percentage fall in value?

⑬ Mr Tell bought 10 DVDs for £6 each.
He sold 8 of them for £10 each and 2 of them for £5 each.
Work out his percentage profit.

Problem solving exercise

① Last year, Stan and Henry were looking at ways to reduce their heating bills.
Stan had wall insulation installed in his house.
Henry had loft insulation installed in his house.
For this year, Stan's heating bill is reduced from £640.50 to £544.43.
Henry's heating bill is reduced from £650 to £520.
Compare the percentage saving of wall insulation and loft insulation.

② Nigel's rent increased from £200 to £220 per week.
At the same time, Nigel's wages increased from £450 to £490 per week.
Work out the percentage increase in the amount of money Nigel has left to spend each week.

Do I know it now?

① Work out these percentages.

a 9 as a percentage of 90

b 75 as a percentage of 125

c 54 as a percentage of 72

d 20.8 as a percentage of 26

② Write the first quantity as a percentage of the second quantity.

a 8 mm, 3.2 cm **b** 240 m, 1 km **c** 8.4, 20 **d** 84 ml, 0.75 litres

③ Work out the profit and percentage profit for each item.

	Item	Cost price	Selling price	Profit	Percentage profit
a	Dress	£80	£84		
b	Pencil	80p	£1.08		
c	Dressing gown	£120	£66		
d	Notebook	£2	88p		

④ Sean bought three cars.
Car A cost £5000, car B cost £6100 and car C cost £6300.
He sold the three cars for £5100, £5800 and £6000, respectively.

a Work out his total percentage loss to the nearest whole number.

b On which car did he make the greatest percentage loss?

Can I apply it now?

① The number of members of a sports club increased as given in the table.

Year	2009	2010	2011	2012
Number	581	620	641	672

In which year was the percentage increase in the number of members the greatest?

5.3 Reverse percentages

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① The prices below include VAT at 20%. Work out the price of each item before VAT was added.

a laptop £336 **b** DVD £19.20 **c** printer £132 **d** CD £13.92

If you can do the question above, try this one on problem solving.

② In 2012 the population of a country was 36 million. The population is decreasing at an annual rate of 2.5%. Write your answers to these questions to the nearest thousand.

a What was the population

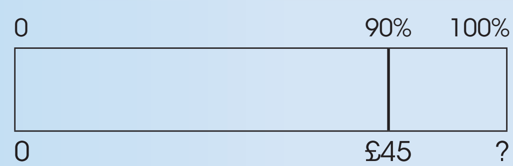
i in 2011 **ii** in 2010 **iii** in 2015?

b When will the population fall below 30 million?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 69 (Problem solving exercise 5.3 Reverse percentages).

What you need to know

Sometimes you know the value of something and the percentage change but not the original value. Working out the original value involves **reverse percentages**, as in this example.



A coat costs £45 in a sale. It has been reduced by 10%. What was its original cost?

This percentage bar shows what you know and what you need to find out.

Using proportionality,

$$\frac{100}{90} \times 45 = 50$$

The original cost was £50.

Alternatively,

90% is £45

1% is $\frac{45}{90} = 0.5$

100% is $0.5 \times 100 = £50$

The original cost was £50.

How to do it

When an amount is reduced

The sale price of a pair of designer sunglasses is £96. The reduction in the sale is 20%.

a What percentage of the original price is the sale price?

b Work out the original price.

Solution

a The sale price is 80% of the original price. 100% − 20% = 80%

b 80% is £96

1% is $\frac{96}{80}$ = £1.20

100% is £1.20 × 100 = £120

So the original price was £120.

When an amount is increased

A shop sells boots for £56 a pair. The shop makes a profit of 40%.
What price did the shop pay for the boots?

Solution

Cost price = 100%, profit = 40%, so the selling price = 140%.

140% is £56

1% is $\frac{56}{140}$ = £0.40

100% is 0.4 × 100 = £40

So the shop paid £40 for the boots.

Learning exercise

1. A shirt costs £48 which includes VAT at 20%. This means 120% = £48.
   Work out the cost of the shirt without VAT.
2. A tie costs £12 following a reduction of 20%. This means 80% = £12.
   Work out the cost of the tie before the reduction.
3. A lamp costs £27 following a reduction of 10%.
   Work out the cost of the lamp before the reduction.
4. A table costs £138 which includes VAT at 20%.

   a Work out the cost of the table without VAT.

   b How much VAT was paid?

5 Work out the original length for each of these.

a ☐ cm was increased by 30% to give 91 cm.

b ☐ m was reduced by 25% to give 96 m.

c ☐ km was reduced by 14% to give 387 km.

d ☐ m was increased by 4% to give 5.2 m.

e ☐ km was reduced by 38% to give 527 km.

6 **a** Increase £200 by 15%.

b An amount was increased by 15% to give £200. What was the original amount?

c Increase your answer to part **b** by 30%.
Is your answer the same as in part **a**?
Explain your answer.

7 The table shows the sale price and the percentage discount for some items. Work out the original price of each item.

	Item	Sale price	Discount	Original price
a	Necklace	£63	10%	
b	Watch	£102	15%	
c	Bracelet	£57	5%	
d	Earrings	£12	40%	

8 The table shows the costs of some household bills. Each bill includes VAT at 20%. Work out the cost of each bill without VAT.

	Bill	Cost with VAT	Cost without VAT
a	Telephone	£96	
b	Satellite TV	£21.60	
c	Insurance	£210	
d	Carpet	£1500	

9 The amount of juice in a standard carton was reduced by 12%. The carton now contains 792 ml. How much did it contain before the reduction?

10 A nurse worked 52 hours this week. This was an increase of 30% from last week. How many hours did he work last week?

11 Joseph negotiated a reduction of 15% on the cost of his new car. He paid £7990. What was the original cost?

12 House prices have risen by 3% this year compared with last year. A house is valued at £185 400 this year. What was its value last year?

13 A jeweller buys a watch and makes 55% profit when he sells it for £124. How much profit did he make?

14 Ned travelled 60% more miles in September than he did in August. He travelled 2240 miles in September. What was his total mileage for August and September?

15 Naomi bought a computer for £432, a TV for £270 and a mobile phone for £138. All three items included VAT at 20%. How much VAT did she pay in total?

Problem solving exercise

1. Ken shops at CashLimited wholesale warehouse where all prices are displayed without VAT. Ken buys a box of printing paper, 4 ink cartridges and a pack of folders.

   A box of printing paper is priced at £11.00, a pack of folders is priced at £9.00 and there is no price on the ink cartridges.

   Including VAT, at 20%, Ken pays £84.
   Work out the price of an ink cartridge.

2. Morgan and Rowan buy identical cars. Morgan buys his car from Car Market Sales and Rowan buys his car from Jeff's Autos.

   Car Market Sales offers Morgan a 12% reduction on the showroom price for the car. Morgan accepts the offer and buys the car. At Jeff's Autos, Rowan buys the identical car for £6250.

   Morgan pays £86 more than Rowan.
   What was the showroom price of the car at Car Market Sales?

Do I know it now?

1. An electricity bill is £126, which includes VAT at 5%.
   - **a** Work out the cost of the bill before VAT was added.
   - **b** How much VAT was paid?
2. Thabo's parents measure his height every birthday. His height is 147 cm on his 14th birthday. His father says 'You grew 5% last year.'
   - **a** How tall was Thabo on his 13th birthday?
   - **b** How much did he grow when he was 13 years old?
3. This year there are 204 registered players in a cricket league. This is a reduction of 15% compared with last year.
   - **a** How many registered players were there last year?
   - **b** What is the reduction in the number of registered players?

Can I apply it now?

1. There is 10% off coats and 25% off shirts in T.C. Clothing. Neil paid £54 for a coat and £48 for a shirt when the offers were on.

   How much did he save in total?

5.4 Repeated percentage increase/decrease

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① £350 is invested in a bank account that pays 7% compound interest each year. How much is in the account after five years?

If you can do the question above, try this one on problem solving.

② In 2009, the value of David's house was £240 000. In the same year the value of Joe's house was £350 000.

For each of the following three years, the value of David's house decreased by 9% of its value the previous year. During the same three years, the value of Joe's house decreased by 12% of its value the previous year.

At the end of 2012, David sells his house for its current value. David then buys Joe's house at its current value.

How much extra money will David need?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 73 (Problem solving exercise 5.4 Repeated percentage increase/decrease).

What you need to know

Did you know?

If a savings account pays interest at a rate of 2% and you invest £500 for 3 years, the interest gets added to your investment so that it grows more quickly. £500 becomes £530.60. The 60p is the interest on the interest in years 2 and 3.

A common use of **repeated percentage increase** is for savings where interest is added to an account and the interest for the next year is calculated on the new balance. This is called **compound interest**.

A common use of **repeated percentage decrease** is for **depreciation**. Assets such as cars reduce in value each year by a percentage of their value at the start of the year.

It is important to recognise when to use these techniques.

How to do it

Repeated percentage increase

£2000 is invested at 3.5% interest. Calculate the amount at the end of four years when

a the interest is not added to the investment

b the interest is added to the investment.

Solution

a 3.5% of £2000 $= \frac{3.5}{100} \times 2000$

$= £70$

This is the interest earned each year.

At the end of four years, the total amount of the investment is the original amount plus four years of interest:

£2000 + 4 × £70 = £2280

b Each year the investment increases by 3.5%, i.e. 103.5% of the investment at the start of that year.

Year 1 103.5% of £2000 $= \frac{103.5}{100} \times £2000$

$= £2070$

Year 2 103.5% of £2070 $= \frac{103.5}{100} \times £2070$

$= £2142.45$

Year 3 103.5% of £2142.45 $= \frac{103.5}{100} \times £2142.45$

$= £2217.44$

Year 4 103.5% of £2217.44 $= \frac{103.5}{100} \times £2217.44$

$= £2295.05$

This can be done in one calculation:

$2000 \times \left(\frac{103.5}{100}\right)^4 = £2295.05$

or $2000 \times 1.035^4 = £2295.05$

1.035 is referred to as the multiplier.

Learning exercise

① Asaph buys a field for £6000. For each of the next three years it increases in value by 10%.

a What is its value at the end of three years?

b What is the percentage increase in its value since Asaph bought it?

c Explain why the increase is greater than 30%.

② Pepe buys a car for £5000. For each of the next three years the car decreases in value by 20%.

a What is the value of Pepe's car at the end of three years?

b What is the percentage decrease in its value since Pepe bought it?

c If it continues to depreciate at 20% per year, after how many more years is Pepe's car worth less than £2000?

③ Match each percentage change with the equivalent decimal multiplier.

% change
20% increase
60% decrease
12% increase
12% decrease
150% increase

Decimal multiplier
$\times$ 1.5
$\times$ 0.88
$\times$ 1.2
$\times$ 0.4
$\times$ 1.12

④ Start with the number 200.

a Increase it by 50%.
Then decrease the answer by 40%.
Then decrease that answer by 10%.

b Find the percentage change from 200 to the final result in part **a**.

c Work out $200 \times 1.5 \times 0.6 \times 0.9$.

d What do you notice about the answers to parts **a** and **c**? Explain the connection.

⑤ Peter invests £500 at 6% per annum.

a How much interest does Peter receive at the end of the first year?

b He reinvests the £500 but not the interest. What is the total of this investment after one year?

c In the second year his reinvestment earns interest at 6%. How much interest does he receive at the end of the second year?

⑥ **a** Interest is paid on the following investments but not reinvested.
How much interest is received in total on

i £1000 at 5% p.a. for 2 years

ii £2000 at 10% p.a. for 5 years

iii £500 at 3.5% p.a. for 3 years?

b Work out the interest paid on each of the investments when the interest is reinvested each year.

⑦ A riverbank has been colonised by mink. They are an alien species that attacks local wildlife. The river authority traps the mink and removes them. Each year it reduces the population of mink by 60%.
What percentage of mink remain after

a 1 year

b 2 years

c 3 years

d 4 years

e 5 years?

⑧ Hannah buys her first car for £3000.

a After she has owned it for a year, she is told that its value has depreciated by 20%.

i How much is the car worth after one year?

ii How much value has the car lost in the first year?

b The rate of depreciation continues at 20% per year.

i Show that after two years the car is worth £1920.

ii How long will it be before Hannah's car is worth less than £500?

⑨ One Monday, 100 people have a highly infectious disease.
The number of people with the disease increases by 20% each day.
How many have the disease the following Monday?

⑩ In 2002 Mike bought a house for £87 000. The house increased in value by 6% each year.

a How much was the house worth in 2003?

b How much was the house worth in 2004?

c In which year did the value of the house become greater than £100 000?

⑪ Look at this information about compound interest rates.

Allied Avon savings accounts compound interest rates	
Standard saver	6% p.a.
Junior saver	7.5% p.a.
Super saver	11% p.a.

Work out the compound interest paid and the total amount for each investment.

a £580 invested for 3 years in a standard savings account.

b £1650 invested for 4 years in a junior savings account.

c £24 000 invested for 10 years in a super savings account.

⑫ Nathan increases the number of kilometres he runs each day by 25%. He went running on Monday, Tuesday and Wednesday. On Wednesday he ran 18.75 km. How far did he run on each of the previous two days?

Problem solving exercise

① In 2013, a young footballer playing in the Premier League was earning £30 000 per week. At the end of 2013, he signed a new two-year contract giving him an increase of 25% in the first year and a further increase of 30% in the second year.
He tells his mum that by the end of 2015, he will have earned over £4 million in these two years.
Is he correct?

② Dan wants to invest £5000 for three years in the same bank.
At the end of three years, Dan wants to have as much money as possible.
Which bank should Dan invest his £5000 with?

The International Bank	**The Friendly Bank**
Compound interest 4.5% for the first year, 1% for each extra year	Compound interest 5.8% for the first year, 0.5% for each extra year

③ The price of oil fluctuates daily. This is often due to political unrest in oil-producing countries.
One day during 2014, the price of oil was £56.79 per barrel. Later that week, the price of oil increased by 5%.
The following week, oil prices decreased by 5% and a newspaper headline read

'Oil returns to same cost as last week'

Explain why this headline is incorrect.

④ Tina invests £1500 in a bank account for four years. The bank pays compound interest at an annual rate of 2.5% for the first year and 1.5% for each of the next three years.
Andy also invests £1500 in a bank account for four years. The account pays a variable rate of compound interest. The interest is 2% for the first year, 1.8% for the second year and 1.7% for the third and fourth years.
Who has made the better investment?

⑤ Jodie buys a painting for £800. The painting increases in value by 12% in the first year and by a further 10% in the second year.
Jodie says that after two years, the value of the painting has increased by 22%.
Is she right?

Do I know it now?

① For each investment, compare the simple interest paid with the compound interest.
- **a** £16 500 invested at 8% p.a. for 4 years.
- **b** £24 000 invested at 12% p.a. for 10 years.

② Margaret buys a car for £8000.
- **a** One year later, its value has depreciated by 15%. What is its value now?
- **b** For each of the next two years, its value depreciates by 10%. What is its value three years after Margaret buys it?

Can I apply it now?

① Dimitri weighs 20 stones. He wants to lose 3 stones in 3 months. He sees the following advertisement for a diet plan.

Special Diet Plan Formula

Lose 6% of body weight in ONE month and 4% of body weight in each subsequent month.

Will Dimitri reach his target weight in 3 months using this diet plan?

CHAPTER 6

ESSENTIAL TOPICS – NUMBER

Ratio and proportion

6.1 Sharing in a given ratio

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the questions below.

① Divide 100 in the ratio 2 : 3 : 5.

② 7.2 kg of fertiliser is spread evenly over three small lawns. The areas of the lawns are 7 m^2, 8 m^2 and 9 m^2. How much fertiliser is used on each lawn?

If you can do the questions above, try this one on problem solving.

③ A new TV channel broadcasts chat shows, reality shows and soaps. They are committed to broadcasting these in the ratio 4 : 3 : 2.
The channel is currently on air for only 12 hours a day.

a How many hours of each type of programme do they broadcast?

b Next month, they plan to increase Saturday broadcasting to 18 hours. How many extra minutes of each programme type will they have to broadcast?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 78 (Problem solving exercise 6.1 Sharing in a given ratio).

What you need to know

In sharing problems, first work out the total number of 'parts'. Then decide how the parts are allocated.
You can use a rectangular bar as in this example.
Kate and Pam share the cost of their £8.40 pizza in the ratio 3 : 4. How much does each pay?
The ratio is 3 : 4.
3 + 4 = 7, so there are 7 parts.

Draw a bar with 7 equal parts.

K	K	K	P	P	P	P
£1.20	£1.20	£1.20	£1.20	£1.20	£1.20	£1.20

Kate has 3 shares and Pam has 4.

Divide £8.40 into 7 shares.
£8.40 ÷ 7 = £1.20

So Kate's share is 3 × £1.20 = £3.60
Pam's share is 4 × £1.20 = £4.80

Alternatively, you can use proportions.

These fractions are called proportional.

Kate pays $\frac{3}{7}$ of £8.40 = £3.60

Pam pays $\frac{4}{7}$ of £8.40 = £4.80

$\frac{1}{7}$ of £8.40 is £1.20.

How to do it

Sharing in a given ratio

Matthew and Harry go to a car boot sale.
They have a lucky find and buy a box of old Star Wars figures for £10.
Matthew pays £4 and Harry pays £6.
There are 40 figures in the box.

Matthew

Harry

a Do you agree with Harry? Why?

b The ratio of their money is 2 : 3.
They share the figures in this ratio.
Find how many each of them gets using

i proportions

ii a rectangular bar.

Solution

a Yes. Harry paid more than Matthew so it seems right that Harry should get more of the figures than Matthew.

b The ratio is 2 : 3.
2 + 3 = 5, so there are 5 parts.

$\frac{1}{5}$ of 40 is 8.

i Matthew gets $\frac{2}{5}$ of 40 = 16 figures

ii Harry gets $\frac{3}{5}$ of 40 = 24 figures

M	M	H	H	H	
2	2	2	2	2	10
8	8	8	8	8	40

Matthew: $8 + 8 = 16$ Harry: $8 + 8 + 8 = 8 \times 3 = 24$

There are two parts for Matthew and three parts for Harry so there are five parts altogether.

This row shows how the £10 they paid in total was shared between them. Matthew paid £4 and Harry paid £6.

There are 40 figures to share. $10 \times 4 = 40$ so multiply each number in the row above by 4.

Matthew gets 16 figures.

Harry gets 24 figures.

Learning exercise

1. £36 is shared between Joe and Jane in the ratio 1 : 2.
   - **a** How many equal parts are there?
   - **b** How much is each equal part?
   - **c** How much does Joe get?
   - **d** How much does Jane get?
   - **e** What is the total of Joe and Jane's amounts?
2. Jared and Raheem share a packet of 45 stickers in the ratio 2 : 7.
   - **a** How many equal parts are there?
   - **b** How many stickers are in each equal part?
   - **c** How many does Jared get?
   - **d** How many does Raheem get?
   - **e** What is the total of Jared and Raheem's amounts?
3. Jalel mixes blue and yellow paint in the ratio 1 : 5.
   He ends up with 480 ml of green paint.
   - **a** How many equal parts are there?
   - **b** How much is each equal part?
   - **c** How much blue paint did he use?
   - **d** How much yellow paint did he use?
   - **e** How much less blue than yellow did he use?
4. Work these out.
   - **a** Share £60 in the ratio 2 : 3.
   - **b** Share £96 in the ratio 5 : 1.
   - **c** Share 160 in the ratio 3 : 5.
   - **d** Share 126 ml in the ratio 7 : 2.
5. Work these out.
   - **a** Share £90 in the ratio 3 : 2.
   - **b** Share £120 in the ratio 1 : 7.
   - **c** Share 225 in the ratio 5 : 4.
   - **d** Share 480 m in the ratio 7 : 3 : 2.

6. £336 is shared between Alex and Abbie in the ratio 5 : 7.
   How much more does Abbie get than Alex?

7. A café sold 168 meals one day.
   The ratio of beef meals to chicken to fish was 4 : 2 : 1.
   How many more beef than fish meals were sold on that day?

8. Jeff is 12 years old and Joseph is 20 years old.
   Plan A is to share £5280 in the ratio of their ages now.
   Plan B is to wait until Jeff is 18 years old and to share £5280 in the ratio of their ages then.
   How much will Jeff gain by waiting until he is 18 years old?

9. In a rectangle the ratio of the length to width is 9 : 5.
   The perimeter of the rectangle is 196 cm.
   What are the length and width of this rectangle?

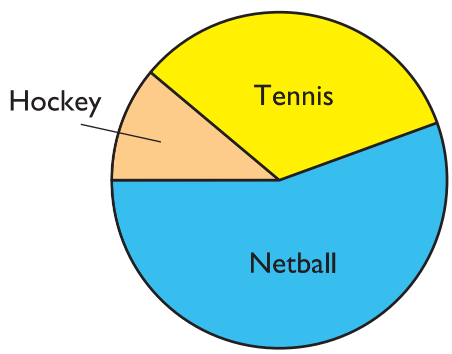
Problem solving exercise

1. Ninety girls are asked what their favourite sport is from hockey, tennis and netball.
   The ratio of the results of hockey to tennis to netball is 1 : 3 : 5.

   a How many girls chose tennis as their favourite sport?

   The pie chart shows these results.

   b Work out the angle of each sector of the pie chart.



2. Christine and Jenny are sisters.
   At Christmas 2004, Christine was 3 years old and Jenny was 12 years old.
   Their father gave them a present of £300 shared in the ratio of their ages.

   a How much money did Christine receive?

   Each Christmas, their father gave them a sum of money shared in the ratio of their ages.

   b In which year did Jenny receive twice as much money as Christine?

Do I know it now?

① The ratio of oranges to lemons in a recipe for marmalade is 1 : 4.

a How many equal parts of fruit are there?

b What fraction of the fruit are oranges?

c What fraction of the fruit are lemons?

② **a** Share 105 g in the ratio 4 : 3.

b Share 330 in the ratio 7 : 8.

c Share 286 g in the ratio 1 : 2 : 10.

d Share 245 in the ratio 1 : 2 : 4.

Can I apply it now?

① Andrew goes to the gym on Mondays and Fridays.
He always divides his time between using the treadmill and doing weights in the ratio 3 : 2.
On Monday he was there for $1\frac{1}{2}$ hours.
On Friday he spends the same time on weights as he had spent on the treadmill on Monday.
How long is he at the gym on Friday?

6.2 Working with proportional quantities

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Karen uses a recipe for cream of mushroom soup which serves four people.

Copy and complete the table to help Karen work out how much of each ingredient she will need for six people.

Ingredients	For 4	For 6
Mushrooms	240 g	
Stock	300 ml	
Small onions	1	
Plain flour	30 g	
Milk	400 ml	
Egg yolks	2	

If you can do the question overleaf, try this one on problem solving.

② Sunita drives 148 miles and uses 17 litres of petrol to reach her holiday destination.
While she is there, she drives a further 58 miles.
She visits her friend on the way home and so travels an extra 53 miles.
Sunita has budgeted for 50 litres of petrol.
Will she be under or over her budget? By how much?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 83 (Problem solving exercise 6.2 Working with proportional quantities).

What you need to know

Did you know?

Supermarkets often give the price per 100 g so that you can compare the unit cost of different brands. They use proportion to calculate this.

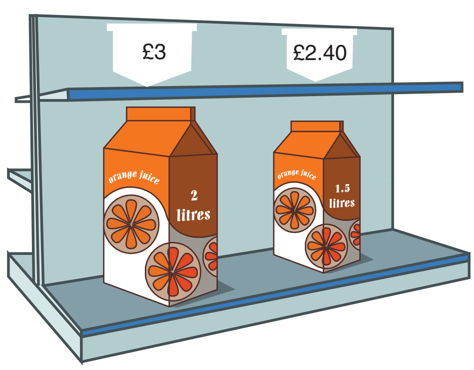
It is important to choose the most suitable method to solve a problem involving ratio and proportion.

- A **ratio table** can be very helpful for understanding a situation.
- The **unitary method** is good when the numbers are not quite straightforward and can be helpful for comparing quantities using unit costs.

How to do it

Comparing unit costs

Orange juice can be bought in different-sized cartons.
Holly needs 6 litres of orange juice for a party.
Which size carton should Holly buy?



Solution

Smaller carton

1.5 litres cost £2.40

1 litre costs $\frac{1}{1.5} \times £2.40 = £1.60$

Unit cost = £1.60

Larger carton

2 litres cost £3.00

1 litre costs $\frac{1}{2} \times £3.00 = £1.50$

Unit cost = £1.50

The larger carton has a lower unit cost, so Holly should buy the larger carton.

Using the unitary method

These ingredients for apple crumble make enough to serve two people.

Inga needs to make an apple crumble for five people.

How much of each ingredient should she use?

Apple Crumble (serves 2)
1 large cooking apple
25 g white sugar
$\frac{1}{4}$ teaspoon cinnamon
90 g wholemeal flour
40 g butter
75 g brown sugar

Solution

In the unitary method, you first find the quantity for 1.

Then you scale up by multiplying.

Ingredients	For 2	For 1 (÷2)	For 5 (×5)
Cooking apple	1	$\frac{1}{2}$	$2\frac{1}{2}$
White sugar	25 g	12.5 g	62.5 g
Cinnamon	$\frac{1}{4}$ tsp	$\frac{1}{8}$ tsp	$\frac{5}{8}$ tsp
Wholemeal flour	90 g	45 g	225 g
Butter	40 g	20 g	100 g
Brown sugar	75 g	37.5 g	187.5 g

This column gives all the answers.

Solving problems with a ratio table

40 blank CDs cost £20.

a Find the cost of 50 blank CDs.

b Find the cost of 24 blank CDs.

Solution

a

Number of CDs	40	20	10	50
Cost (£)	20	10	5	25

40 + 10 = 50

20 + 5 = 25

50 CDs cost £25.

b

Number of CDs	40	20	4	24
Cost (£)	20	10	2	12

20 + 4 = 24

10 + 2 = 12

24 CDs cost £12.

Learning exercise

① 4 apples cost 96p.

a How much does 1 apple cost?

b How much do 3 apples cost?

② 6 carrots cost 90p.

a How much does 1 carrot cost?

b How much do 8 carrots cost?

③ 100 litres of heating oil cost £60.

a How much do 10 litres of oil cost?

b How much do 230 litres cost?

④ Look at the prices on these two market stalls.

a **i** How much does 1 can of cola cost in Bev's Bargains?

ii How much does 1 can of cola cost in Dan's Discounts?

iii Which market stall is better value for cans of cola?

b **i** How much do 4 bananas cost in Bev's Bargains?

ii How much do 4 bananas cost in Dan's Discounts?

iii Which is better value for bananas?

c **i** How much do 12 oranges cost in Bev's Bargains?

ii How much do 12 oranges cost in Dan's Discounts?

iii Which is better value for oranges?

⑤ Vincent has a recipe for shepherd's pie which serves 5 people.
Copy and complete this table to help him find the quantities needed for 8 people.

Ingredient	Quantity for 5 people	Quantity for 1 person	Quantity for 8 people
Minced beef	900 g		
Stock	480 ml		
Onion	2		
Tin of tomatoes	1		
Potatoes	700 g		
Worcestershire sauce	40 ml		

⑥ 3 concert tickets cost £42.
How much do 8 of these tickets cost?

⑦ 9 cups of coffee cost £15.30.
How much do 4 cups of coffee cost?

8 For each of these, work out which is better value. Explain your answer.

a 6 kg for £14.70 or 7 kg for £17.50

b 100 ml for £18 or 150 ml for £24

c 80 g for £16.40 or 60 g for £12.06

9 A window cleaner charges a fixed rate of £5 plus an amount for each window he cleans.
He cleaned a house with 6 windows and charged £9.80.
Tom's house has 12 windows.
He thinks the window cleaner will charge him £19.60.
Is Tom right?
Explain your answer.

Problem solving exercise

1 Farah works in the retail industry. The table shows some information about the hours she worked and her pay for the last three weeks.

Day	Hours worked		
	Week 1	**Week 2**	**Week 3**
Monday	8	9	8
Tuesday	6	4	8
Wednesday	10	10	
Thursday	4	8	10
Friday	10	12	4
Total pay	£475		£450

a Work out Farah's pay for Week 2.

b How many hours did Farah work on Wednesday of Week 3?

2 The ingredients for apple crumble make enough to serve two people.

Apple Crumble (serves 2)
1 large cooking apple
25 g white sugar
$\frac{1}{4}$ teaspoon cinnamon
90 g wholemeal flour
40 g butter
75 g brown sugar

Inga is going to make some apple crumble using this recipe.
Inga has the following quantities of each ingredient:

24 large cooking apples
200 g of white sugar
6 teaspoons of cinnamon
1.2 kg of wholemeal flour
300 g of butter
500 g of brown sugar.

Work out the maximum number of servings of apple crumble that Inga can make.

③ Here are three boxes of the same type of chocolates.

Melanie and her friend go shopping for chocolates.
Melanie says, 'You always get the best value for money by buying the largest box.'
Is Melanie right? Show all your working.

Do I know it now?

① Five adult bus tickets cost £11.70.

a How much does one adult bus ticket cost?

b How much do nine adult bus tickets cost?

② For each of these, work out which is better value. Explain your answer.

a 2 m for £8.10 or 60 cm for £2.49

b 600 g for £5.40 or 750 g for £7.20

c 2 litres for £22.12 or 800 ml for £8.92

Can I apply it now?

① A particular brand of shampoo comes in 3 sizes.
Which size is the best value?

CHAPTER 7

ESSENTIAL TOPICS – NUMBER

Number properties

JUST IN CASE

Multiples

To find the multiples of 5, start at 5 and count up in 5s, 5, 10, 15, 20, and so on.

To find the multiples of 8, start at 8 and count up in 8s, 8, 16, 24, 32, ...

So the multiples are the numbers you find in a times table.

Large numbers also have multiples, it is not just those you know the tables for.

For example, the multiples of 53 are

$1 \times 53 = 53$

$2 \times 53 = 106$

$3 \times 53 = 159$

$4 \times 53 = 212$, and so on.

Multiples of 2 are called **even numbers**.

Whole numbers that are not even are called **odd numbers**.

A factor of a number divides into it exactly.

A factor is also called a divisor of the number.

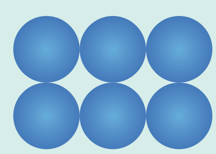
The factors of a number come in pairs that multiply together to make the given number.

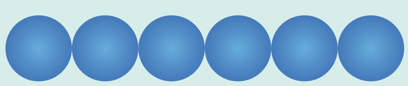
Think of the number as a set of dots.

If the dots are arranged to make a rectangle, then you have found factors.

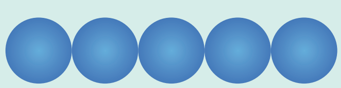
For example, 2 and 3 are factors of 6 because $2 \times 3 = 6$.

1 and 6 are also factors of 6 because $1 \times 6 = 6$.





Every number has at least one pair of factors. Every number of dots can be made into a rectangle that is one dot high and has all of the dots in a row. The only way that five dots can be made into a rectangle is

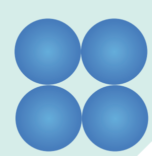


This means that the only factors of 5 are 1 and 5. Such numbers, with only two factors, are called **prime numbers**.

The prime numbers up to 50 are: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43 and 47.

1 is not a prime number because 1 has only one factor.

For some numbers, 4 for example, the dots can be arranged into a square.



These numbers are called **square numbers**. $2 \times 2 = 4$, which is written $2^2 = 4$.
2 squared is 4.
The square numbers up to 100 are 1, 4, 9, 16, 25, 36, 49, 64, 81 and 100.
$2^3 = 2 \times 2 \times 2 = 8$; 8 is called a cube number.
The **cube numbers** up to 100 are 1, 8, 27 and 64.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Multiples

a Write down all the multiples of 5 between 34 and 61.

b Write down all the multiples of 4 between 27 and 45.

c Write down all the multiples of 9 between 35 and 82.

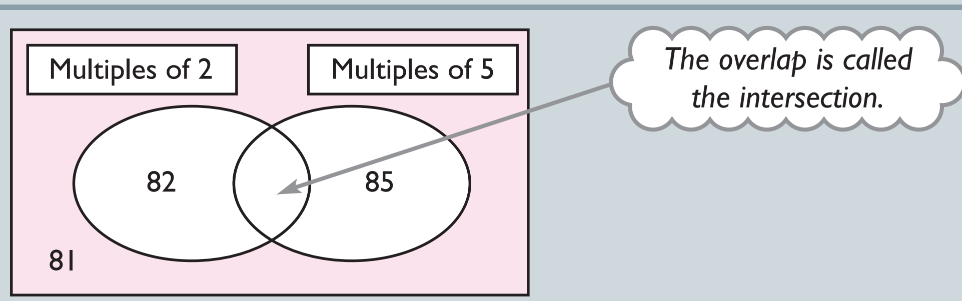
Factors, primes and powers

a List the factors of 16.

b Which of these factors are

- **i** square numbers
- **ii** prime numbers
- **iii** cube numbers?

Divisibility tests



a On a copy of this diagram, sort all the numbers from 50 to 100.

b What can you say about the numbers in the intersection?

Applying the knowledge

① Decide whether each statement is true or false.

a 1 is a prime number.

b 1 is a square number.

c 1 is a factor of 9 and 11.

d 1 is the square of −1.

e 1 is divisible by 2.

② Which numbers have all the properties listed in **a**–**c**?

a a square number less than 40

b adding 1 gives a prime number

c a multiple of 4

7.1 Index notation

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Write each of these calculations as a single power.

a $4^5 \times 4^9$

b $7^9 \div 7^2$

c $(13^7)^8$

What you need to know

Repeated multiplications can be written using **index notation** like this:

- $5 \times 5 \times 5 = 5^3 = 125$
- $3 \times 3 \times 3 \times 3 \times 3 = 3^5 = 243$

Numbers written using index notation can be multiplied and divided easily.

This example shows how to multiply numbers written using index notation.

$3^5 \times 3^4 = (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3)$

$= 3^9$

The powers have been added: 5 + 4 = 9.

This example shows how to divide numbers written using index notation.

$3^5 \div 3^4 = (3 \times 3 \times 3 \times 3 \times 3) \div (3 \times 3 \times 3 \times 3)$

$= \dfrac{3 \times \cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}{\cancel{3} \times \cancel{3} \times \cancel{3} \times \cancel{3}}$

$= 3^1$

The powers have been subtracted: 5 − 4 = 1.

$= 3$

Notice that $3^1 = 3$.

Using brackets with index notation means that the powers are multiplied.

$(3^5)^4 = 3^5 \times 3^5 \times 3^5 \times 3^5$

$= (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3)$
$\times (3 \times 3 \times 3 \times 3 \times 3) \times (3 \times 3 \times 3 \times 3 \times 3)$

$= 3^{20}$ ← 4 lots of 5 is 20.

How to do it

Equivalent amounts

Match these cards into pairs of the same value.

4×4^3	$4^9 \div 4^2$
$4^9 \div 4^3$	$4^5 \times 4^2$
$(4^2)^3$	$(4^2)^2$

Solution

Simplify each amount.

$4 \times 4^3 = 4 \times 4 \times 4 \times 4$
$= 4^4$

$4^9 \div 4^3 = \dfrac{\cancel{4} \times \cancel{4} \times \cancel{4} \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{\cancel{4} \times \cancel{4} \times \cancel{4}}$
$= 4^6$

$(4^2)^3 = (4 \times 4) \times (4 \times 4) \times (4 \times 4)$
$= 4^6$

$4^9 \div 4^2 = \dfrac{\cancel{4} \times \cancel{4} \times 4 \times 4 \times 4 \times 4 \times 4 \times 4 \times 4}{\cancel{4} \times \cancel{4}}$
$= 4^7$

$4^5 \times 4^2 = (4 \times 4 \times 4 \times 4 \times 4) \times (4 \times 4)$
$= 4^7$

$(4^2)^2 = (4 \times 4) \times (4 \times 4)$
$= 4^4$

So $4 \times 4^3 = (4^2)^2$
$4^9 \div 4^3 = (4^2)^3$
$4^9 \div 4^2 = 4^5 \times 4^2$

Unless otherwise stated, answer the questions in the following exercise without the use of a calculator. You may, however, wish to use a calculator to check some of your answers.

Learning exercise

① Write each of these as a power of 3.

a 3×3 **b** $3 \times 3 \times 3 \times 3 \times 3$

c $3 \times 3 \times 3 \times 3$ **d** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

② Write these in index form.

a $5 \times 5 \times 5$ **b** $2 \times 2 \times 2 \times 2 \times 2 \times 2$

c $2 \times 5 \times 2 \times 5$ **d** $7 \times 7 \times 7 \times 7 \times 7$

e $11 \times 11 \times 11 \times 17 \times 17$ **f** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

g $2 \times 2 \times 2 \times 2 \times 2 \times 19$ **h** $5 \times 5 \times 7 \times 5 \times 7 \times 7$

③ Use a calculator to work out the value of these.

a 5^3 **b** 2^{10} **c** 7^4 **d** 1^{30}

e 3^6 **f** 6^5 **g** 4.5^2 **h** 8^7

i 29^1 **j** 15^4 **k** 2^{16} **l** $(-6)^4$

④ Write each of these as a single power of 2.

a $2^2 \times 2^2$ **b** 16 **c** 2×8 **d** 64

e $2^3 \times 2^4 \times 2$ **f** $2 \times 2 \times 2 \div 2$ **g** $2^3 \div 2$ **h** $2 \times 8 \times 16$

⑤ Write each of these as a single power of 3.

a $3^{10} \div 3^2$ **b** $3^7 \div 3^4$ **c** $3^6 \div 3$

d $3^9 \div 3^3$ **e** $3^8 \div 3^2$ **f** $3^{11} \div 3^5$

g $3^{10} \div 3$ **h** $\frac{3^6}{3^2}$ **i** $\frac{3^9}{3^4}$

⑥ Write each of these as a single power of 5.

a $(5^3)^2$ **b** $(5^2)^4$ **c** $(5^6)^2$

d $(5^4)^3$ **e** $(5^6)^6$ **f** $(5^7)^3$

g $(5^5)^4$ **h** $(5^2)^9$ **i** $(5^8)^4$

⑦ Write down the equal pairs in this list.

2^{10}	8^3	9^3	2^9	4^5	3^6

⑧ **a** How much greater is 2^6 than 6^2?

b How much smaller is 5^3 than 5^4?

c What is the sum of 2^8 and 3^4?

d What is the product of 6^3 and 1^{10}?

⑨ Work out the value of these. Give your answers as ordinary numbers.

a $3^2 + 3$ **b** $5^3 - 5$ **c** 4×4^2 **d** $10^3 - 10$

e $7^2 \div 7$ **f** $2^3 + 3^2$ **g** $2^3 \times 3^2$ **h** $2^3 - 3^2$

⑩ Write each of these as a single power of 2.

a $2^3 \times 2^4$ **b** $2^8 \div 2^4$ **c** $(2^4)^2$ **d** $(2^5)^3$

e 2×2^9 **f** $2^{12} \div 2^3$ **g** $\frac{2^{10}}{2^2}$ **h** $2^3 \times 2 \times 2^4$

⑪ Work out the value of the missing numbers.

a $3^4 \times 3^{\square} = 3^{10}$ **b** $7^6 \div 7^{\square} = 7^3$ **c** $(5^{\square})^2 = 5^{12}$ **d** $2^6 = 2^{\square} \div 2$

Do I know it now?

① Write each of these in index form.

a $17 \times 17 \times 17$ **b** $2 \times 2 \times 5 \times 5 \times 5$

c $3 \times 5 \times 3 \times 3 \times 3 \times 5$ **d** $2 \times 3 \times 3 \times 11 \times 3 \times 11$

② Write each of these as a single power of 2. Do not use a calculator.

a $2^6 \times 2^4$ **b** 32 **c** $2^2 \times 2^5 \times 2^2$ **d** $2 \times 4 \times 8$

③ Work out the value of these. Give your answers as ordinary numbers.

a $4^3 + 4^3$ **b** $6^3 - 6^2$ **c** $2^3 \times 2^2$ **d** $19^2 \div 19^2$

④ Work out the missing numbers.

a $7^8 \times 7^2 = 7^{\square} \div 7^2$ **b** $3^{\square} \times 3^2 = (3^2)^4$

c $5^9 \div 5^2 = 5^{\square} \times 5$ **d** $(2^6)^3 = 2^{\square} \div 2^2$

7.2 Prime factorisation

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① **a** Write 312 as a product of its prime factors using index notation.

b Find the LCM and HCF of 12 and 64.

If you can do the question above, try this one on problem solving.

② Amy is in hospital. Her medication consists of:

- paracetamol, 2 tablets to be taken every 4 hours
- antibiotics, 1 tablet every 3 hours
- steroids, 1 injection every 6 hours.

Amy had all three of her medications at 8.30 a.m. When will Amy next have all three medications at the same time?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 94 (Problem solving exercise 7.2 Prime factorisation).

What you need to know

Did you know?

Data are encrypted on the internet by multiplying them by very large prime numbers. In order to hack into the data, you need to identify the prime factors of each number in the encrypted data by dividing by all of the prime numbers less than the square root of the encrypted number.

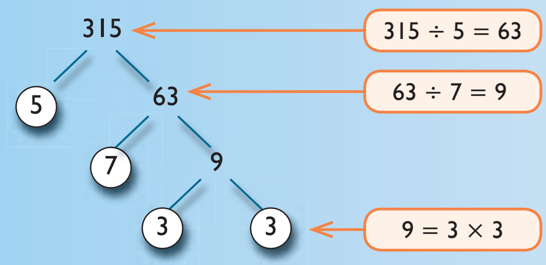
The **factors** of a number divide into it exactly.

The factors of 12 are 1, 2, 3, 4, 6 and 12.

Prime factors are the factors of a number that are also prime numbers. The prime factors of 12 are 2 and 3.

Every number can be written in terms of its prime factors.
$12 = 2 \times 2 \times 3 = 2^2 \times 3$ Remember 1 is not a prime number.

A factor tree is often used to write a number as a **product of its prime factors**.



From this diagram, you can see that $315 = 5 \times 7 \times 3 \times 3 = 3^2 \times 5 \times 7$.

The set of prime factors for each number is unique. A different set of prime factors will give a different number.

The **highest common factor (HCF)** of two numbers is the largest factor that they share.

You can find the HCF of two numbers by listing their factors, as in this example.

Factors of 20	1	2	4	5	10	20		
Factors of 30	1	2	3	5	6	10	15	30

The highest number in both lists is 10 and this is the HCF.

The HCF is often quite a small number.

The **lowest common multiple (LCM)** of two numbers is the lowest multiple that they share.

You can find the LCM of two numbers by listing their multiples, as in this example.

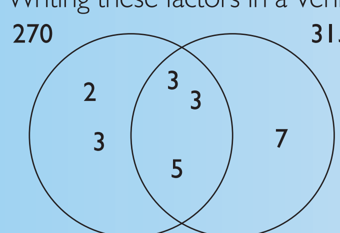
Multiples of 20	20	40	60	80	100
Multiples of 30	30	60	90	120	150

The lowest number in both lists is 60 and this is the LCM. The LCM is often quite a large number.

Writing the prime factors of two numbers in a Venn diagram will help you to find the HCF and LCM of those numbers. Look at this example for 315 and 270.

$315 = 3 \times 3 \times 5 \times 7 \quad 270 = 2 \times 3 \times 3 \times 3 \times 5$

Writing these factors in a Venn diagram gives



The HCF is found by multiplying the numbers in the intersection:
$3 \times 3 \times 5 = 45$
The LCM is found by multiplying all of the numbers in the diagram:
$2 \times 3 \times 3 \times 3 \times 5 \times 7 = 1890$

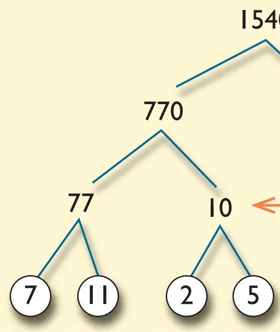
How to do it

Finding prime factors using a factor tree

Write 1540 as a product of its prime factors.

Solution

Using a factor tree



Find any product that gives 1540, e.g. 770×2.

Find any product that gives 770, e.g. 77×10.

Stop when all the factors are prime numbers.

So $1540 = 7 \times 11 \times 2 \times 5 \times 2$

$= 2^2 \times 5 \times 7 \times 11$

Prime factors

Find the HCF and LCM of 24 and 90 by

a using a Venn diagram

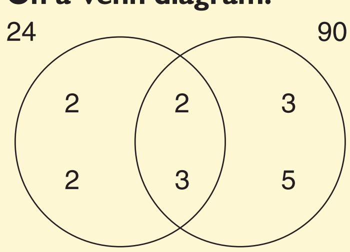
b listing factors and multiples.

Solution

a The prime factors of 24 are $2 \times 2 \times 2 \times 3 = 2^3 \times 3$.

The prime factors of 90 are $2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$.

On a Venn diagram:



HCF $= 2 \times 3 = 6$

Multiplying the numbers in the intersection gives the HCF.

LCM $= 2 \times 2 \times 2 \times 3 \times 3 \times 5 = 360$

Multiply all the numbers to find the LCM.

b Factors of 24   1   2   3   4   6   8   12   24

Factors of 90   1   2   3   5   6   9   10   15   18   30   45   90

The HCF is 6.

The highest number which is in both lists is 6.

Multiples of 24   24   48   72   96   120   144   168   192   216   240   264   288   312   336   360   384

Multiples of 90   90   180   270   360   450 …

The LCM is 360.

360 is the first number which is in both lists.

Learning exercise

1. Write these prime factorisations as ordinary numbers.

   a $2 \times 3 \times 5$ **b** $2 \times 5 \times 11$ **c** $2^2 \times 3$

   d 2×3^2 **e** 3×5^2 **f** 2×5^3

   g $3 \times 5 \times 7^2$ **h** $3^3 \times 5^2 \times 7$

2. Write each of these numbers as a product of its prime factors, in index form.

   a 60 **b** 126 **c** 100 **d** 54

   e 225 **f** 154 **g** 105 **h** 495

   i 300 **j** 405 **k** 500 **l** 624

3. **a** Write down all the factors of 12.

   b Write down all the factors of 20.

   c Which numbers are common factors of 12 and 20?

   d What is the highest common factor (HCF) of 12 and 20?

4. Find the HCF of each pair of numbers.

   a 6 and 10 **b** 15 and 20 **c** 24 and 30

   d 9 and 18 **e** 16 and 24 **f** 26 and 52

   g 40 and 75 **h** 36 and 54 **i** 48 and 60

   j 29 and 37 **k** 66 and 154 **l** 51 and 85

5. **a** Write down the first ten multiples of 6.

   b Write down the first ten multiples of 8.

   c Which numbers are common multiples of 6 and 8?

   d What is the lowest common multiple (LCM) of 6 and 8?

6. Find the LCM of each pair of numbers.

   a 6 and 9 **b** 5 and 8 **c** 7 and 10

   d 12 and 20 **e** 6 and 14 **f** 15 and 30

   g 16 and 10 **h** 5 and 13 **i** 24 and 40

   j 50 and 60 **k** 18 and 27 **l** 30 and 36

7. Work out the missing numbers in these.

   a $2 \times 3^{\square} \times 7^2 = 882$ **b** $2^{\square} \times 3 \times 5^2 = 1200$

   c $3 \times \square^3 \times 11 = 4125$ **d** $\square^{\square} \times 7 \times \square = 5824$

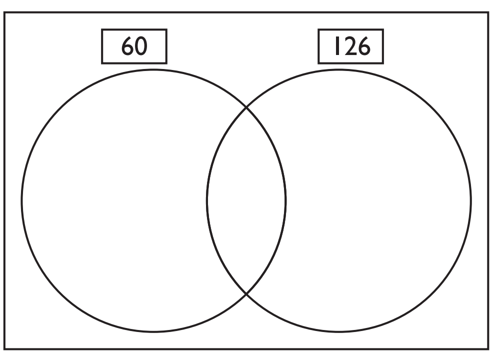
8. **a** Write 60 as a product of its prime factors.

   b Write 126 as a product of its prime factors.

   c Write the prime factors in a copy of the Venn diagram.

   d Find the HCF of 60 and 126.

   e Find the LCM of 60 and 126.



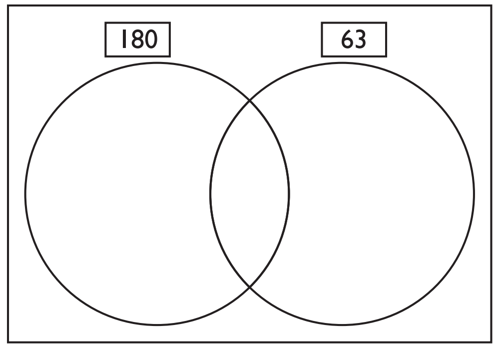
9 **a** Write 180 as a product of its prime factors.

b Write 63 as a product of its prime factors.

c Write the prime factors in a copy of the Venn diagram.

d Find the HCF of 180 and 63.

e Find the LCM of 180 and 63.



Problem solving exercise

1 Nadir is planning a party. She wants to buy some samosas, some sausage rolls and some cakes.

The table shows the quantities and costs of each item.

	Samosas	Sausage rolls	Cakes
Number in a box	24	16	18
Cost per box	£5.80	£2.50	£7.10

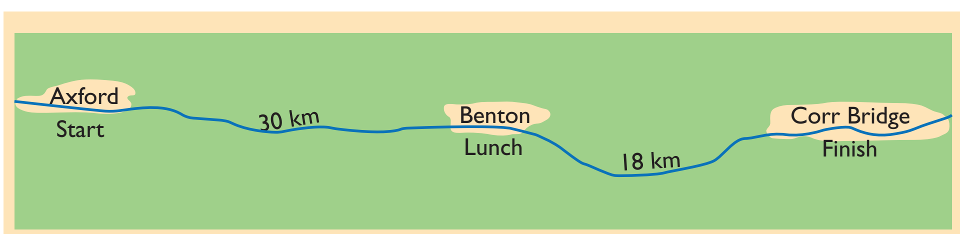
Nadir wants to buy the smallest number of packets that will give her the same number of samosas, sausage rolls and cakes. She has £120 to spend on this food.

Does she have enough money to buy the food she needs?

2 Bradley and Mark cycle around a track. Each lap Bradley cycles takes him 50 seconds. Each lap Mark cycles takes him 80 seconds.

Mark and Bradley leave the start line at the same time. By how many laps will Mark be behind Bradley when they are next at the start line together?

3 The map shows the route of a charity walk.



Marshals stand at equally-spaced intervals along the route, including the lunch stop, the start and the finish. The distance between marshals is always a whole number of kilometres.

What is the smallest possible number of marshals?

Do I know it now?

① Write these as ordinary numbers. (They are given as the products of their prime factors.)

a $2^2 \times 5$

b $2^4 \times 5^2$

c $2 \times 3 \times 5 \times 7 \times 11$

d $2^2 \times 3 \times 5^2 \times 7 \times 11$

e $2^4 \times 3 \times 5^4 \times 7 \times 11$

② Write each number as a product of its prime factors, using index form.

a 90 **b** 165 **c** 770 **d** 819

e 1400 **f** 1750 **g** 1584 **h** 2912

③ **a** Write down all the prime factors of 54 and 60, using index form.

b Find the highest common factor (HCF) of 54 and 60.

c Find the lowest common multiple (LCM) of 54 and 60.

Can I apply it now?

① Linda has her car serviced every 6000 miles. Here are some of the checks they carry out. How many miles does the car travel before it has a service that includes all five checks?

Check	Required every
Brake fluid	6000 miles
Change oil filter	12 000 miles
Tyres	6000 miles
Wiper blades	18 000 miles
Change timing belt	24 000 miles

CHAPTER 8

ESSENTIAL TOPICS – ALGEBRA

Starting algebra

JUST IN CASE

Using letters

A formula is a rule for working something out.
It is important to say what each of the letters in a formula stands for.
For example, in the formula $W = 10h$
W means 'total wage in pounds' and h means 'number of hours worked'.
The letters stand for numbers. They are not abbreviations for words.

h = hours ✘

h = number of hours worked ✔

The letters used in a formula are called variables.
They represent numbers which can change (or vary).
In the formula $W = 10h$, because there is no sign between the number and the variable, they should be multiplied.

$W = 10h$

When $h = 25$, $W = 10h$

$= 10 \times 25$

$= 250$.

The total wage is £250.

When $h = 40$, $W = 10 \times 40$

$= 400$.

The total wage is £400.
This is called **substituting into formulae**.
Equivalent formulae show the **same** information in different ways.
The following formulae can be used to calculate the perimeter of a rectangle:

$P = 2l + 2h$

$P = l + l + h + h$

Combining variables

An expression is a collection of terms.

$4a + 6b + a - 2b + 5$

6b means 6 x b.

You usually write this as a, not 1a.

Write 4a, not a4.

A **term** is a single number or a **variable**.
A term can also be a product of numbers or variables.

Here are some examples.

$t \qquad 5 \qquad 3n \qquad ab \qquad \frac{1}{2}m \qquad 7pq$

Terms that contain exactly the same variable are known as **like terms**.

- $5T$ and $3T$ are like terms as they contain the same letter.
- $3a^2$ and $6a^2$ are also like terms.
- $6w$ and $3v$ are not like terms because they have different letters.
- $5a$ and $6a^2$ are not like terms as they have different powers of a.

Like terms can be combined into a single term.

This is called *collecting* or *gathering* terms.

$4a + 6b + a - 2b + 5 = 5a + 4b + 5$

Simplify these expressions.

a $6v - 2v + 4v$

b $7w + 12w^2 - 2w$

Solution

a $6v - 2v + 4v = 8v$ ← The variable is the same in all three terms.

b $7w + 12w^2 - 2w = 5w + 12w^2$ ← $7w$ and $2w$ are like terms.
Make sure you keep the sign with its term. $7w - 2w = 5w$
$12w^2$ uses a different power of w so you cannot combine it with the other two terms.

Simplify this expression.

$2e \times 3f \times 4e$

Solution

$2e \times 3f \times 4e = 2 \times 3 \times 4 \times e \times f \times e$ ← When multiplying terms, multiply the numbers together and multiply the variables together separately.

$= 24e^2f$ ← $e \times e = e^2$ and $e^2 \times f = e^2f$ and $24 \times e^2f = 24e^2f$

Working with formulae

Number machines can help you work with formulae.

The cost, C pence, of a bus ticket for a journey of m miles is given by the formula $C = 20m + 50$.

You can write this using a number machine.

$m \longrightarrow \boxed{\times 20} \xrightarrow{20m} \boxed{+50} \longrightarrow 20m + 50 = C$

For a journey of 7 miles, $m = 7$.

$7 \longrightarrow \boxed{\times 20} \xrightarrow{140} \boxed{+50} \longrightarrow 190$

The cost is 190 pence or £1.90.

You can use a number machine in reverse.

This will tell you the number of miles you can travel for a certain amount of money.

Work from right to left with the **inverse operations**.

So, for a fare of £1.30, the number machine looks like this.

$$4 \longleftarrow \boxed{\div 20} \xleftarrow{80} \boxed{-50} \longleftarrow 130$$

The Williams family are going on holiday to Florida.

a The exchange rate is 1.6 dollars to the pound.
Write a formula to represent this information.

b They wish to take \$1200 with them.
How many pounds should they change into dollars?

Solution

a Number of dollars = number of pounds × 1.6

If D is the number of dollars and P is the number of pounds then $D = 1.6P$

b The formula is used in reverse to find the number of pounds.

Number of pounds = number of dollars ÷ 1.6

To find the number of dollars you multiplied by 1.6, so to find the number of pounds you must divide by 1.6.

$= 1200 \div 1.6$

$= 750$

They need to change £750 into dollars.

This formula gives p in terms of e and g.

$$p = \frac{3e - g}{4}$$

Find the values of p when

a $e = 4$ and $g = 0$

b $e = 5$ and $g = 7$.

Solution

a $\dfrac{3e - g}{4} = \dfrac{3 \times 4 - 0}{4}$

$= \dfrac{12 - 0}{4} = \dfrac{12}{4} = 3$

b $\dfrac{3e - g}{4} = \dfrac{3 \times 5 - 7}{4}$

$= \dfrac{15 - 7}{4} = \dfrac{8}{4} = 2$

Solving simple equations

An **equation** says that one expression is equal to another.

For example:

$4x - 3 = 17$

Solving an equation means finding the value of x that makes the equation true.

You can solve an equation using the balance method.

You must keep the equation balanced, like a pair of weighing scales, by doing the *same* operation to *both* sides.

$$\begin{aligned} 4x - 3 &= 17 \quad &{+3}\\ 4x &= 17 \quad &{\div 4}\\ x &= 5 \end{aligned}$$

The inverse of subtract 3 is add 3. Make sure you add 3 to *both* sides.

The inverse of multiply by 4 is divide by 4.

$x = 5$ is the solution.

Using brackets

When you **expand** an expression you multiply out the brackets.

When you rewrite an expression using brackets you are **factorising**.

$5a + 10 = 5(a + 2)$

- $5a + 10$ is the expanded expression.
- $5(a + 2)$ is the factorised expression.

When you expand (or multiply out) brackets you must multiply every term inside the bracket by the term outside the bracket.

$$b \times (a + 3b - 2) = b \times a + b \times 3b + b \times -2$$
$$= ab + 3b^2 - 2b$$

There are three terms inside the bracket so there will be three terms in your answer.

When you factorise an expression, look at the numbers first and then the letters.

$$2fg + 6f^2 = 2 \times fg + 2 \times 3f^2$$
$$= 2f \times g + 2f \times 3f$$
$$= 2f(g + 3f)$$

2 goes into 2 and 6.
f goes into fg and f^2
so $2f$ goes outside the bracket.

$2f$ is the highest common factor (HCF) of both terms.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Making and using word formulae

Look at this advert.

Mobile Phones
Calls cost 25p per minute

a Copy and complete this formula.

Cost (in pence) = ________ × number of minutes

b What is the cost of a six-minute call?

Using letters

Work out the value of $2a + 3b - 4c$ when

a $a = 2, b = 3$ and $c = 3$

b $a = 1, b = 2$ and $c = 0$

c $a = 0, b = 4$ and $c = 3$

d $a = 2, b = 3$ and $c = 2$.

→ Combining variables

Simplify these expressions.

a $6d + 7d - 5d$

b $8m - 10m$

c $g^2 + 3g^2$

d $5r + 11r^2 - 2r$

e $5h - 5 - 5gh + 3hg$

f $2c \times 2b + 2c \times 2b \times c$

→ Working with formulae

Ken is changing a distance from miles to kilometres.
He divides the number of miles by 5 and then multiplies by 8.

a Draw a number machine to show this.

b Use your number machine to convert 75 miles into kilometres.

c Draw the inverse number machine.

d Ken drives 120 km. Use your inverse machine to find out how many miles this is.

→ Setting up and solving simple equations

Solve these equations.

a $3x = 18$

b $5x - 7 = 13$

c $3 - 4t = -17$

→ Using brackets

a Expand the brackets in these expressions.

i $3(x + 2y)$ **ii** $6(2 - 3r)$ **iii** $3x(4x - 7)$

b Factorise these expressions fully.

i $6a^2 + ab$ **ii** $15ab + 10a$ **iii** $16cd - 12d^2$

→ Applying the knowledge

① Match the words with the algebra. One expression in words and one expression in algebra do not match. Write a matching expression for each one.

Multiply b by 2 then subtract from a	$a(b - c)$
Subtract c from a then multiply by b	$\frac{a}{c} - b$
Add b to a then divide into c	$\frac{ab}{c}$
Multiply a by b then divide by c	$\frac{c}{a + b}$
Divide c by b then multiply by a	abc
Multiply a by c then divide into b	$b(a - c)$
Add b to a then divide by c	$a - 2b$
Multiply ab by c	$\frac{b}{ac}$
Multiply a by 4 then subtract c	$\frac{a + b}{c}$
Subtract c from b then multiply by a	$\frac{c}{b} \times a$

② A square has a perimeter of $(40x + 60)$ cm.

A regular pentagon has the same perimeter as the square.

Show that the difference between the lengths of the sides of the two shapes is $(2x + 3)$ cm.

8.1 Working with more complex equations

SKILLS CHECK

→ Do I need to do this section?

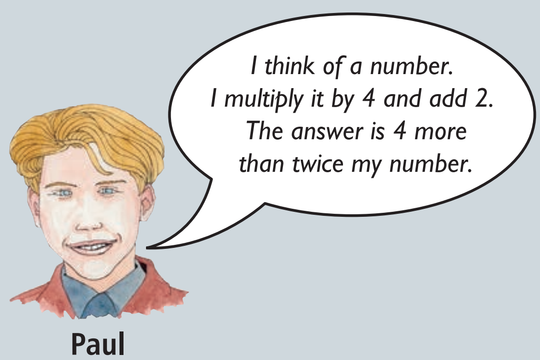
Complete this section if you need help with the questions below.

① Solve these equations and check your answers.

a $3x - 2 = 4x - 5$

b $2x + 9 = 3x - 5$

② Paul thinks of a number, x.



a Write down expressions for

i 'multiply my number by 4 and add 2'

ii 'four more than twice my number'.

b Write down an equation for Paul's number.

c Solve your equation to find the value of x.

If you can do the questions above, try this one on problem solving.

③ Jamie and Holly spend the same amount of time exercising at the youth club. Jamie spends 1 hour playing table tennis and does some swimming. Holly only goes swimming. She swims for four times longer than Jamie.

How long do they spend at the youth club exercising?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 105 (Problem solving exercise 8.1 Working with more complex equations).

What you need to know

You can use the **balance method** to solve equations with an unknown on both sides.

The first step is to get all the unknowns onto the same side of the equation:

$$3x + 8 = 33 - 2x$$

The inverse of subtract $2x$ is add $2x$.

$+ 2x \qquad + 2x$

Remember to do the same thing to both sides of the equation so it balances.

$$5x + 8 = 33$$

The inverse of add 8 is subtract 8.

$- 8 \qquad - 8$

Subtract 8 from both sides of the equation.

$$5x = 25$$

The inverse of multiply by 5 is divide by 5.

$\div 5 \qquad \div 5$

Divide both sides by 5.

$$x = 5$$

Check: $15 + 8 = 33 - 10 = 23$ ✓

Always check your work.

How to do it

➤ Solving an equation with an unknown on both sides

Solve $3x - 8 = 20 - x$.

Solution

$3x - 8 = 20 - x$ ← Add x to both sides.

$4x - 8 = 20$ ← Add 8 to both sides.

$4x = 28$ ← Divide both sides by 4.

$x = 7$

Check: $3x - 8 = 20 - x$

$3 \times 7 - 8 = 20 - 7$

$21 - 8 = 13$

$13 = 13$ ✓

➤ Solving word problems

Jamie and Holly both had the same amount of credit on their mobile phones.
Jamie sent 18 texts and has £1.40 credit left.
Holly sent 12 texts and has £2 credit left.
They both pay the same amount for one text.

a Work out the cost of one text message.

b How much credit did Jamie and Holly start with?

Solution

a Let t represent the cost of one text message in pence. Change pounds to pence because whole numbers are easier to work with.

Jamie's credit = Holly's credit

$18t + 140 = 12t + 200$ — Subtract $12t$ from both sides.

$6t + 140 = 200$ — Subtract 140 from both sides.

$6t = 60$ — Divide both sides by 6.

$t = 10$

So a text message costs 10p.

b Substitute $t = 10$ into the expression for Jamie's credit.

$$\begin{aligned} 18t + 140 &= 18 \times 10 + 140 \\ &= 320 \end{aligned}$$

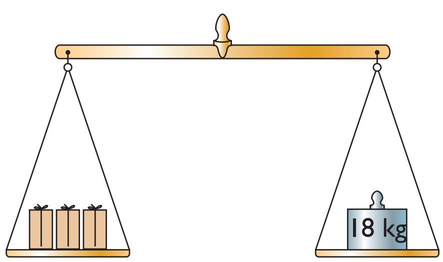
So Jamie's starting credit was 320p or £3.20.
Check that Holly's credit is also 320p.

$$\begin{aligned} 12t + 200 &= 12 \times 10 + 200 \\ &= 320 \checkmark \end{aligned}$$

Learning exercise

① Write an equation for each balancing problem and solve it.

a



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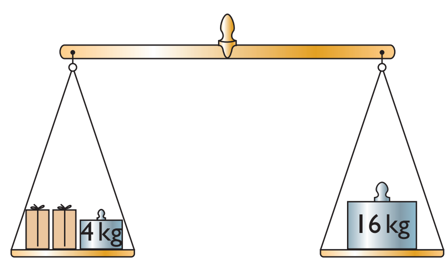
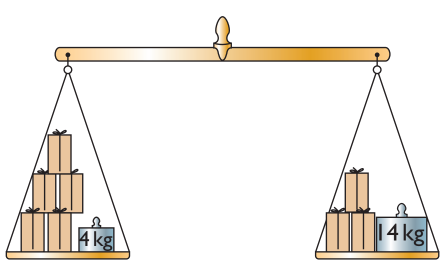
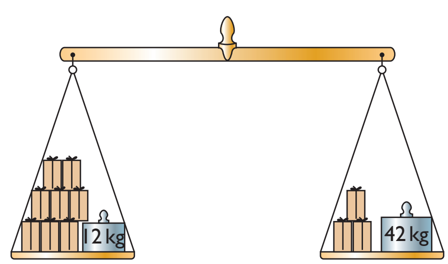


c



d



② Solve these equations.

a $6s = 27$ **b** $5x + 14 = 49$ **c** $18 - 3h = 3$ **d** $\frac{y}{4} - 6 = 9$

e $12 = 2 - 5m$ **f** $14 = 29 + 6g$ **g** $8 = 5 + \frac{1}{2}p$ **h** $-8 = -5 - \frac{1}{2}p$

③ Solve these equations.

a $4a + 7 = 3a + 3$ **b** $7f + 3 = 2f + 18$

c $7x - 3 = 6 - 2x$ **d** $17 + 2y = 8 - y$

④ Solve these equations.

a $5a - 8 = 7a + 22$ **b** $6b + 15 = -2b - 9$

c $5c - 3 = c - 1$ **d** $11 - 2f = 8 - 8f$

⑤ Julie works out that if she buys 6 apples she will have 20p left over but if she buys only 4 apples she will have 64p over.

a Write an equation to represent this solution.

b Solve the equation to find the cost of an apple.

⑥ Jan tries to solve this equation.

$11 - 4x = 6x + 5$

a Here is her attempt.

$$
\begin{aligned}
11 - 4x &= 6x + 5 \\
11 + 5 &= 6x - 4x \\
16 &= 2x \\
16 \div 2 &= x \\
x &= 8 \quad ✗
\end{aligned}
$$

Explain what she has done wrong.

b Using the same steps as Jan, write out a correct solution.

c Check your solution by substituting it in the original equation.

⑦ Sally and Tara need to solve this equation.

$4.1x - 3.7 = 3.6x - 2.2$

Here is how they start.

Sally

$4.1x - 3.7 = 3.6x - 2.2$

$4.1x - 3.6x = -2.2 + 3.7$

$0.5x = 1.5$

$x = \frac{\ldots\ldots\ldots}{\ldots\ldots\ldots}$

$x = \ldots\ldots\ldots$

Tara

Multiply both sides by 10.

$41x - 37 = 36x - 22$

$41x - 36x = 37 - 22$

....

....

....

a Copy and complete their answers.

b Whose method do you prefer? Say why.

8. Solve these equations.

a $2.1a + 3.6 = 1.1a + 5.7$

b $4.5b - 2.9 = 3.6b + 4.3$

c $3.7 - 2.2c = 1.8c - 5.4$

d $8.5 - 5.3d = 3.6 - 1.8d$

e $2\frac{1}{2}x - 6 = 3 - \frac{1}{2}x$

f $2\frac{1}{2}x - 6 = 3 + \frac{1}{2}x$

g $4.11x - 2 = 4.1x - 1$

h $0.001x - 0.005 = 0.003 - 0.003x$

9. Sandra and Zoe are sisters. They are making dresses from the same material. Their parents give them the same amount of money to buy it. The cost of 1 metre of the material is £m.

Sandra buys 3.5 metres and has £4.25 left over.

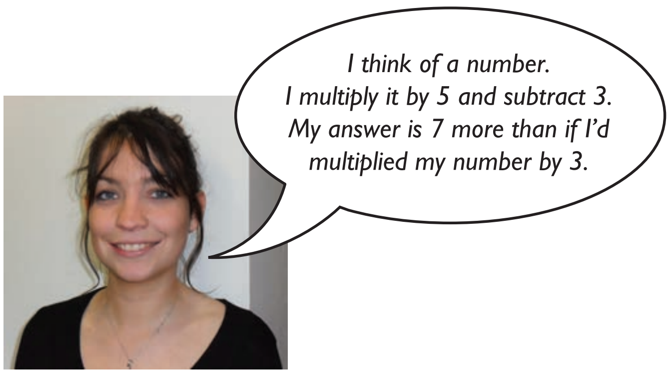
Zoe buys 2 metres and has £11.00 left over.

a Write this information as an equation for m.

b Solve the equation.

c How much money was each girl given?

10.



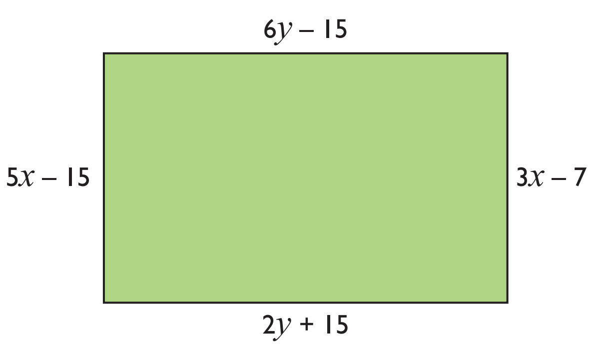
Ava

a Write this information as an equation.

b What number did Ava think of ?

Problem solving exercise

1. Work out the area of this rectangle. All the measurements are in centimetres.



2. Jack gives £400 to his three grandchildren.

He gives Ellie twice as much as Harry.

He gives Tom £40 less than Harry.

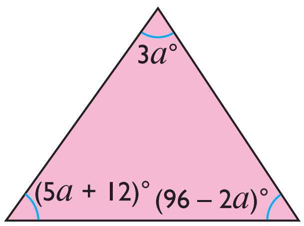
a Let £h stand for the amount of money that Harry gets. Write down expressions for the amount of money given to

i Ellie **ii** Tom.

b How much money does Jack give to each grandchild?

③ The angles in this triangle are $3a°$, $(5a + 12)°$ and $(96 - 2a)°$.

Show that the triangle is isosceles.



Do I know it now?

① Solve these equations.

a $8g - 5 = 5g + 40$ **b** $8 + 2d = d + 2$

c $9t - 4 = t - 8$ **d** $24 + 5c = 16 - 3c$

② Solve these equations.

a $23 - 4d = 3d + 2$ **b** $7e + 4 = 18 - 3e$

c $17 + g = 6 - g$ **d** $5 - 3h = 1 + h$

③ Solve these equations.

a $1.4x + 2.1 = 0.5x + 5.7$ **b** $3.88 - 1.02x = 1.38 + 1.48x$

c $4\frac{1}{4}x + 1\frac{1}{2} = 16\frac{1}{2} - \frac{3}{4}x$ **d** $5 - \frac{3}{4}x = 1\frac{1}{2} - \frac{1}{4}x$

Can I apply it now?

① Zorro is buying USB pens for his computer. They cost £p each.

Zorro could buy 5 USB pens and have £4.25 of the money in his pocket left.

Instead, he buys 2 USB pens and spends £16.50 on a game. This leaves him with just 20 pence.

a What is the cost of a USB pen?

b How much money did Zorro have?

8.2 Solving equations with brackets

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Solve these equations.

a $4(k + 2) = 24$

b $7(6 - 2r) = 28$

c $5t + 8 = 2(2t + 9)$

d $3(n - 1) = 3 - 3n$

e $4(g + 1) = 3(g + 3)$

f $3(2 - 3r) + 2 = 2(5 - r)$

If you can do the question above, try this one on problem solving.

② The head of a fish is 6 cm long. The body of the fish is twice as long as the head and tail together. In total, the fish is 54 cm long. How long is the tail?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 110 (Problem solving exercise 8.2 Solving equations with brackets).

What you need to know

When an equation has a bracket, it is usually easiest to **expand the brackets** first.

$5(3x - 6) = 12$ ← First multiply out the brackets.

$5 \times 3x - 5 \times 6 = 12$

$15x - 30 = 12$ ← Add 30 to both sides.

$15x = 42$ ← Divide both sides by 15.

$x = 2.8$

Alternatively, you can choose to divide both sides of the equation by the number outside the brackets.

$4(n + 13) = 80$ ← Divide both sides by 4.

$n + 13 = 20$ ← Subtract 13 from both sides.

$n = 7$

How to do it

Solving equations with an unknown on one side

Solve these equations.

a $6(5a - 2) = 33$

b $5(4b + 3) = -25$

Solution

a $6(5a - 2) = 33$ ← First multiply out the brackets.

$6 \times 5a - 6 \times 2 = 33$ ← Simplify.

$30a - 12 = 33$ ← Add 12 to both sides.

$30a = 45$ ← Divide both sides by 30.

$a = 1.5$

b $5(4b + 3) = -25$ ← It is easiest to divide both sides by 5 first.

$4b + 3 = -5$ ← Subtract 3 from both sides.

$4b = -8$ ← Divide both sides by 4.

$b = -2$

Solving equations with an unknown on both sides

Solve $2(3 - x) = 9 + x$.

Solution

$2(3 - x) = 9 + x$ ← First multiply out the brackets.

$2 \times 3 - 2 \times x = 9 + x$

$6 - 2x = 9 + x$ ← You can subtract x from both sides or add $2x$ to both sides.

Subtracting x from both sides gives a negative x term so it is more straightforward to add $2x$ to both sides.

$6 = 9 + 3x$ ← Subtract 9 from both sides.

$-3 = 3x$ ← Divide both sides by 3.

$-1 = x$ ← Swap the two sides of the equation over. $-1 = x$ means the same as $x = -1$.

$x = -1$

Check by substituting $x = -1$ back into the original equation.

$2(3 - x) = 9 + x$

$2 \times (3 - -1) = 9 + -1$

$2 \times 4 = 8$ ✓

Learning exercise

① First expand the brackets, then solve these equations.

a $5(2x + 3) = 75$ **b** $3(x + 2) = 33$

c $4(5x - 3) = 18$ **d** $8(5 - x) = 16$

② Solve these equations by dividing both sides by the number outside the brackets first.

a $10(3a + 6) = 180$ **b** $2(5b - 3) = 74$

c $5(4c + 7) = 195$ **d** $4(2d - 9) = 52$

③ **a** Copy and complete the table to build an equation to find the number.

Instruction	Algebra
I think of a number, n.	n
I multiply it by 5.	
I add 6.	
I multiply it by 3.	
The answer is 123.	

b Solve your equation to find the value of n.

c Work through the instructions to check your answer is correct.

④ Solve these equations.

a $4(x + 6) - 3x = 38$ **b** $2(3x - 5) - 3x + 4 = 6$

c $5(2x + 1) - x - 3 = 56$ **d** $4x + 9 - 2(x - 4) = 27$

⑤ Solve these equations.

a $8(x + 3) = 3x - 11$ **b** $2(4x - 2) = 11 - 2x$

c $3(5x - 6) = 4x + 15$ **d** $10(3x - 5) = 5x$

⑥ Solve these equations.

a $7(3x - 4) = 4(2x + 3) - 1$ **b** $6(2x - 5) = 3(x + 2)$

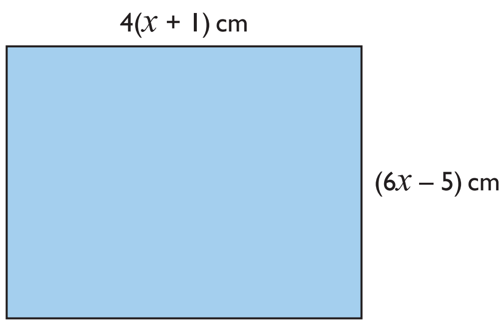
c $5(x + 4) = 3(2x - 8)$ **d** $3(4x + 2) = 4(5x + 2)$

⑦ The perimeter of this rectangle is 28 cm.

a Write down an expression for the perimeter of the rectangle.

b Write down and solve an equation for x.

c Work out the area of the rectangle.



⑧ Isobel thinks of a number, n.

When she subtracts 2 and then multiplies the result by 3, she gets the same answer as when she subtracts her number from 30.

a Write down an expression for 'subtracts 2 and then multiplies the result by 3'. Use n for Isobel's number.

b Write down an equation for n.

c What number is Isobel thinking of?

d Show how you can check your answer is correct.

9 The sum of the ages of Peter and his dad is 42.

a Peter is p years old now.

Copy and complete this table.

	Age now	Age in 4 years
Peter	p	
Dad		

In 4 years' time, Peter's dad will be four times as old as Peter.

b Write down an equation for p.

c Solve the equation.

d How old are Peter and his dad now?

e Show how you can check you answer is correct.

10 **a** Expand the brackets.

i $\frac{1}{2}(4x + 2)$ **ii** $\frac{1}{3}(3x + 6)$

b Solve $\frac{1}{2}(4x + 2) = \frac{1}{3}(3x + 6)$

11 **a** Simplify

i $6 \times \frac{1}{2}(4x + 2)$ **ii** $6 \times \frac{1}{3}(3x + 6)$

b Multiply both sides of the equation $\frac{1}{2}(4x + 2) = \frac{1}{3}(3x + 6)$ by 6.

c Using your answer to part **b**, solve the equation $\frac{1}{2}(4x + 2) = \frac{1}{3}(3x + 6)$.

Problem solving exercise

1 There are 54 people on a coach trip to a theme park.

There are 30 adults on the coach trip.

A child's ticket costs £25 less than an adult's.

The total cost of the tickets is £1560.

How much is a child's ticket for the trip?

2

Louise

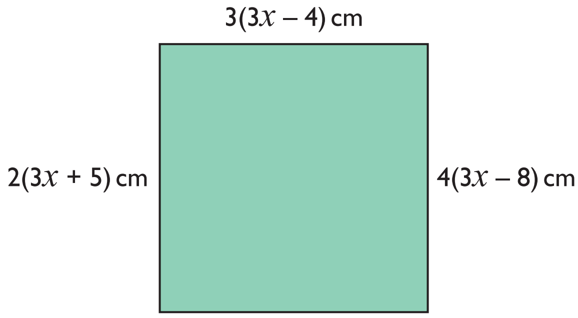
I think of a number and call it n.
I add 12 to the number.
I then multiply by 3.
This gives me the same answer as when I subtract n from 60.

a Write this as an equation for n.

b What number is Louise thinking of?

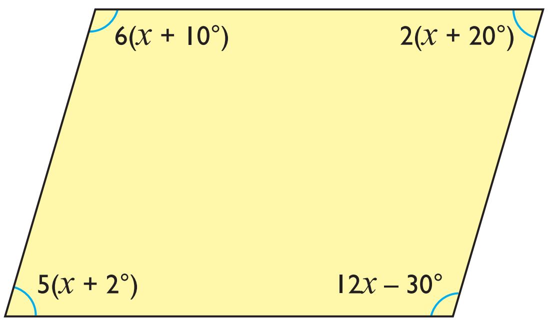
③ Here is a quadrilateral.

Show that the quadrilateral is not a square.



④ Here is a quadrilateral.

Use the given angles to decide whether or not this quadrilateral is a parallelogram.



Do I know it now?

① Solve these equations.

a $6(5a - 1) = 114$

b $3(2b + 4) = 6$

c $5(4c - 3) = 165$

d $7(4d - 5) = 0$

② Solve these equations.

a $3(2x - 5) = 5x - 14$

b $2(4x + 1) = 7x + 4$

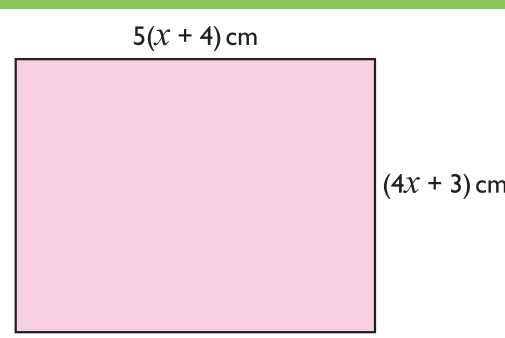
c $5(3x - 4) = 2(5 - x) - x$

d $4(2x - 3) = 5(x - 1) + 2$

Can I apply it now?

① Here is a rectangle.

The perimeter of the rectangle is 136 cm.
What is the area of the rectangle?



CHAPTER 9

ESSENTIAL TOPICS – ALGEBRA

Sequences

JUST IN CASE

What is a sequence?

A number **sequence** is a list of numbers that are in order.

There must be a rule to calculate each successive number.

Each number is referred to as a **term** of the sequence.

By finding the rule for moving from one term to another, you can predict the next or missing terms.

In this sequence, the rule is 'add 2' to get the next term: 11, 13, 15, 17, 19, … .

In this sequence, the rule is 'add 1 to the denominator' to get the next term: $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$.

Generating sequences

Sequences can be generated using either **term-to-term rules** or **position-to-term rules**.

- Term-to-term rules are given in the form of instructions such as 'add 5'.

  If the first term of a sequence is 4, then using the rule 'add 5', you could continue the sequence to get 4, 9, 14, 19, … .

  To define a sequence, you must give the term-to-term rule and the first term.

- Position-to-term rules relate each term to its position in the sequence.
  They are formulae.
  The rule 'each term is twice its position in the sequence' generates the sequence 2, 4, 6, 8, 10, … .

Position (n)	1	2	3	4
Term	2	4	6	8

The letter n is usually used to represent the position.

This rule can be written as nth term $= 2n$.

To calculate any term, for example, the 23rd, substitute the position number, n, into the formula

$$n\text{th term} = 2 \times 23$$
$$= 46.$$

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

What is a sequence?

Write down the missing terms in these sequences.

a $\square$, 8, 16, 32, 64, $\square$, ...

b 4, 1, 4, 1, $\square$, $\square$, ...

c 5, 8, 12, $\square$, $\square$, 30, ...

Defining sequences

Write down the first four terms of these sequences.

a nth term $= 2n + 1$

b nth term $= 3n - 1$

c nth term $= n^2$

d nth term $= n^2 + 1$

Applying the knowledge

1. Work out the next three terms in these sequences.

   a 1160, 580, 290, 145, $\square$, $\square$, $\square$, ...

   b -60, 30, -15, 7.5, $\square$, $\square$, $\square$, ...

   c -128, -192, -288, -432, $\square$, $\square$, $\square$, ...

2. At Samil's bus stop, the 208 bus is due at 10:00 and then every 10 minutes. The 119 bus is due at 9:47 and then every 7 minutes.

   When is the first time after 10:00 that both buses are due at the same time?

9.1 Linear sequences

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① **a** Work out the values of the missing terms of these linear sequences.
b Write down the position-to-term formula for each sequence.

i 16, □, □, 22, □, 26, … **ii** 50, □, □, 35, □, 25, □, …

iii −20, □, −16, −14, □, □, …

If you can do the question above, try this one on problem solving.

② Avonford High School's canteen uses trapezium-shaped tables.

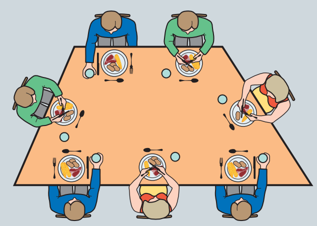
Each table can seat seven people.

The tables are laid out in long lines.

a Draw the next two stages.

b Copy and complete this table.

Number of tables	1	2	3	4	5
Number of seats	7	12			



Stage 1

c Predict how many people can sit at

i six tables **ii** eight tables.

d Find the formula for n tables.

e How many people can sit at a line of 20 tables?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 118 (Problem solving exercise 9.1 Linear sequences).

What you need to know

The sequence 4, 7, 10, 13, … is a **linear sequence**.

This is because the gap between successive terms, known as the **difference**, is always the **same**, in this case 3.

All sequences where the **term-to-term rule** is an addition or subtraction of a constant amount are linear sequences.

The sequence 4, 7, 10, 13, … has a constant difference of 3. This means the sequence is linked to the 3 times table.

The **position-to-term rule** is therefore of the form

nth term $= 3n + c$ where c is a number.

The first term will be

$3 \times 1 + c \quad = 3 + c$ So in this case, $c = 1$

How to do it

Finding the position-to-term formula

For each of the sequences below

- **i** find the next three terms
- **ii** find the position-to-term formula.

a 8, 16, 24, 32, 40, ...

b 27, 25, 23, 21, 19, ...

Solution

a **i** To get the next term, add 8 each time.
So the next three terms are 48, 56, 64.

The difference is 8 so the sequence is related to the 8 times table.

ii Since the sequence is linear, nth term $= 8n$ + a number.
Look at the first term of the sequence.
It is 8 so there is no need to add a number in this case.
So nth term $= 8n$.

b **i** To get the next term, subtract 2 each time.
So the next three terms are 17, 15, 13.

ii Since the sequence is linear,
nth term $= -2n$ + a number.

The difference is -2 so the sequence is related to the -2 times table.

The first term of the sequence is 27.

$n = 1$ for the first term.

So nth term $= -2n + 29$.
Or, more neatly, nth term $= 29 - 2n$.

Generating a sequence using a position-to-term rule

Write down the first five terms of the sequence with this position-to-term rule.

nth term $= 14n + 45$

Solution

Using nth term formula for all terms

When $n = 1$, 1st term $= 14 \times 1 + 45 = 59$

Substitute the term number into the nth term formula.

When $n = 2$, 2nd term $= 14 \times 2 + 45 = 73$

When $n = 3$, 3rd term $= 14 \times 3 + 45 = 87$

When $n = 4$, 4th term $= 14 \times 4 + 45 = 101$

When $n = 5$, 5th term $= 14 \times 5 + 45 = 115$

Using position-to-term rule for first term only

When $n = 1$, 1st term $= 14 \times 1 + 45 = 59$

From the formula, the common difference is 14.

2nd term $= 59 + 14 = 73$

Add 14 to the previous term.

3rd term $= 73 + 14 = 87$

4th term $= 87 + 14 = 101$

5th term $= 101 + 14 = 115$

Learning exercise

① Write down the first five terms of the sequence for each of these position-to-term rules.

a nth term $= 3 \times$ position

b nth term $= 4 \times$ position $+ 2$

c nth term $= 2 \times$ position $+ 25$

② Look at these linear sequences.

A 30, $\square$, $\square$, 54, 62, …

B 84, $\square$, $\square$, 75, 72, …

a Work out the missing terms.

b The 100th term in sequence **A** is 822 and in sequence **B** is –213.

Write down the 101st term for each sequence.

c Copy and complete these position-to-term rules for each sequence.

i **A** $\square \times$ position $+ 22$

ii **B** $-3 \times$ position $+ \square$

③ Here are the position-to-term formulae for three sequences.

a $3n + 1$ **b** $6n - 2$ **c** $4n + 3$

For each sequence:

i Write down the first four terms, and the 100th term.

ii Write down the first term and the difference between consecutive terms.

iii What is the connection between your answer to part **ii** and the formulae for the nth term?

④ Write down the position-to-term formula for each of these sequences.

a 5, 8, 11, 14, … **b** 4, 6, 8, 10, … **c** 5, 9, 13, 17, …

⑤ In each case, use the position-to-term formula to work out the first five terms and the 20th term of the sequence.

a $3n + 6$ **b** $2n + 5$ **c** $7n - 3$

⑥ For each of these sequences

i write down the position-to-term formula

ii work out the 100th term.

a 11, 15, 19, 23, …

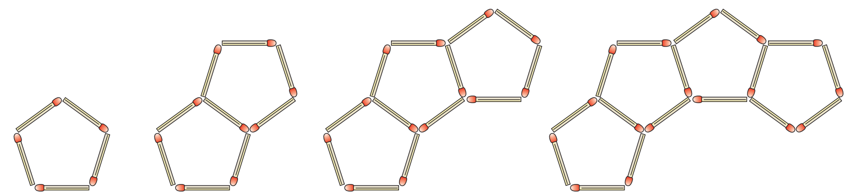
b 2, 12, 22, 32, …

c 11, 18, 25, 32, …

⑦ Match the sequences to their rules.

3, 6, 9, 12, …	$5n + 1$
7, 9, 11, 13, …	$3n$
6, 11, 16, 21, …	$2n + 5$

⑧ Here is a sequence of pentagonal matchstick patterns.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

a Draw the next two patterns in the sequence.

b Copy and complete this table for the first six patterns.

Number of pentagons	1	2	3	4	5	6
Number of matchsticks	5					

c Predict the number of matchsticks for seven pentagons.

Explain how you found your answer.

d Write down the position-to-term formula.

e Predict the number of matchsticks for

i 10 pentagons **ii** 20 pentagons.

f How many pentagons will 101 matchsticks make?

⑨ Here are the first four terms of a sequence.

6, 10, 14, 18

a **i** Write down the next term.

ii Explain how you found your answer.

b Write down the position-to-term formula for this sequence.

c Use your formula to work out the 40th term.

d Which position is the term 350?

⑩ Here are the first four terms of a sequence.

3, 9, 15, 21

a **i** Write down the next term.

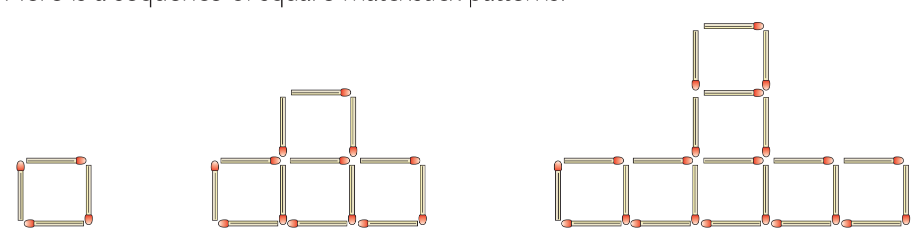
ii Explain how you found your answer.

b Write down the position-to-term formula for this sequence.

c Use your formula to work out the 40th term.

d Which position is the term 267?

⑪ Here is a sequence of square matchstick patterns.



Pattern 1 Pattern 2 Pattern 3

a Draw the next two patterns.

b Copy and complete the table for the first six patterns.

Pattern number	1	2	3	4	5	6
Number of matchsticks	4					

c **i** Predict the number of matchsticks for pattern number seven.

ii Explain how you found your answer.

d Write down the position-to-term formula.

e Predict the number of matchsticks for

i pattern 10 **ii** pattern 20.

f Why is the number of matchsticks for pattern 20 not double that for pattern 10?

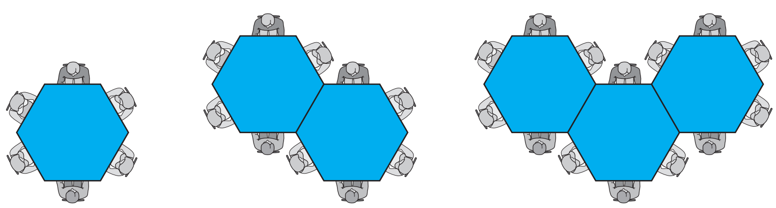
g How many squares will 85 matchsticks make?

Problem solving exercise

① A school uses hexagonal tables in its dining hall.

The tables are always laid out according to this pattern.

One chair is placed at each open edge of each table.



6 chairs 10 chairs 14 chairs

a How many chairs are needed for a pattern with four tables?

b How many chairs are needed to fit around a pattern with n tables?

c 77 students need to be seated on tables and chairs laid out in this pattern.

Show that there will be one empty chair if the least number of tables is used.

② Andy makes a sequence by counting back from 40 in 3s. The sequence starts 40, 37, 34, …

a Write down the next two terms in Andy's sequence.

b State which one of the following is the formula for the nth term of Andy's sequence.

i $40 - 3n$ **ii** $43 - 3n$ **iii** $40 + 3n$ **iv** $43 + 3n$

c Will 2 be a term in this sequence? Explain why or why not.

Do I know it now?

① Here are the first few terms of a sequence.

9, ☐, ☐, ☐, 41, ☐, 57, …

a Work out the term-to-term rule.

b Write down the position-to-term formula for this sequence.

c Find the value of the 50th term.

d Which position is the term 449?

② For each of these sequences

i write down the position-to-term formula

ii work out the 100th term.

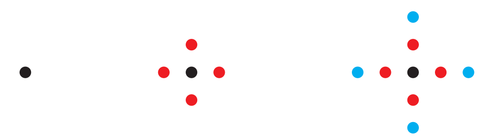
a 80, 76, 72, 68, …

b 100, 95, 90, 85, …

c 60, 58, 56, 54, …

d 80, 86, 92, 98, …

③ Here are the first three of a sequence of cross-shaped patterns.



Pattern 1 Pattern 2 Pattern 3

a How many dots are there in patterns 4 and 5?

b Give a formula for the number of dots, d, in pattern number n.

c How many dots are there in

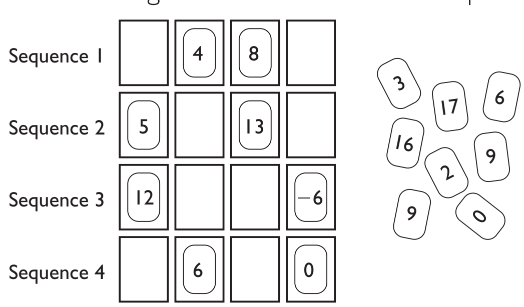
i pattern 90

ii pattern 120?

d Which pattern has 645 dots?

Can I apply it now?

① Maria is using number cards to make sequences.



a Where could she place the remaining cards so that each line is a sequence?

You can use each card once only.

b Write down the term-to-term rule for each of the sequences you make.

c Which sequences are linear?

d Write down the position-to-term rule for each linear sequence.

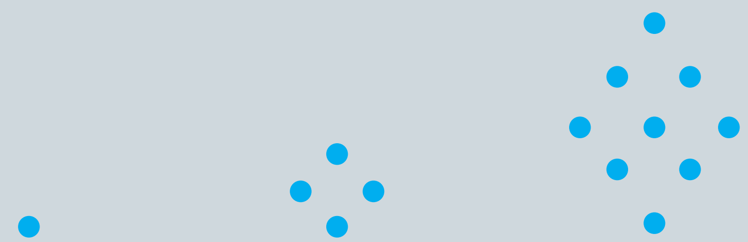
9.2 Special sequences

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Look at this sequence of patterns.



Pattern 1　　Pattern 2　　Pattern 3

a Draw the next two patterns.

b Copy and complete this table.

Pattern	1	2	3	4	5
Number of dots	1	4			

c Write down a formula for the number of dots in the nth pattern.

If you can do the question above, try this one on problem solving.

② Sequence A is the triangular numbers.

Sequence B is the triangular numbers but starting with 0: 0, 1, 3, 6, …

a Write down the first six terms of each sequence.

b Add the first terms of sequences A and B together to make the first term of a new sequence, sequence C. Repeat to form the first six terms of sequence C.

c Describe sequence C.

d Draw patterns of dots to explain your answer to part **c**.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 124 (Problem solving exercise 9.2 Special sequences).

What you need to know

Did you know?

The number of petals on a flower often corresponds to a number in the Fibonacci sequence. The sections of a pine cone are in spirals. The numbers of spirals also correspond to Fibonacci numbers. Similar patterns can be found in sunflower seed heads.

There are some important number sequences that are not linear (i.e. there is not a constant difference between consecutive terms).

- The **triangular numbers**: 1, 3, 6, 10, ...
  (that is, 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4, ...).
  The difference between successive terms increases by one each time.
  The position-to-term formula for the triangular numbers is
  $$n\text{th term} = \frac{n(n+1)}{2}.$$
- The **square numbers**: 1, 4, 9, 16, 25, ...
  (that is $1 \times 1, 2 \times 2, 3 \times 3, 4 \times 4, 5 \times 5, \ldots$).
  The difference between successive terms is 1, 3, 5, 7, 9, ... and the position-to-term formula is nth term $= n^2$.
- The Fibonacci numbers: 1, 1, 2, 3, 5, 8, 13, 21, ... where each term (after the initial two) is the sum of the two previous terms.
  The difference between successive terms is the Fibonacci sequence itself!

Many sequences are variations of these three sequences.

For example,

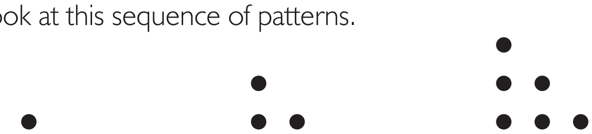
101, 104, 109, 116, ... is the sequence of square numbers plus 100.

3, 12, 27, 48, ... is the sequence of square numbers multiplied by 3.

How to do it

Investigating triangular numbers

Look at this sequence of patterns.



Triangle 1 Triangle 2 Triangle 3

a Draw the next three triangles.

b How many dots are there in each triangle?

c What pattern do you notice in the number sequence?

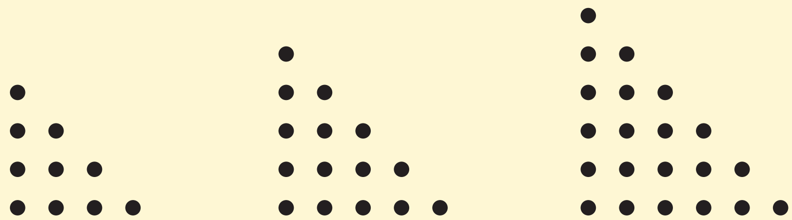
d The formula for the nth term of the sequence is $\frac{n(n+1)}{2}$.

Use this formula to check your answer for triangle 5.

e Work out and write down the 10th triangular number.

Solution

a



Triangle 4 Triangle 5 Triangle 6

b Triangle 1 has 1 dot.
Triangle 2 has 3 dots – 2 dots are added.
Triangle 3 has 6 dots – 3 dots are added.
Triangle 4 has 10 dots – 4 dots are added.
Triangle 5 has 15 dots – 5 dots are added.
Triangle 6 has 21 dots – 6 dots are added.

c The number of dots added increases by one each time.

d For $n = 5$, $\frac{n(n+1)}{2} = \frac{5 \times 6}{2} = 15$. ← Substitute $n = 5$ into the formula.

So the answer for triangle 5 is correct.

e When $n = 10$, $\frac{n(n+1)}{2} = \frac{10 \times 11}{2} = 55$.

So the 10th triangular number is 55.

Learning exercise

① **a** Write down the first five terms of each sequence.

i $n^2 + 10$ **ii** $n^3 - 1$ **iii** $\frac{(n^2 + n)}{2}$

b What name is given to the numbers in **a iii**?

② Write down the position-to-term formula for each sequence.

a 1, 4, 9, 16, 25, … **b** 2, 5, 10, 17, 26, …

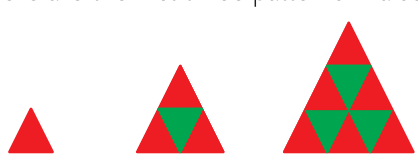
c 0, 3, 8, 15, 24, … **d** 2, 8, 18, 32, 50, …

③ Write down the position-to-term formula for each sequence.

a 11, 14, 19, 26, 35, … **b** 1, 8, 27, 64, 125, …

c 6, 13, 32, 69, 130, … **d** 2, 16, 54, 128, 250, …

④ Here are the first three patterns in a sequence made from triangles.



Pattern 1 Pattern 2 Pattern 3

a Draw pattern number 4.

b Copy and complete the table.

Pattern number	1	2	3	4	5
Number of red triangles	1	3			
Number of green triangles	0	1			
Total number of triangles, T	1				

c What are the names of the sequences in the table?

d **i** Work out the total number of triangles in the 10th pattern.

ii How many of the triangles are red?

iii How many of the triangles are green?

e Write down a formula for the total number of triangles, T, in pattern n.

⑤ Here are the first four patterns in a sequence made from coloured counters.

Pattern 1 Pattern 2 Pattern 3 Pattern 4

a Write down how many counters are added to

i pattern 1 to make pattern 2

ii pattern 2 to make pattern 3

iii pattern 3 to make pattern 4.

b How many counters need to be added to make pattern 5?

c How many counters are in pattern 10?

d How many counters are in the nth pattern?

e Which pattern number has $(1 + 3 + 5 + 7 + 9 + 11 + 13)$ counters?

f What is the sum of the first 100 odd numbers?

⑥ **a** Write down the first five terms of the sequence with this position-to-term formula.

nth term $= n(n + 1)$

b **i** Use your answer to part **a** to write down the position-to-term formula for this sequence.

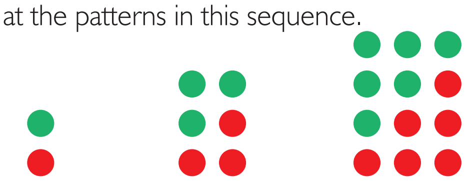
1, 3, 6, 10, 15, …

ii Describe the numbers in this sequence.

iii Draw the first five patterns in a sequence that gives these numbers.

iv Find the value of the 40th term.

c Now look at the patterns in this sequence.



Pattern 1 Pattern 2 Pattern 3

i Draw patterns 4 and 5 of this sequence.

ii The number of circles in pattern 1 is 1×2.

In pattern 2 it is 2×3.

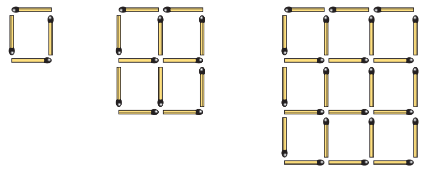
How many circles are there in pattern n?

iii How many circles are red in pattern n?

How many circles are green?

iv How is this connected to part **b**?

⑦ Look at these matchstick patterns.



Pattern 1 Pattern 2 Pattern 3

a Draw pattern number 4.

b Copy and complete the table.

Pattern number	1	2	3	4	5
Number of matchsticks, *M*	4	12			

c Work out the total number of matchsticks in the 8th pattern.

d **i** Write down the first five triangular numbers.

ii How is the sequence for the number of matchsticks related to the triangular numbers?

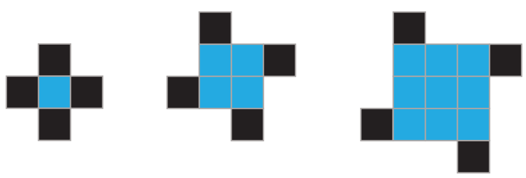
iii The nth triangular number is $\frac{1}{2}n(n + 1)$.

How many matchsticks are in the 20th pattern?

iv Write down a formula for the total number of matchsticks, M.

Problem solving exercise

① Lucy is making tiling patterns.



Pattern 1 Pattern 2 Pattern 3

a Draw pattern number 4.

b Copy and complete the table.

Pattern number	1	2	3	4	5
Number of black tiles	4	4			
Number of blue tiles	1	4			
Total number of tiles, *T*	5				

c Work out the total number of tiles in the 10th pattern.

d Which pattern uses 229 tiles?

e Write down a formula for the total number of tiles, T, in pattern n.

f Lucy has 400 tiles.

i Can she use them all to make a single pattern?

Give a reason for your answer.

ii Write down the pattern number she can make.

iii How many tiles (if any) does she have left over?

② Here is a sequence of Fibonacci numbers.

0, 1, 1, 2, 3, 5, 8, 13, …

a Continue the pattern for all Fibonacci numbers less than 200.

b List the Fibonacci numbers under 200 that are prime numbers.

c Write down the Fibonacci numbers under 200 that can be factorised using other Fibonacci numbers.

Do I know it now?

① Write down the first five terms of each sequence.

a $n^2 + 5$ **b** $2n^2 - 1$ **c** $n^3 + 3$ **d** $n(n - 1)$

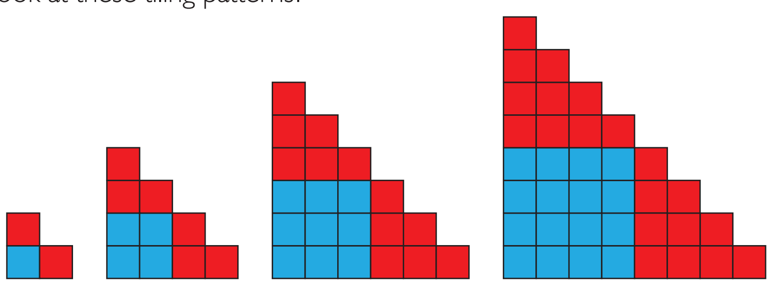
② Write down the position-to-term formula for each sequence.

a 3, 6, 11, 18, 27, … **b** −2, 1, 6, 13, 22, …

c 2, 9, 28, 65, 126, … **d** 3, 12, 27, 48, 75, …

Can I apply it now?

① Look at these tiling patterns.



a Copy and complete the table.

Pattern number	1	2	3	4	5
Number of blue squares					
Number of red squares					
Total number of squares					

b Write down an expression for the number of blue squares in pattern n.

Ben realises that the total number of red squares follows this pattern:

Number of red squares in pattern 1 = 1 × 2 = 2

Number of red squares in pattern 2 = 2 × 3 = 6

Number of red squares in pattern 3 = 3 × 4 = 12

c How many red squares are in pattern 10?

d Write down an expression for the number of red squares in pattern n.

e Ben says that it is impossible for a pattern to have 500 red squares.

Is Ben right? Give a reason for your answer.

f Hence find a rule for the total number of squares. What type of numbers are these?

CHAPTER 10

ESSENTIAL TOPICS – ALGEBRA

Functions and graphs

JUST IN CASE

Real-life graphs

There are some crucial pieces of information you must use to read the story that a graph tells.

- The title explains what the story is about.
- The labels on the axes tell you what the points represent.
- The shape of the graph tells you how the relationship changes.

Look at this graph showing the height of a hot air balloon.

It is called a travel graph or a distance–time graph.

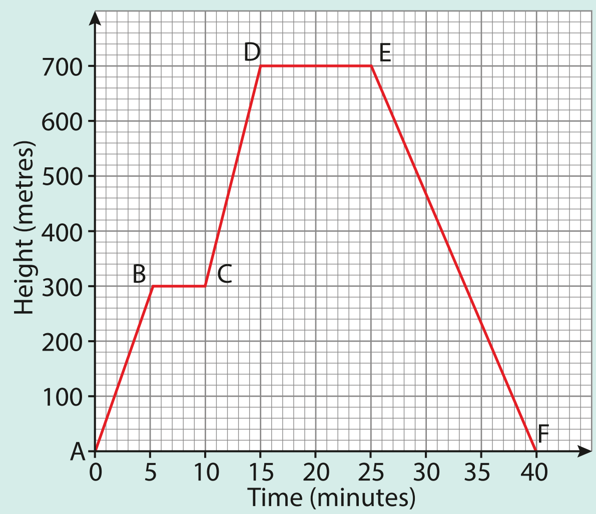
a How high does the balloon go?

b What is shown by the lines BC and DE?

c For how long is the balloon at a height of 300 m?

d How long does the balloon take to land?

e When is the balloon at a height of 500 m?



Solution

a The balloon reaches a maximum height of 700 m.

The maximum height is the largest value of y that the graph reaches.

b The lines BC and DE represent times when the balloon remains at the same height.

The lines BC and DE are horizontal lines.

c The balloon is at 300 m for 5 minutes.

Given by the length of line BC.

d It takes 15 minutes to come down.

The line EF begins 25 minutes into the flight and finishes at 40 minutes.

e It is at a height of 500 m at $12\frac{1}{2}$ minutes on the way up and at 29 minutes on the way down.

Draw a line horizontally from 500 m on the y axis and read off the x value of the point where it crosses the graph.

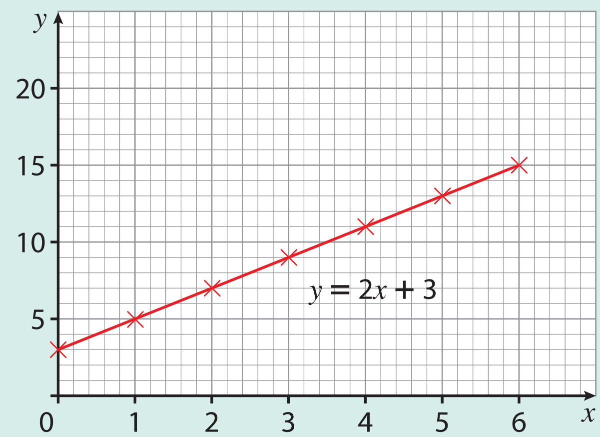
Plotting graphs of linear functions

You can represent a linear function by plotting its graph.

Start by drawing a table of values.

This table is for the function $y = 2x + 3$.

x	1	2	3	4	5	6
$2x$	2	4	6	8	10	12
$+3$	3	3	3	3	3	3
$y = 2x + 3$	5	7	9	11	13	15



A linear function is represented by a straight line graph.

Conversion graphs are an example of linear functions.

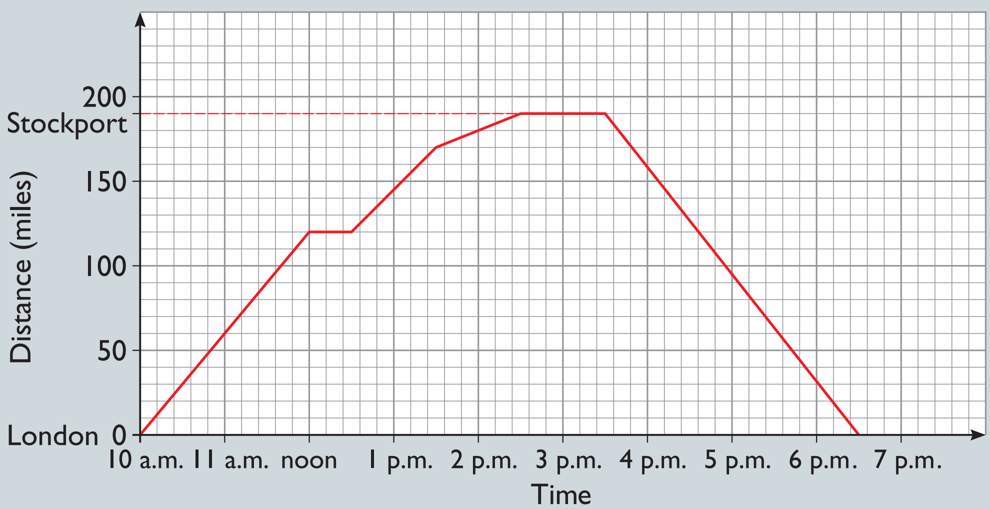
SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Real-life graphs

The graph represents Mr Watson's journey from London to Stockport.



a Mr Watson stopped for a break at a service station.

i At what time did he stop?

ii For how long did he stop?

b He met heavy traffic on one part of his journey.

i For how long was he in heavy traffic?

ii How far did he travel during this time?

c At what time did Mr Watson arrive in Stockport?

d How long did he spend in Stockport?

e **i** What is the distance between Stockport and London?

ii It took Mr Watson 3 hours to drive home. What was his average speed?

➜ Plotting graphs of linear functions

a Copy and complete this table of values for $y = 2x - 1$.

x	1	2	3	4	5	6
$2x$	2	4				
-1	-1	-1				
$y = 2x - 1$	1	3				

b Draw an x axis from 0 to 6 and a y axis from -1 to 12. Plot the points from the table of values and join them with a straight line. Extend the line so that it crosses the y axis.

c Where does the graph of $y = 2x - 1$ cross the y axis?

d Will the point (7, 13) be on the graph of $y = 2x - 1$? How do you know?

➜ Applying the knowledge

① Here are the distances it takes a car to stop when travelling at speeds from 20 mph to 70 mph.

It shows the stopping distances in dry conditions and in wet conditions.

Speed in mph	Stopping distance in feet (dry)	Stopping distance in feet (wet)
20	40	60
30	75	120
40	120	200
50	175	300
60	240	420
70	315	560

a Draw a graph to show the stopping times in dry and wet conditions.

b Find the stopping distance of a car that is travelling at

i 35 mph in dry conditions

ii 45 mph in wet conditions.

② The cost, £y, of x invitation cards to Christina's wedding is given in the table.

Number of invitation cards, x	20	40	60	80	100
Cost in £, y	16	26	36	46	56

a Plot a graph of this information.

b Use your graph to estimate

i the cost of 55 invitation cards

ii how many cards can be bought for £42.

10.1 The equation of a straight line

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Here are the equations of eight lines.

$y = 2x - 3$	$x + y = 5$	$3y = 6x + 10$	$y + 3x = 0$
$y = -3$	$y = 3 - x$	$y = 3x + 5$	$y = 4x - 7$

Choose the equations of the lines which have

a a y-intercept at (0, 0)

b a gradient of 0

c a negative gradient

d a gradient of 3

e a gradient of 2

f a gradient of -3

g the steepest slope

h a y-intercept at (0, 5).

If you can do the question above, try this one on problem solving.

② **a** Draw x and y axes. Take values from -2 to 8 on each axis.

b Plot the line whose gradient is 1 and whose intercept on the y axis is 2.

c Plot the line whose gradient is 2 and whose intercept on the y axis is -2.

d Write down the co-ordinates of the point where these two lines meet.

e **i** Plot the line whose gradient is $\frac{1}{2}$ and whose intercept on the y axis is 4.

ii What do you notice about this line?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 135 (Problem solving exercise 10.1 The equation of a straight line).

What you need to know

Straight lines have an equation of the form $\boldsymbol{y = mx + c}$

- The value of m represents the **gradient** or 'steepness' of the line, or the rate of change of y with respect to x.
- The value of c tells you where the line crosses the y axis (**y-intercept**).

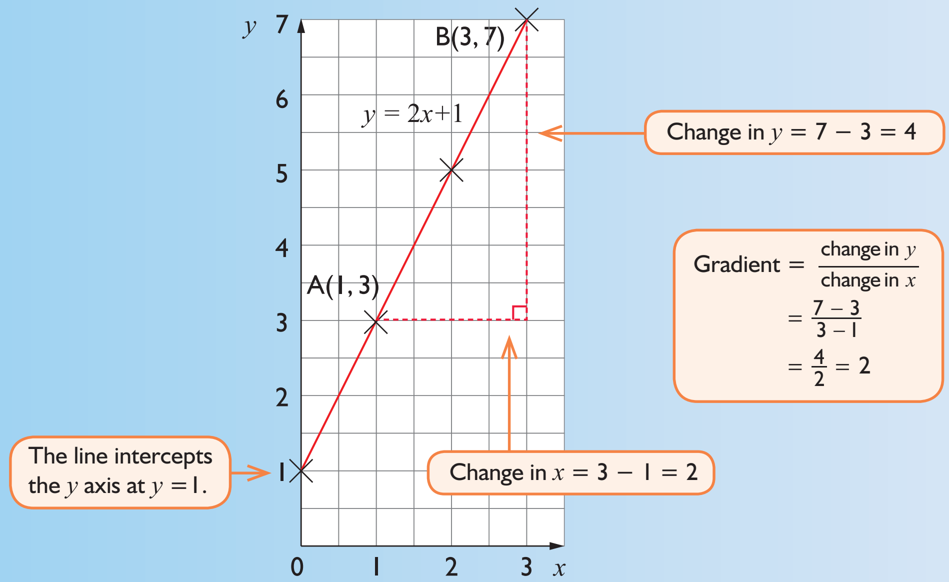
Special cases are:

- Horizontal lines with an equation of the form $y = a$ where a is a number.
- Vertical lines with an equation of the form $x = b$ where b is a number.

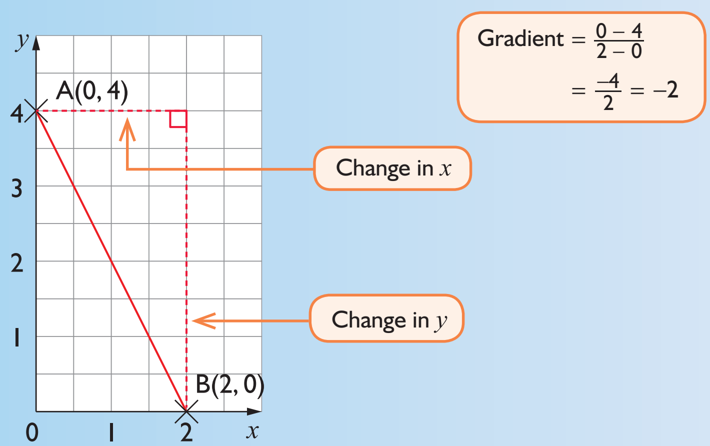
Parallel lines have the same gradient.

To find the gradient of a straight line:

1 Choose any two points.
2 Subtract the y co-ordinates.
3 Subtract the x co-ordinates.
4 The gradient is the change in y divided by the change in x.



If the line slopes down from left to right, the gradient is negative.



How to do it

Finding the gradient from the equation of a line

Here are the equations of ten lines.
Write down the pairs of parallel lines.

a $x = 7$
b $y = 3 + 7x$
c $y = 2x + 3$
d $y = -5$
e $y = 2x - 4$
f $x = -2$
g $y = 5 - x$
h $y = -x$
i $y = 7x + 5$
j $y = 4$

Solution

a and **f** ← Vertical lines

b and **i** ← Gradient is 7

c and **e** ← Gradient is 2

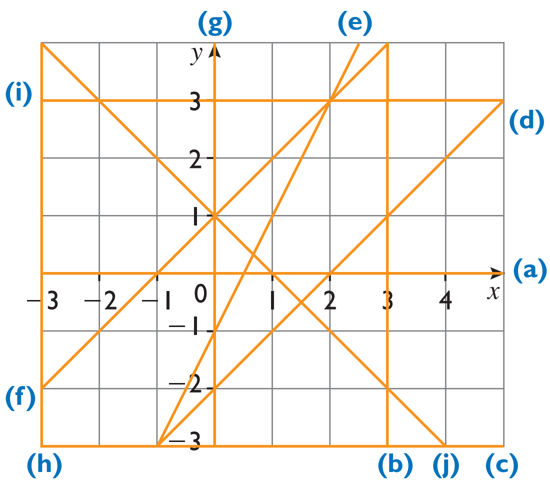
d and **j** ← Horizontal lines

g and **h** ← Gradient is -1

➤ Finding an equation from a graph

Match the lines in this diagram with the correct equations.

$x = 3$	$x = -3$	$x = 0$	$y = 3$	$y = -3$
$y = 0$	$y = x + 1$	$y = x - 2$	$y = 2x - 1$	$y = 1 - x$



Solution

a is the x axis. Its equation is $y = 0$.

b is a vertical line. Every point on the line has an x co-ordinate of 3. The equation of the line is $x = 3$.

c is a horizontal line. Every point on the line has a y co-ordinate of -3. The equation of the line is $y = -3$.

d has a gradient of 1 and a y-intercept of -2. The equation of the line is $y = x - 2$.

e has a gradient of 2 and a y-intercept of -1. The equation of the line is $y = 2x - 1$.

f has a gradient of 1 and a y-intercept of 1. The equation of the line is $y = x + 1$.

g is the y axis. Its equation is $x = 0$.

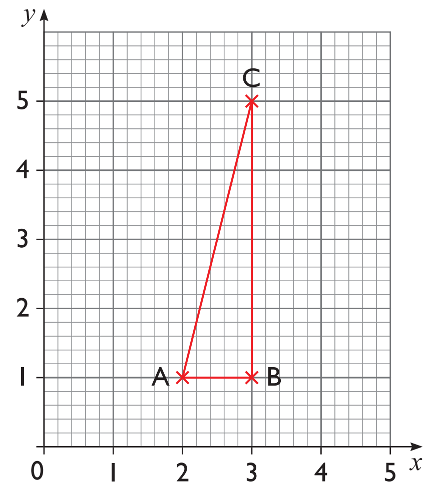
h is a vertical line. Every point on the line has an x co-ordinate of -3. The equation of the line is $x = -3$.

i is a horizontal line. Every point on the line has a y co-ordinate of 3. The equation of the line is $y = 3$.

j has a gradient of -1 and a y-intercept of 1. The equation of the line is $y = -x + 1$ or $y = 1 - x$.

Learning exercise

① Look at this graph.



a Write down the co-ordinates of

i A **ii** B **iii** C.

b Write down the change in

i the y co-ordinate from B to C

ii the x co-ordinate from A to B.

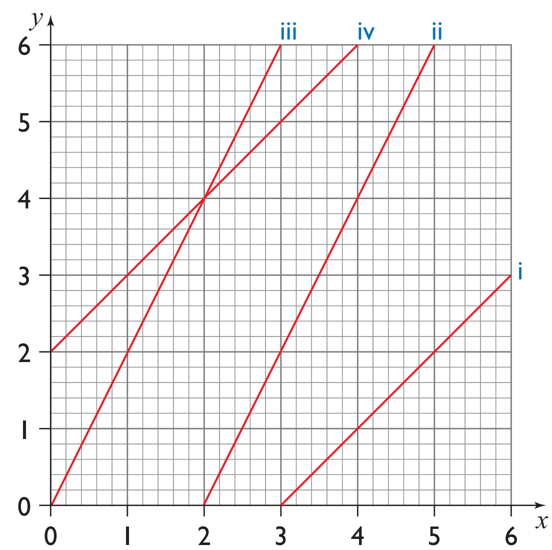
c Work out the gradient of AC.

② **a** Write down the co-ordinates of two points on each of these lines.

b Work out the gradient of each line.

c **i** Which pairs of lines are parallel?

ii What can you say about the gradients of parallel lines?



③ For each part, work out the gradient and the equation of the line that passes through both points.

a (1, 2) and (3, 8)

b (5, 6) and (6, 7)

c (3, 0) and (5, 0)

d (−1, −2) and (1, 6)

e (0, 2) and (2, 0)

f (0, 5) and (3, −1)

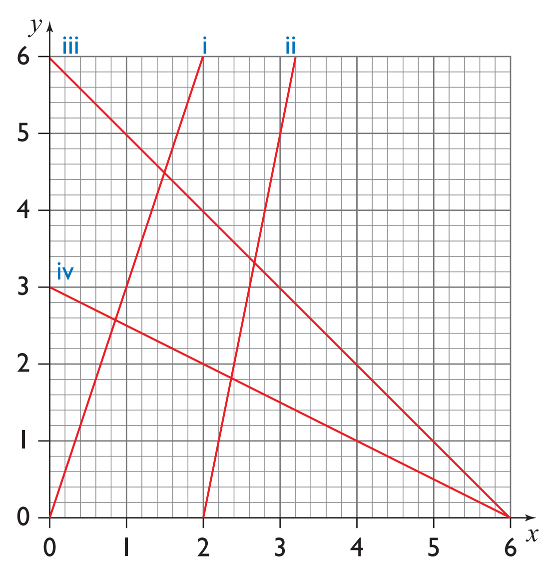
g (0, −3) and (5, −18)

h (0, 8) and (2, −2)

④ **a** Write down the co-ordinates of two points on each of these lines.

b Work out the gradient of each line.

c What can you say about the gradients of lines that slope down from left to right?



5 **a** Find the co-ordinates of two points on the line $y = 2x - 3$.

b Draw the graph of $y = 2x - 3$ taking values of x from -1 to 4.

c Write down

i the gradient of the line

ii the y-intercept (the value of y where it crosses the y axis).

d Where can you see the answers to part **c** in the equation of the line?

6 **a** Copy and complete these tables.

x	-2	-1	0	1	2
$4x$	-8				
-3	-3				
$y = 4x - 3$	-11				

x	-2	-1	0	1	2
$5x$	-10				
$+2$	$+2$				
$y = 5x + 2$	-8				

b Draw the two lines on the same pair of axes.

c Work out the gradient of each line.

d Write down the y-intercept of each line.

e Explain why you did not need to draw the graphs of the two lines to answer parts **c** and **d**.

f Write down the gradient and the y-intercept of the line $y = 8x - 5$.

7 Look at the five lines on this graph.

a For each line

i write down two points

ii work out the gradient

iii write down the y-intercept.

b Match each line with one of these equations.

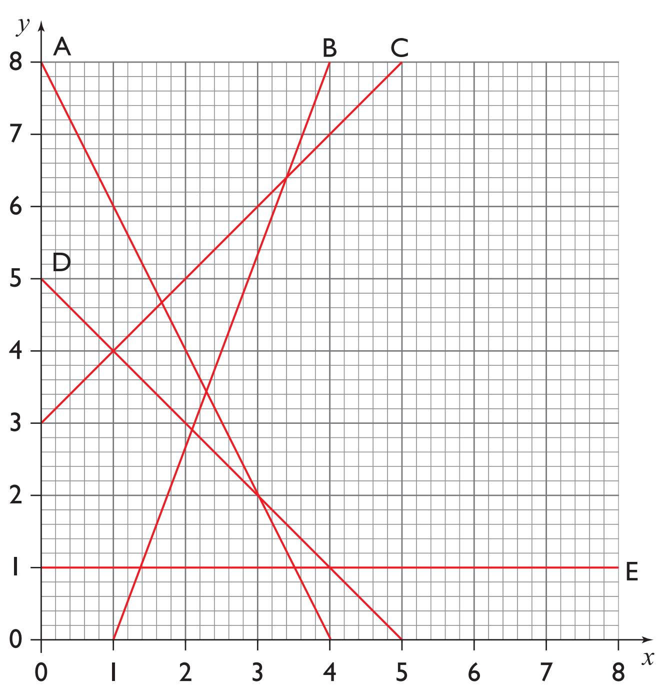
i $y = \frac{8}{3}x - \frac{8}{3}$

ii $y = -x + 5$

iii $y = -2x + 8$

iv $y = x + 3$

v $y = 1$ This is $y = 0x + 1$.



8. Here are the equations of some lines.

A $y = 2x + 7$ **B** $y = -2x + 7$ **C** $y = 7x$

D $y = 7x + 2$ **E** $y = -7x - 2$ **F** $y = 2x - 7$

Which line or lines has each property?

a goes through the origin

b parallel to $y = 2x$

c y-intercept of $+7$

d parallel to $y = -7x + 2$

e y-intercept of -2?

9. **a** Draw the lines with these equations on the same axes.

A $y = x + 5$ **B** $y = 3x - 5$ **C** $y = 5x + 3$

D $y = 3x$ **E** $y = x - 5$ **F** $y = 5x + 5$

b Which lines have the same gradient (are parallel)?

c Which lines have the same y-intercept?

10. Write down the equation of the line that passes through each pair of points.

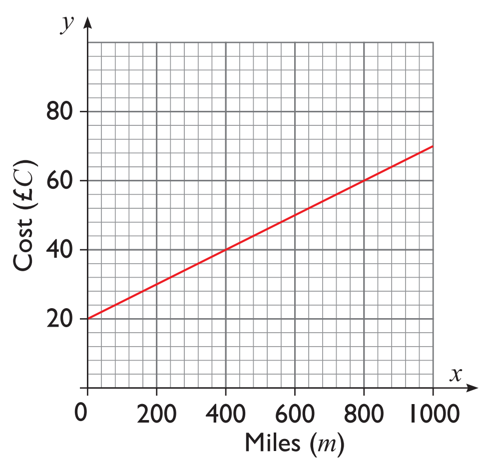
a (0, 0) and (5, 15)

b (0, 3) and (4, 11)

c (0, 1) and (10, 41)

d (0, −2) and (6, 4)

11. This graph shows the charges (£C) that AB Cars make for hiring a car for m miles.



a Write down the formula in the form $C =$ ________.

b Work out the cost for 1200 miles.

c How many miles can you drive a hire car for a cost of £100?

Problem solving exercise

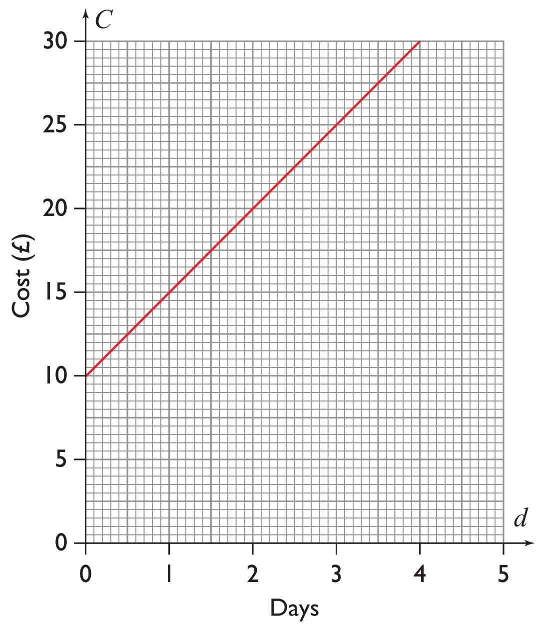
1. Here is the graph that can be used to find the cost C, in pounds, of hiring a hedge-trimmer for d days.

   a Write down the equation of the straight line in terms of C and d.

   b Write in words the cost, C, of hiring this hedge-trimmer for d days.

   c **i** Explain what the gradient of the straight line represents.

   ii Explain what the intercept with the vertical axis represents.



2. Here are the equations of some straight lines.

   A $y = -x + 6$ **B** $y = 2x + 3$ **C** $y = 3 - 2x$

   D $y = -2x + 6$ **E** $y = 2x - 6$

   a Which of these lines are parallel to each other?

   b Which of the lines meet at (0, 3)?

3. Ayesha is comparing the daily cost of van hire. She compares these three firms.

   A £50 plus 10p a mile

   B £25 plus 30p a mile

   C £65 a day with no mileage charge

   a Ayesha writes the cost, £C, for m miles for company A as the formula $C = 50 + 0.1m$.

   Write similar formulae for the other two companies.

   b Draw a graph showing the three formulae.

   Put m on the horizontal axis with values from 0 to 200.

   Put C on the vertical axis.

   c Ayesha thinks she will drive the van about 160 miles.

   Which company should she choose?

Do I know it now?

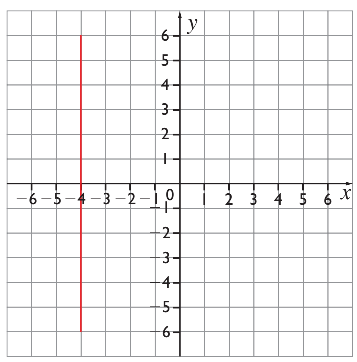
① The table gives information about some lines.

Line	Gradient	y-intercept
A	4	3
B	8	2
C	2	-2
D	-5	4
E	-1	-5
F	6	-3

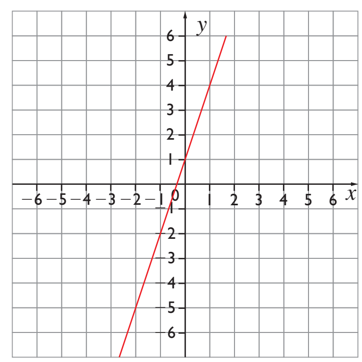
Write down the equation of each line.

② Write down the equation of each line.

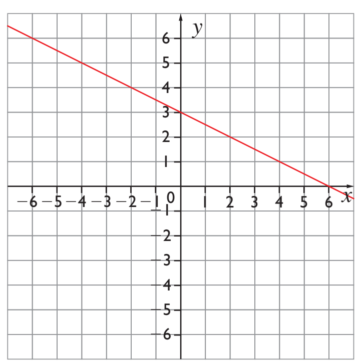
a



b



c



Can I apply it now?

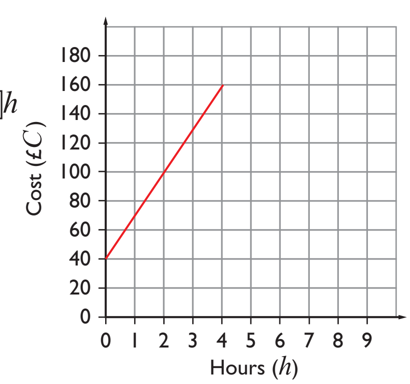
① A plumber has a call out fee and then charges for his time.
The cost, £C, of a job lasting h hours is shown on this graph.

a Write down the plumber's formula in the form $C = \square + \square h$

b **i** What is his call out fee?

ii What is his hourly rate?

c The bill for one job is £250. For how many hours did the plumber work?



10.2 Plotting quadratic and cubic graphs

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① **a** Copy and complete this table of values.

x	-3	-2	-1	0	1	2	3
x^2	9	4					
$-2x$	6	4					
-3	-3	-3					
$y = x^2 - 2x - 3$	12	5					

b Plot the points and join them with a smooth curve.

c What is the minimum value that y takes?

If you can do the question above, try this one on problem solving.

② A stone is thrown out of a window. Its height, h metres, is given by the equation $h = 20 + 15t - 5t^2$.

a Draw the graph of h against t for values of t (in seconds) from 0 to 4.

b Use your graph to estimate the maximum height the stone reaches.

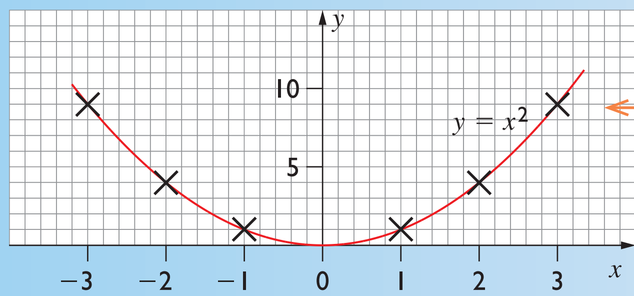
If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 143 (Problem solving exercise 10.2 Plotting quadratic and cubic graphs).

What you need to know

Equations which contain terms with powers of x greater than 1 (e.g. x^2, x^3 or higher powers of x) are not linear functions; instead they produce curved lines.

Equations with x^2 terms but no x^3 or higher terms produce **quadratic curves**.

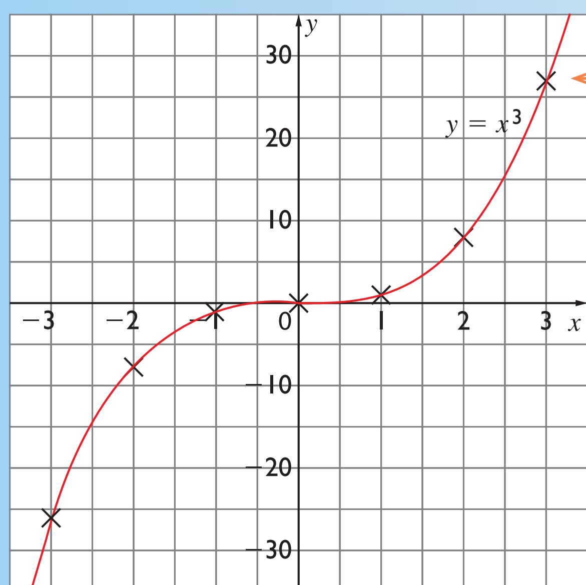
x	−3	−2	−1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9



The points are joined by a smooth curve.

Equations with x^3 terms as the highest power of x produce **cubic curves**.

x	−3	−2	−1	0	1	2	3
$y = x^3$	−27	−8	−1	0	1	8	27



The points are joined by a smooth curve.

How to do it

Drawing and using quadratic curves

a Make a table of values for $y = x^2 - 4x + 1$, taking values of x from -1 to 5.

b Draw the graph of $y = x^2 - 4x + 1$.

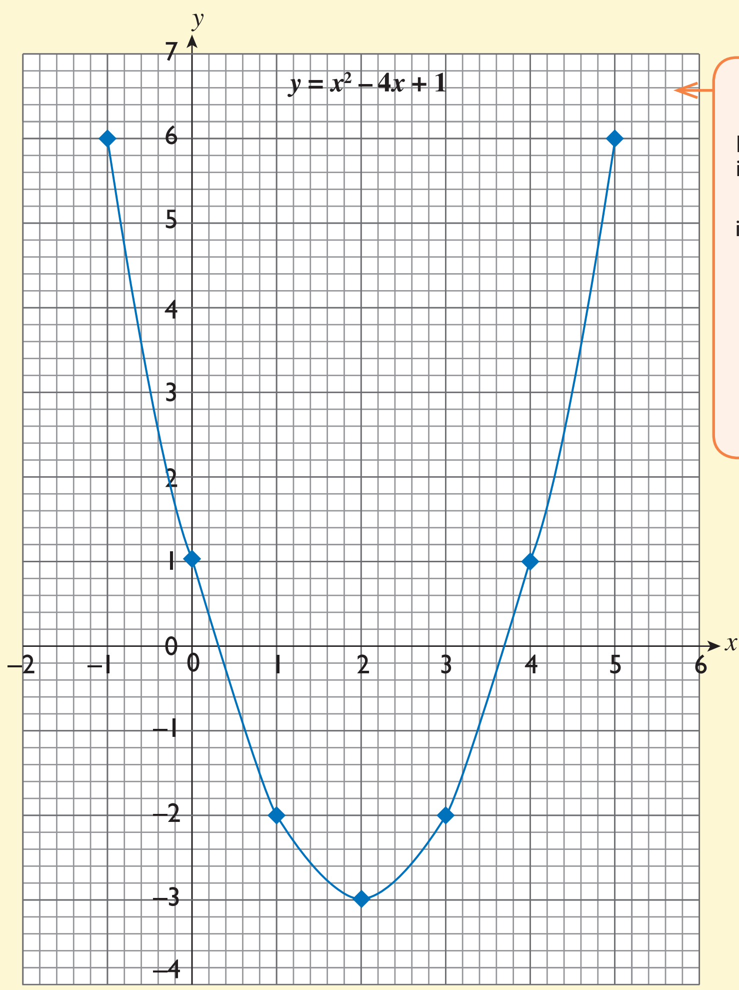
c Solve the equation $x^2 - 4x + 1 = 0$ using your graph.

Solution

a

x	-1	0	1	2	3	4	5
x^2	1	0	1	4	9	16	25
$-4x$	4	0	-4	-8	-12	-16	-20
$+1$	1	1	1	1	1	1	1
$y = x^2 - 4x + 1$	6	1	-2	-3	-2	1	6

b



All quadratic equations produce a curve in the shape of a parabola. Draw it as smoothly as you can. The curve crosses the line $y = 0$ at approximately $x = 0.3$ and $x = 3.7$.

c $x = 0.3$ or $x = 3.7$

The solution to the equation $x^2 - 4x + 1 = 0$ is given by the x values at the points where the curve crosses the x axis (the line $y = 0$). Values taken from the graph are only approximate.

Drawing and using cubic curves

a Make a table of values for $y = x^3 - 5x$, taking values of x from -3 to 3.

b Draw the graph of $y = x^3 - 5x$.

c Find the values of x where this curve crosses the x axis.

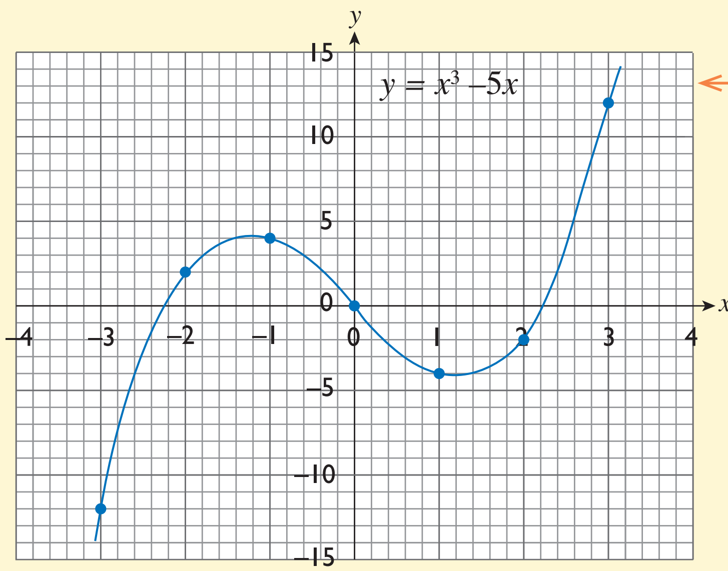
d Write down the solution of the equation $x^3 - 5x = 0$.

Solution

a

x	-3	-2	-1	0	1	2	3
x^3	-27	-8	-1	0	1	8	27
$-5x$	15	10	5	0	-5	-10	-15
$y = x^3 - 5x$	-12	2	4	0	-4	-2	12

b



All cubic equations produce an S-shaped curve. Draw it as smoothly as you can. The curve crosses the line $y = 0$ at $x = 0$ and approximately $x = -2.2$ and $x = 2.2$.

c $x = 0$, $x = -2.2$ or $x = 2.2$

The value $x = 0$ is exact and can be found from the table. The other two values are only approximate.

Learning exercise

① **a** Copy and complete this table for the quadratic curve $y = x^2 + 1$.

x	-3	-2	-1	0	1	2	3
x^2	9						
$+1$	1						
$y = x^2 + 1$	10						

b Draw axes with values of x from -3 to 3 and values of y from 0 to 10. Plot the points from your table and join them with a smooth curve.

c What is the equation of the line of symmetry of the curve?

d What are the co-ordinates of the minimum point of the curve?

e Use your graph to find the values of x when $y = 2.5$.

② **a** Copy and complete this table for $y = 12 - x^2$.

x	−4	−3	−2	−1	0	1	2	3	4
12	12								
$-x^2$	−16								
$y = 12 - x^2$	−4								

b Draw axes with values of x from −4 to 4 and values of y from −5 to 15.

Plot the points from your table and draw the curve.

c What is the equation of the line of symmetry of the curve?

d What are the co-ordinates of the maximum point of the curve?

e Use your graph to find the values of x where the curve crosses the x axis.

③ **a** Copy and complete this table for $y = x^2 - 4x + 2$.

x	−1	0	1	2	3	4	5
x^2							
$-4x$							
+2							
$y = x^2 - 4x + 2$							

b Draw axes with values of x from −1 to 5 and values of y from −3 to 7. Draw the curve.

c What is the equation of the line of symmetry of the curve?

d What are the co-ordinates of the minimum point of the curve?

e What are the values of x when $y = -1$?

④ **a** Copy and complete this table for $y = (x - 2)^2$.

x	−2	−1	0	1	2	3	4	5	6
$x - 2$	−4								
$y = (x - 2)^2$	16								

b Draw axes with values of x from −2 to 6 and values of y from 0 to 16. Draw the curve.

c What is the equation of the line of symmetry?

d What are the co-ordinates of the minimum point of the curve?

e What are the values of x when $y = 7$?

⑤ **a** Copy and complete this table for $y = x^3 + 2$.

x	−3	−2	−1	0	1	2	3
x^3	−27						
+2	+2						
$y = x^3 + 2$	−25						

b Draw axes with values of x from −3 to 3 and values of y from −25 to 30. Draw the curve.

c Use your graph to find the co-ordinates of the points where the curve crosses the x axis and the y axis.

d Describe the symmetry of the graph.

⑥ **a** Draw the graph of $y = x^2 - 4x + 3$, taking values of x from -1 to 5.

b Use your graph to solve the equation $x^2 - 4x + 3 = 0$.

c Use your graph to estimate the values of x when $y = 6$.

d Explain why it is not possible to find a value of x for which $y = -2$.

⑦ **a** Draw the graphs of $y = x^3 - 8$ and $y = -x^3 + 8$ on the same pair of axes, taking values of x from -2 to 3.

b Use your graphs to solve the equation $x^3 - 8 = 0$.

c For what values of x is $x^3 - 8$ greater than $-x^3 + 8$?

d Describe the relationship between the two curves.

⑧ **a** Copy and complete this table of values for the curve $y = x^3 - 6x^2 + 11x - 6$.

x	0	1	2	3	4
x^3					
$-6x^2$					
$+11x$					
-6					
$y = x^3 - 6x^2 + 11x - 6$					

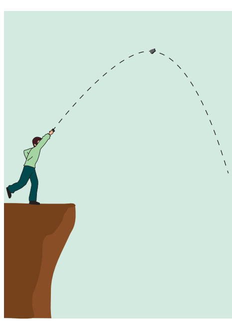
b Draw the curve.

c Use your graph to solve the equation $x^3 - 6x^2 + 11x - 6 = 0$.

d For how many values of x is $y = 1$? Use your graph to find them to 1 decimal place.

e What are the values of x (to 1 decimal place) when $y = 4$?

⑨ Mo stands on top of a cliff 60 m high. He throws a stone into the air.



The equation for the height, y metres, of the stone above the beach is $y = 60 + 20t - 5t^2$, where t is the time in seconds.

a Draw the graph of y against t. Take values of t from 0 to 6 and y from 0 to 100.

b What is the maximum height of the stone above the beach?

c For how long is the stone in the air?

⑩ Use a co-ordinate grid with values of x from -3 to $+3$ with 2 cm intervals and values of y from -15 to 15 with 5 units every 2 cm.

a Draw the graphs of $y = (x - 1)^2 - 4$ and $y = 5x - x^3$.

b Write down the values of x where the two graphs cross. Give your answers to 1 decimal place.

⑪ Alex stands at the edge of a 10 m high cliff and throws a pebble into the sea. The height above sea level of the pebble at time t seconds is modelled by the equation

$$h = 10 + 8t - 5t^2.$$

a Copy and complete this table of values.

t	0	0.5	1	1.5	2	2.5
10						
$+8t$						
$-5t^2$						
$h = 10 + 8t - 5t^2$						

b Draw the graph of h against t.

c What is the maximum height reached by the pebble?

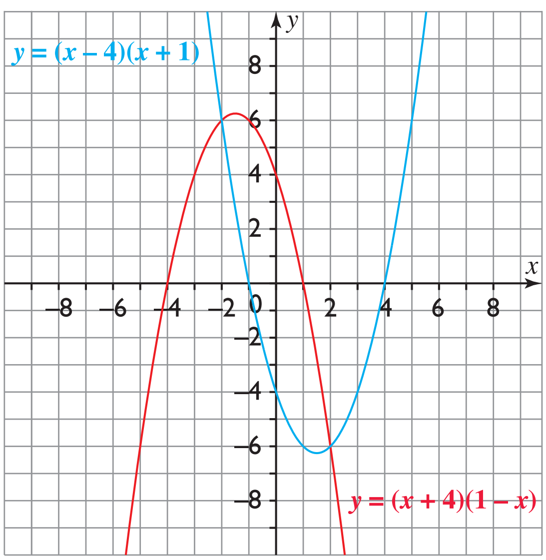
d For how long is the pebble above the height of the cliff?

e **i** When does the pebble hit the sea?

ii Does your graph have any meaning after this time?

Problem solving exercise

① Here are the graphs of $y = (x - 4)(x + 1)$ and $y = (x + 4)(1 - x)$:



a For which values of x is $y = (x + 4)(1 - x)$ always positive?

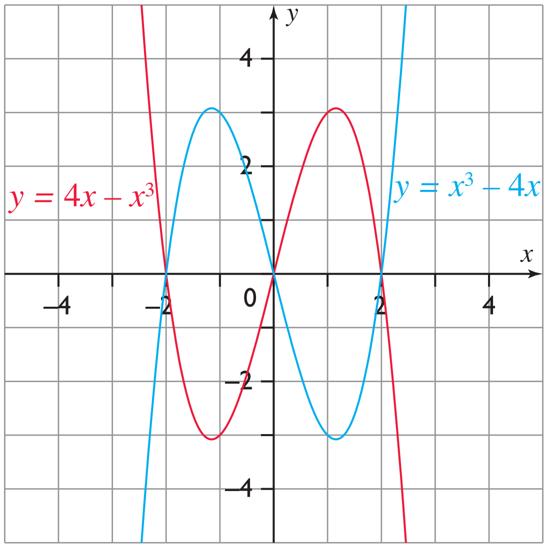
b For which values of x is $y = (x - 4)(x + 1)$ always negative?

c Find the values of x when the blue line is below the red line.

d What is the minimum point of $y = (x - 4)(x + 1)$?

e What is the maximum point of $y = (x + 4)(1 - x)$?

② Here are the graphs of $y = x^3 - 4x$ and $y = 4x - x^3$:



a For which values of x is $y = 4x - x^3$ positive?

b For which values of x is $y = x^3 - 4x$ negative?

c Find the values of x when $4x - x^3$ is greater than $x^3 - 4x$.

d Describe the relationship between the two curves.

e Describe the symmetry of $y = x^3 - 4x$.

③ Janey's company manufactures golf balls. Janey and Pete tested one of the balls by throwing it into the air from the top of the cliff. Pete made a video.

Back at work, they found the height of the ball at 2 second intervals.

Time, t (s)	2	4	6	8
Height, h (m)	40	40	0	−80

a Show that the path of the ball is consistent with the quadratic function $h = 30t - 5t^2$.

b Draw the graph of h against t, for values of t from 0 to 8.

c For how long was the ball more than 25 m above the cliff?

d The ball landed 8 seconds after Janey threw it in the air. How high was the cliff?

④ Narinder makes rectangular table mats. The length of each mat is x cm and its area is A cm^2. Each mat he makes has a perimeter of 80 cm.

a Show that $A = x(40 - x)$.

b Draw the graph of A against x for values of x from 0 to 40.

c Work out the length of a table mat with area 375 cm^2.

d Explain how your graph shows you that the table mat with the greatest area is square-shaped.

⑤ José makes garden ornaments out of concrete. They stand on a base of radius x cm. The volume of concrete needed, V cm^3, is given by the formula

$$V = 2x^2(10 - x).$$

a Draw a graph of V against x, taking values of x from 0 to 10.

b A base needs 200 cm^3 of concrete. Use your graph to estimate its radius.

c One day, José has an order for five ornaments that need a base of radius 6.2 cm. Use your graph to estimate how much concrete he needs.

Do I know it now?

① **a** Copy and complete this table for $y = 5x - x^2$.

x	-1	0	1	2	3	4	5	6
$5x$	-5							
$-x^2$	-1							
$y = 5x - x^2$	-6							

b Draw axes with values of x from -1 to 6 and values of y from -8 to 8. Draw the curve.

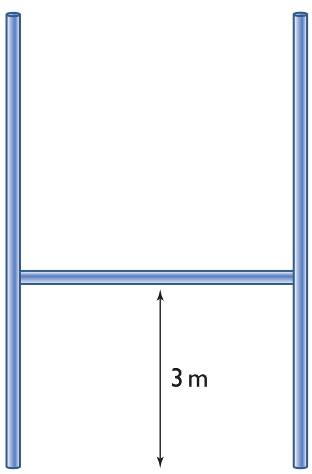
c Describe the main features of the curve.

d Use your graph to solve the equation $5x - x^2 = 0$.

Can I apply it now?

① Fauzia is a junior rugby player. She is learning to take place kicks. The ball must go between the posts and above the bar; the bar is 3 m high.

When the ball has travelled x m horizontally, its height is y m.
For Fauzia's kicks, $y = 0.7x - 0.02x^2$.



a Make a table of values of y, taking x to be 0, 10, 15, 20, 25 and 35.

b Draw the graph of y against x.

c How far from the posts can Fauzia make a successful kick?

Assume she kicks the ball straight so that it goes through the posts.

CHAPTER 11

ESSENTIAL TOPICS – ALGEBRA

Algebraic methods

You will need to know the content of Chapter 8 before you attempt this chapter.

11.1 Linear inequalities

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① Solve these inequalities.

a $3x < 15$

b $2x + 5 \geqslant 17$

c $4x - 6 \leqslant 38$

d $\frac{x}{3} + 4 > 24$

e $2 - 6x < 14$

f $2(x + 4) < 3x + 1$

If you can do the question above, try this one on problem solving.

② The perimeter of this rectangle is at least 18 cm and less than 42 cm. The length is double the width.

a Write down an expression involving w for the perimeter, p.

b Copy and complete this inequality for w.

$\square \leqslant w < 7$

c The width is a whole number of centimetres. Work out the minimum and maximum possible area of the rectangle.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 150 (Problem solving exercise 11.1 Linear inequalities).

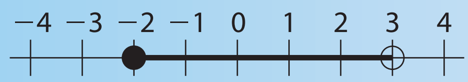
What you need to know

Did you know?

Many manufacturing companies rely on having a minimum amount of various ingredients to fulfil an order. Inequalities are used to express those constraints and solve the problem of how much of the product can be made.

You can use a number line to show an inequality.

This number line represents $-2 \leqslant x < 3$.



The integer (whole number) values of x are -2, -1, 0, 1 and 2.

These numbers are said to 'satisfy' the inequality.

You solve linear inequalities in the same way that you solve equations, but remember:

- **Keep the inequality sign pointing the same way.**

  $x + 6 < 7$ $\quad$ $x + 6 < 7$

  $x < 1$ ✓ $\quad$ $x > 1$ ✗

- **When you multiply or divide by a negative number, turn the inequality sign round.**

  $-3x > 12$ $\quad$ $-4x > 12$ $\quad$ $-\frac{1}{2}x \leqslant 2$

  $x < -4$ ✓ $\quad$ $x > -3$ ✗ $\quad$ $x \geqslant -4$ ✓

How to do it

Solving a word problem with an inequality

An animal shelter has enough kennels to keep 20 dogs.

They never have fewer than 8 dogs.

a Write an inequality for the number of dogs, d, at the shelter.

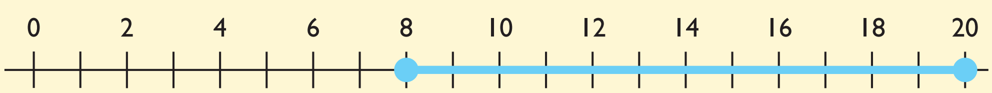
b Show the inequality on a number line.

Solution

a $8 \leqslant d \leqslant 20$ ← Use 'less than or equal to' signs to include 8 and 20.

b On the number line, draw circles at 8 and 20 and join them with a straight line.

d can equal 8 and 20, so fill in both of the circles.



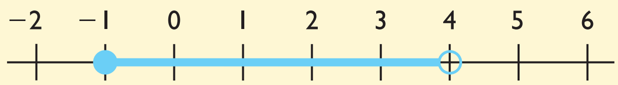
Using a number line

a Show the inequality $-1 \leqslant x < 4$ on a number line.

b Write down the possible integer (whole number) values of x.

Solution

a



x must be less than 4, so leave this circle open.

x can equal -1, so fill in this circle.

b Include -1, but not 4.

The integer values of x are -1, 0, 1, 2 and 3. ← Remember that 0 is a whole number.

➤ Solving inequalities

Solve these inequalities.

a $5(2x + 3) \geqslant 20$

b $8(x + 1) \leqslant 3x + 18$

c $6 - 3x < 9$

Solution

a $5(2x + 3) \geqslant 20$ ← Expand the brackets.

$10x + 15 \geqslant 20$ ← Subtract 15 from both sides.

$10x \geqslant 5$ ← Divide both sides by 10.

$x \geqslant \frac{5}{10}$

$x \geqslant \frac{1}{2}$ ← Choose any number greater than or equal to $\frac{1}{2}$.

Check: Let $x = 1$.

$5(2 \times 1 + 3) \geqslant 20$

$5(2 + 3) \geqslant 20$ ← Calculate the brackets first.

$5 \times 5 \geqslant 20$

$25 \geqslant 20$ ✓

b $8(x + 1) \leqslant 3x + 18$ ← Expand the brackets.

$8x + 8 \leqslant 3x + 18$ ← Subtract $3x$ from both sides.

$5x + 8 \leqslant 18$ ← Subtract 8 from both sides.

$5x \leqslant 10$ ← Divide both sides by 5.

$x \leqslant 2$

Check: Let $x = 2$. ← Choose any number less than or equal to 2.

$8(2 + 1) \leqslant 3 \times 2 + 18$

$8 \times 3 \leqslant 6 + 18$

$24 \leqslant 24$ ✓ ← Notice that $\leqslant$ means less than or equal to so it is true that $24 \leqslant 24$.

Let $x = 1$. ← Choose another number less than or equal to 2.

$8(1 + 1) \leqslant 3 \times 1 + 18$

$8 \times 2 \leqslant 3 + 18$

$16 \leqslant 21$ ✓

c In this inequality there is a negative x term.

There are two methods you can use to deal with this.

Method 1

$6 - 3x < 9$ — Subtract 6 from both sides.

$-3x < 3$ — Divide both sides by -3 and change the direction of the inequality sign.

$x > \frac{3}{-3}$

$x > -1$

Method 2

$6 - 3x < 9$ — Add $3x$ to both sides so the x term becomes positive.

$6 < 9 + 3x$ — Subtract 9 from both sides.

$-3 < 3x$ — Divide both sides by 3.

$\frac{-3}{3} < x$

$-1 < x$ — Turn the inequality around.

$x > -1$

Check: Let $x = 1$. — Choose a value of x that is greater than -1.

$6 - 3 \times 1 < 9$

$6 - 3 < 9$

$3 < 9$ ✓

Learning exercise

① Match each of the following statements to one of the inequalities below.

$x > 5$	$x \leqslant 5$	$x \geqslant 5$	$x < -5$	$x \geqslant -5$

a x is bigger than 5.

b x is greater than or equal to 5.

c x is at most 5.

d x is at least 5.

e x is lower than -5.

f The lowest x can be is -5.

g x has a minimum value of 5.

h x has a maximum value of 5.

② Write down the inequalities shown on these number lines.

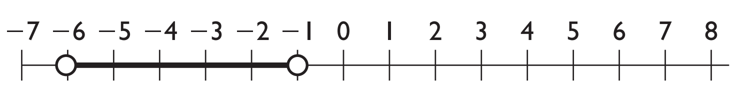
a

b

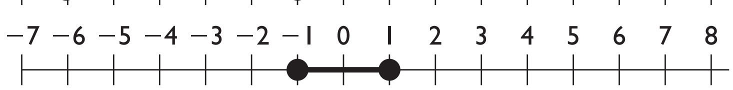
c

d

e



f



③ Show each inequality on a number line.

a $2 < x < 9$ **b** $6 \leqslant x < 8$ **c** $-4 < x \leqslant -1$ **d** $-2 \leqslant x \leqslant 5$

④ Solve these inequalities.

a $x + 5 < 8$ **b** $x - 2 > -5$ **c** $4x < 20$

d $3x \geqslant -18$ **e** $2x + 1 > 9$ **f** $3x - 5 < 22$

⑤ For each inequality, list all the possible whole-number values of x.

a $2 < x < 7$ **b** $-3 < x < -1$ **c** $1 \leqslant x \leqslant 4$ **d** $-5 \leqslant x < 2$

⑥ Solve these inequalities.

a $5x - 3 \geqslant 2x + 9$ **b** $24 + 2x < 30 - 4x$

c $7(x - 3) < 5(x + 6)$ **d** $3(x + 2) + 2(x + 1) \geqslant 4(x + 5)$

⑦ Solve these inequalities.

a $4x < 12$ **b** $-4x < 12$ **c** $4x < -12$ **d** $-4x < -12$

⑧ Trish and Wendy are solving the inequality $-2x < 8$.

Trish says the answer is $x < -4$.

Wendy says the answer is $x > -4$.

Who is correct? Explain why.

⑨ **a** **i** Draw the line $x = 3$ on graph paper. **ii** Shade the region $x < 3$.

b Draw graphs to show these regions.

i $x > -3$ **ii** $y > 4$

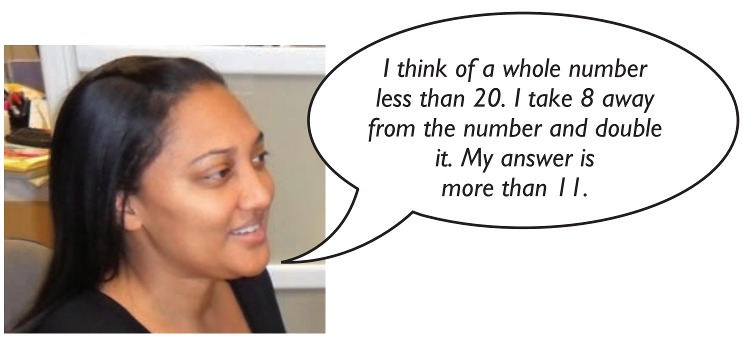
⑩ Solve these inequalities.

a $4 - 3x > 19$ **b** $2 - x < -5$ **c** $6 - 4x \leqslant 12$

d $5(1 - 2x) \geqslant 15$ **e** $2x + 7 < 5x - 5$ **f** $4 + \frac{x}{2} > 7$

Problem solving exercise

①



Aimee

a Write this information as an inequality.

b Which numbers could Aimee have thought of?

2 Sue is x years old. She is over 2.

Ben is twice as old as Sue.

Ceri is 4 years older than Sue.

The total of their three ages is less than 28.

a Write this information as an inequality.

b What age could Sue be?

c What are the corresponding ages of Ben and Ceri?

3 Here is the cost of hiring a van from two companies.

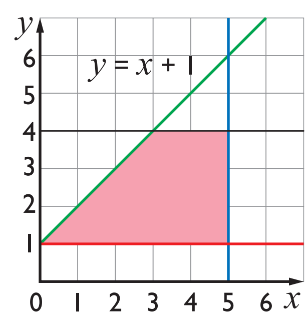
Vans 2 go	**Vans r Us**
£80 fixed cost + 50p a mile	£1 a mile No other charges

Dave wants to drive m miles. Vans 2 go will cost him less.

a Write down an inequality for m.

b Solve it to find m.

4 In this diagram, the equation of the green line is $y = x + 1$.



a Write down the equation for

i the blue line

ii the red line

iii the purple line.

b Write down the inequalities that fully describe the shaded region. The boundary lines are included in the region.

Do I know it now?

1 Solve these inequalities.

a $3x + 2 < 14$

b $2x - 5 > 17$

c $4x - 3 \leqslant 11 - 3x$

d $2 - 12x \geqslant 16 - 19x$

2 Solve these inequalities.

a $2(x + 3) - 5 > 3(2 - x)$

b $x + 5 < 3x + 9$

c $3(x - 8) < 5x - 28$

d $-12x - 3 > -8x - 23$

Can I apply it now?

① Abbi is a years old.

Abbi is 5 years older than Cathy.

Bobbi is twice as old as Abbi.

The total of their ages is less than 35.

a Write this information as an inequality.

b Solve the inequality.

c What is Bobbi's greatest possible age?

Give your answer as a whole number of years.

11.2 Solving pairs of equations by substitution

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Solve each pair of simultaneous equations by substitution.

a $y = x$

$x + 4y = 25$

b $a = 5b$

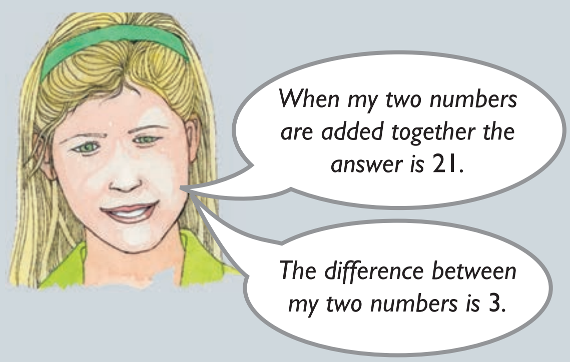
$2a + 3b = 52$

c $5d - 2e + 14 = 0$

$5e = d + 12$

If you can do the question above, try this one on problem solving.

② Kate is thinking of two numbers. Call them m and n.



a Write down two equations for m and n.

b Solve your equations to find the two numbers.

c How can you check that your answer is correct?

d Are there other possible answers for m and n?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 156 (Problem solving exercise 11.2 Solving pairs of equations by substitution).

What you need to know

You can use the substitution method to solve this pair of simultaneous equations.

$y = 6 - x$ ①

$2x + y = 10$ ②

- Substitute $y = 6 - x$ ① into equation ②:

$2x + 6 - x = 10$ ← Solve to find the value of x.

$x + 6 = 10$

$x = 4$

- Make sure you find the value of both unknowns by substituting the found value into the other equation, in this case equation ①:

$y = 6 - 4$

so $y = 2$

- Substitute the values back into equation ② to check your answer.

$2 \times 4 + 2 = 10$ ✓

How to do it

Solving simultaneous equations using the substitution method

Use the substitution method to solve this pair of simultaneous equations.

$y = 3x - 8$

$y = 12 - x$

Solution

$y = 3x - 8$ ① ← As both equations equal y, they are equal to each other.

$y = 12 - x$ ②

$3x - 8 = 12 - x$ ← Add x to both sides.

$4x - 8 = 12$ ← Add 8 to both sides.

$4x = 20$ ← Divide both sides by 4.

$x = 5$

$y = 3x - 8$ ① ← Substitute the value of x you have just found into either equation to find the value of y.

$y = 3 \times 5 - 8$

$y = 15 - 8$

$y = 7$

So the solution is $x = 5$, $y = 7$. ← You need the values of both x and y for the solution.

Check: $y = 12 - x$ ② ← Check your solution by substituting into the equation you didn't use to find the value of y.

$7 = 12 - 5$

$7 = 7$ ✓

➤ Solving a word problem involving simultaneous equations

At the local shop, birthday cards cost five times as much as postcards.
John buys three birthday cards and six postcards.
The shopkeeper charges him £8.40.
How much does each birthday card and postcard cost?

Solution

Let b stand for the cost of a birthday card in pence.

Let c stand for the cost of a postcard in pence.

$$b = 5c \quad (1)$$

Birthday cards cost five times as much as postcards.

$$3b + 6c = 840 \quad (2)$$

Three birthday cards and six postcards cost 840 pence.

$$3 \times 5c + 6c = 840$$

Substitute the value of b ($5c$) in equation (1) into equation (2).

$$15c + 6c = 840$$

Solve the equation to find c.

$$21c = 840$$

$$c = 40$$

$$b = 5c \quad (1)$$

$$b = 5 \times 40$$

Substitute the value you found for c into equation (1) to find b.

$$= 200$$

Check: $3b + 6c = 840$ (2)

$$3 \times 200 + 6 \times 40 = 600 + 240$$

$$= 840 \checkmark$$

So a birthday card costs £2 and a postcard costs 40 pence.

Write 200 pence as £2 in your solution.

Learning exercise

(1) Solve each pair of simultaneous equations by substitution.

a $x = 2y$
$x + 3y = 15$

b $x = 3y$
$2x + y = 28$

c $x = 2y - 1$
$2x + 3y = 12$

d $y = 3x + 5$
$2x + 5y = 8$

(2) Solve each pair of simultaneous equations by substitution.

a $y = x - 2$
$2x - 3y = 8$

b $x = 3y - 2$
$3x - 2y = 15$

c $x = 5y + 3$
$2x = 3y - 1$

d $4x - 5y = 3$
$x = 2y - 3$

③ Solve each pair of simultaneous equations by substitution.

a $5x + 2y = 17$

$y = 2x - 5$

b $4x - y = -9$

$y = 2x + 3$

c $x = y - 6$

$x + y = 14$

d $y = 8x + 1$

$x + y = 10$

④ Solve each pair of simultaneous equations by substitution.

a $x = 3y + 2$

$4x - 5y = 22$

b $2y = 3x + 14$

$x = 5y + 4$

c $x = y + 1$

$2x - 5 = 3y - 6$

d $y = 5x - 6$

$3x - 2y = -2$

⑤ One number, x, is twice another number, y. The two numbers add up to give 15.

a Write two equations for x and y.

b Solve them to find the two numbers.

⑥ The sum of two numbers is 6 and the difference is 1.

Write two equations and solve them to find the two numbers.

⑦ A hardback book costs £h and a paperback book costs £p.

Hardback books are twice the price of paperback books.

Jo buys three hardback books and four paperback books, and the total cost is £35.

a Write this information as two equations for h and p.

b Solve the equations simultaneously.

c Work out the cost of one hardback book and five paperback books.

⑧ At a pet shop, a rat costs £r and a mouse costs £m.

A rat costs £2 more than a mouse.

Billy buys three rats and five mice for £62.

a Write two equations for r and m.

b Solve the equations simultaneously to work out the cost of a rat and the cost of a mouse.

⑨ In a theatre, a seat in the stalls costs £7 more than a seat in the circle.

Patrick buys four tickets for seats in the stalls and seven tickets for seats in the circle, and the total cost is £523.

Write two equations and solve them to find the costs of the two types of ticket.

⑩ Mrs Jones organises a school trip to the theatre for 42 people.

She takes x children and y adults on the trip.

Each adult paid £40 for their ticket.

Each child paid £16 for their ticket.

The total cost of the tickets was £1080.

a Write two equations for x and y.

b How many adults went on the trip? How many children went on the trip?

Problem solving exercise

1 Sadie thinks of two numbers, m and n.

Sadie

a Write this information as two equations for m and n.

b Solve the equations.

What are the values of m and n?

2 Gerry organises a coach trip to the theatre.

There are a adults and c children.

There are five more children than adults.

a Write this as an equation for a and c.

The cost of an adult ticket is £15 and the cost of a child's ticket is £10.

Gerry spends £550 on tickets.

b Write another equation for a and c.

c How many adults went on the trip and how many children?

3 A large tin of beans weighs l g and a small tin weighs s g.

A large tin of beans weighs 100 g more than a small tin of beans.

Six large tins of beans weigh the same as ten small tins of beans.

a Write this information as two equations for l and s.

b Solve the equations.

c Work out the total mass of two large tins and five small tins.

Do I know it now?

1 Solve these simultaneous equations by substitution.

a $x = y + 6$
$x + y = 12$

b $y = 3x - 4$
$2x + 3y = 32$

c $x = 4y + 3$
$3x - 5y = -5$

d $y = 6x + 2$
$6x - 2y = 14$

2 Solve these simultaneous equations by substitution.

a $4x - 2y = -12$
$x = 3y + 2$

b $y = 2x$
$3x - 5 = 39 - 4y$

c $x = 5y + 9$
$3x + y = 11$

d $y = x - 2$
$2y + 3x = 1$

Can I apply it now?

① A train has f first-class coaches and s standard-class coaches.

The number of standard-class coaches is three times the number of first-class coaches.

A first-class coach seats 40 passengers; a standard-class coach seats 80 passengers.

The train seats 560 passengers.

a Use this information to write two equations for f and s.

b Solve the equations.

c How many coaches does the train have?

11.3 Solving simultaneous equations by elimination

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Solve each pair of simultaneous equations.

a $3x - 2y = -17$
$5x + 2y = -7$

b $2x + 3y = 19$
$3x + 2y = 16$

c $2x + 3y = 21$
$4x - 2y = 2$

d $2a - 3b = 2$
$3a - 2b = 8$

If you can do the question above, try this one on problem solving.

② Meena buys six mugs and four plates for £36.

Susan buys four mugs and six plates for £34.

Write and then solve a pair of equations to work out the cost of one mug and the cost of one plate.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 161 (Problem solving exercise 11.3 Solving simultaneous equations by elimination).

What you need to know

A pair of simultaneous equations can be solved by eliminating one of the unknowns.

You either add or subtract the equations to form a third equation with just one unknown.

- To decide whether to add or subtract, look at the signs of the unknown you wish to eliminate.
  If the signs are **different**, you **add**.
  If the signs are **the same**, you **subtract**.
- Sometimes you need to multiply each term in one (or both) equations by a constant so that one of the unknowns has the same coefficients.

When you have found one unknown, substitute it in either equation to find the other.

To check your answer, substitute the values you have found into both equations.

How to do it

Subtracting simultaneous equations

Solve this pair of simultaneous equations.

$5x + 3y = 18$
$5x - 2y = 13$

Solution

Both equations have $5x$, so you can eliminate x.

$5x + 3y = 18$ ①

Subtract $5x - 2y = 13$ ②

The signs in front of both $5x$ are the same $(+5x)$ so you subtract.

$0 + 5y = 5$

Divide both sides by 5.

$y = 1$

Alternatively,

When subtracting one equation from another, you may find it helps to change the signs in the second equation and then add.

$5x + 3y = 18$ ①

Add $-5x + 2y = -13$ ②

$0 + 5y = 5$

Divide both sides by 5.

$y = 1$

$5x + 3y = 18$ ①

To find the value of x, substitute $y = 1$ into equation ①.

$5x + 3 \times 1 = 18$

$5x + 3 = 18$

Subtract 3 from both sides.

$5x = 15$

Divide both sides by 5.

$x = 3$

So the solution is $x = 3$ and $y = 1$.

Check: $5x + 3y = 18$ ①
$5 \times 3 + 3 \times 1 = 18$
$15 + 3 = 18$
$18 = 18$ ✓

$5x - 2y = 13$ ②
$5 \times 3 - 2 \times 1 = 13$
$15 - 2 = 13$
$13 = 13$ ✓

➤ Multiplying simultaneous equations by a constant

Solve this pair of simultaneous equations.

$3s + 2t = 13$
$5s - t = 13$

Solution

$3s + 2t = 13$ ①
$5s - t = 13$ ②

① $3s + 2t = 13$
② × 2 $10s - 2t = 26$ ← Multiply equation ② by 2 so that you can eliminate t.

Add $13s + 0 = 39$

$s = \frac{39}{13}$ ← Divide both sides by 13.

$s = 3$

The signs in front of the $2t$ are different ($+2t$ and $-2t$) so you add.

① $3s + 2t = 13$

$3 \times 3 + 2t = 13$ ← To find the value of t, substitute $s = 3$ into equation ①.

$9 + 2t = 13$ ← Subtract 9 from both sides.

$2t = 4$ ← Divide both sides by 2.

$t = 2$

So the solution is $s = 3$ and $t = 2$.

Check: $3s + 2t = 13$ ①
$3 \times 3 + 2 \times 2 = 13$
$9 + 4 = 13$
$13 = 13$ ✓

$5s - t = 13$ ②
$5 \times 3 - 2 = 13$
$15 - 2 = 13$
$13 = 13$ ✓

➤ Solving a word problem using simultaneous equations

Danni has £4.40 in 2p and 5p coins.
She has 100 coins altogether.
Write down two equations for this information.
How many of each type of coin does she have?

Solution

Let t = number of 2p coins
and f = number of 5p coins. ← Choose letters to represent the unknowns.

$t + f = 100$ ① ← There are 100 coins altogether.

$2t + 5f = 440$ ② ← Danni has 440 pence altogether. This matches the 2 and 5 which are in pence.

Make the coefficients of one of the unknowns the same in both equations.

5 × ① $5t + 5f = 500$
$2t + 5f = 440$ ② ← The signs on $5f$ are positive in both equations, so subtract.

Subtract $3t + 0 = 60$

$t = \frac{60}{3}$ ← Divide both sides by 3.

$t = 20$

① $t + f = 100$

$20 + f = 100$

$f = 80$

To find the value of f, substitute $t = 20$ into equation ①.

So Danni has 20 two-pence coins and 80 five-pence coins.

Check: $t + f = 100$ ①

$20 + 80 = 100$

$100 = 100$ ✓

$2t + 5f = 440$ ②

$2 \times 20 + 5 \times 80 = 440$

$40 + 400 = 440$ ✓

Learning exercise

1. Solve each pair of simultaneous equations by adding them first.

   a $x + 2y = 0$, $x - 2y = 4$
   b $3x + y = 18$, $2x - y = 7$
   c $x - 5y = 2$, $3x + 5y = -14$
   d $6x + 2y = 6$, $x - 2y = 8$
   e $-x + y = 1$, $x + y = -5$
   f $-2x + 3y = 10$, $2x + y = 6$

2. Solve each pair of simultaneous equations by subtracting them first.

   a $2x - 3y = 5$, $2x + 4y = 12$
   b $x + 4y = 1$, $x + 2y = 3$
   c $3x + 4y = 30$, $x + 4y = 26$
   d $2x - 5y = 22$, $3x - 5y = 23$
   e $5x - y = 12$, $3x - y = 8$
   f $x + y = 2$, $3x + y = 12$

3. Solve these simultaneous equations. Decide for yourself whether to add them or subtract them.

   a $3x + y = 7$, $2x - y = 8$
   b $4x - 2y = 14$, $x - 2y = 11$
   c $3x + 4y = 32$, $3x - 2y = 2$
   d $-5x + 3y = 21$, $5x - 2y = -19$
   e $6x - y = 21$, $3x + y = 15$
   f $2x - 4y = 16$, $3x + 4y = 14$

4. Here are two simultaneous equations.

   $3x + 4y = 18$ equation 1

   $5x - 2y = 4$ equation 2

   a Explain why you cannot eliminate either variable just by adding or subtracting.

   b Which equation should be multiplied, and by which value, to ensure you can eliminate a variable?

   c Solve the equations.

5. Solve each pair of simultaneous equations by multiplying one equation first.

   a $4x - y = 19$, $2x + 3y = -1$
   b $3x + 4y = 7$, $x + 2y = 1$
   c $5x - y = -20$, $4x + 3y = 3$
   d $6x - 3y = 51$, $3x + 4y = 20$
   e $x - 5y = 31$, $4x - 2y = 16$
   f $3x - 2y = -2$, $2x - 4y = 12$

⑥ Solve these simultaneous equations.

a $4a + 3b = 18$
$3a + 4b = 17$

b $3x + 2y = 20$
$2x + 5y = 17$

c $3p + 5q = -5$
$2p - 3q = 22$

d $7x - 4y = 39$
$3x + 5y = 10$

e $6x - 5y = 38$
$5x - 3y = 27$

f $-4m + 6n = -4$
$3m + 2n = 29$

⑦ In one week, Trevor made five journeys to the supermarket and three journeys to the park. He travelled a total of 99 kilometres.

During the next week, he made two journeys to the supermarket and four to the park. He travelled a total of 62 kilometres.

a Write down two simultaneous equations. Use s for the distance to the supermarket and p for the distance to the park.

b Solve your equations.

c How far does Trevor travel in a week when he goes seven times to the park and twice to the supermarket?

⑧ The price of tickets for a zoo are £a for adults and £c for children.

Rose buys two adult tickets and three children's tickets for £24.

Amanda buys three adult tickets and five children's tickets for £38.

a Write down two simultaneous equations.

b Solve them to find the price of each ticket.

Problem solving exercise

① A banana costs b pence and an apple costs a pence.

Robin buys four bananas and an apple for 150p.

Marion buys two bananas and an apple for 100p.

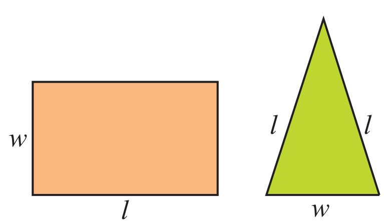
Work out the cost of seven bananas and eight apples.

② In this diagram, the length and width of the rectangle are l cm and w cm respectively.

The equal sides of the isosceles triangle are also l cm and the base w cm.

The perimeter of the rectangle is 32 cm.

The perimeter of the triangle is 26 cm.



a Write this information as two simultaneous equations.

b Find the values of l and w.

③ Peter makes mountain bikes and sports bikes.

It takes him 4 hours to make a mountain bike and 6 hours to make a sports bike. Each mountain bike costs him £45 to make and each sports bike costs him £60 to make.

One week Peter made m mountain bikes and s sports bikes in 54 hours.

The cost of making these bikes was £570.

a Write this information as a pair of simultaneous equations.

b How many of each type of bike did he make?

④ A mobile phone company charges their pay-as-you-go customers c pence per minute for calls and t pence for each text message.

Ben is charged £7.30 for making 10 minutes of calls and sending 40 texts.

Alicia is charged £7.25 for making 5 minutes of calls and sending 50 texts.

a Write this information as two simultaneous equations.

b Solve the equations to find the values of c and t.

c Ruby has £20 of credit. If she makes 15 minutes of calls and sends 100 texts, how much credit will she have left?

Do I know it now?

① Solve each pair of simultaneous equations.

a $3x + 4y = 2$
$3x - 5y = 11$

b $2x - 3y = 2$
$5x - 3y = 14$

c $x - 4y = 15$
$2x + 4y = -6$

② Solve these simultaneous equations.

a $x - y = 7$
$5x + 3y = 75$

b $4x - 6y = 34$
$3x - 3y = 21$

c $7x + 4y = 27$
$3x + 2y = 11$

③ Solve these simultaneous equations.

a $3x + 2y = 6$
$2x + 3y = 4$

b $3x - 5y = 0$
$4x + 3y = 29$

c $4x - 5y = 17$
$3x - 2y = 18$

Can I apply it now?

① Cedric and Bethany go to a garden centre and buy some trees.

Cedric buys four peach trees and three apple trees for £75.

Bethany buys three peach trees and one apple tree for £45.
Work out the cost of three peach trees and seven apple trees.

Chapter 12 covers working with quadratics. It can be found in the Next Steps section which contains material for students aiming to achieve a higher grade.

CHAPTER 13

ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Units and scales

JUST IN CASE

Length

Length is a measure of size. It tells you how long an item is.

To measure or draw a line you use a ruler.

Most rulers are marked in centimetres (cm).

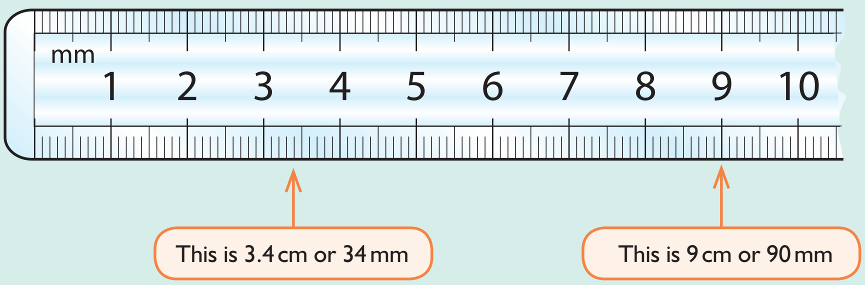
Small divisions are millimetres (mm).

1000 millimetres = 1 metre

1000 metres = 1 kilometre

Before measuring an object, you need to have a rough idea of how big it is so you can choose an appropriate **measuring instrument**.

This could be a ruler, tape measure, trundle wheel, micrometer or odometer (in a car).



Mass

The mass of an object is how heavy it is.

Weight is used to measure the mass of an object.

Scales are used to weigh objects.

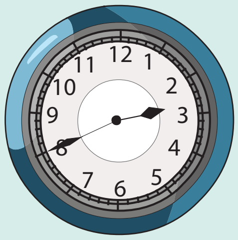
Milligrams (mg), **grams** (g) and **kilograms** (kg) are all units of mass.

1000 milligrams = 1 gram

1000 grams = 1 kilogram

You need to have a rough idea of the mass of an object so you can choose an appropriate **measuring instrument**. This could be a scientific balance, kitchen scales or bathroom scales.

Time



This clock shows 20 minutes to 3. This could be in the morning or the afternoon.

Using a 12-hour clock, this would be written 2:40 am (morning) or 2:40 pm (afternoon).

Using a 24-hour clock, 0240 would be in the morning and 1440 would be in the afternoon.

60 seconds = 1 minute

60 minutes = 1 hour

24 hours = 1 day

7 days = 1 week

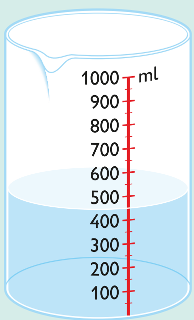
Each month has a different number of days.

Month	Number of days
January, March, May, July, August, October, December	31
April, June, September, November	30
February	28

Every fourth year is a leap year. In a leap year, February has 29 days.

12 months = 1 year

Volume



The **volume** of an object is the amount of space it takes up.

The capacity of a container is the amount inside it.

The volume of a liquid can be measured using a scale on a container or measuring cylinder.

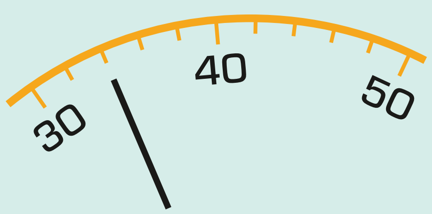
Many containers, such as bottles, cans and measuring spoons, are designed to hold an exact amount.

1000 millilitres (ml) = 1 litre (l)

100 centilitres (cl) = 1 litre (l)

You need to have a rough idea of the volume of an object so you can choose an appropriate **measuring instrument**. This could be a measuring spoon, cylinder and jug, burette or tank.

Interpreting scales



Scales are divided into large divisions and small divisions.

This scale is from a set of scales.

The large divisions are numbers to show 30, 40 and 50 grams.

The small divisions are not labelled but you can work them out.

There are 10 grams between the labelled numbers.

There are five divisions between the labelled numbers, so each division represents $10 \div 5 = 2$ grams.

The metric system

A prefix is used to show a fraction or multiple of the basic unit.

Those commonly used are:

milli- $\frac{1}{1000}$ *centi*- $\frac{1}{100}$ *kilo*- 1000.

You can see examples for length, mass and volume on pages 166–167.

The same pattern is used for all metric units and there are many other prefixes, for example *deci*- for $\frac{1}{10}$.

However, *milli*-, *centi*- and *kilo*- are the most commonly used.

Another unit for mass is the tonne.

1 tonne = 1000 kilograms

- If you are converting to a smaller unit, you multiply.
- If you are converting to a bigger unit, you divide.

Metric-imperial conversions

These will be given in the question and so you use the same idea as above to decide whether to multiply or divide by the conversion number.

Bearings

A **bearing** is given as an **angle** direction.

It is the **angle** measured clockwise from north.

Compass bearings are always given using three figures, so there can be no mistakes.

A **back bearing** is the direction of the return journey.

If a bearing is less than 180°, the back bearing is 180° more than the bearing.

If a bearing is more than 180°, the back bearing is the bearing less 180°.

North is 000°, east is 090°, south is 180° and west is 270°.

The line from a hill to a radio mast is shown in the diagram.

a What is the bearing of the mast from the hill?

b What is the back bearing to return from the mast to the hill?

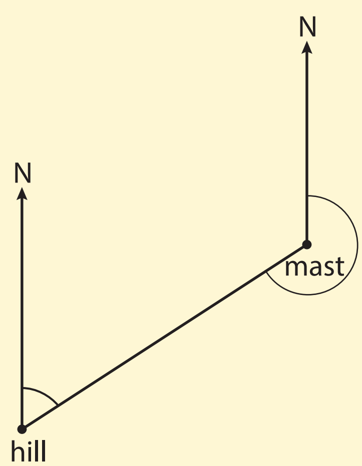
Solution

a Measure the marked angle at the hill. It is 57°.
057°

Remember to write the answer with three figures.

b The back bearing is 180° more than the bearing.
057° + 180° = 237°

You can check your answer by measuring.



SKILLS CHECK

➜ Warming up

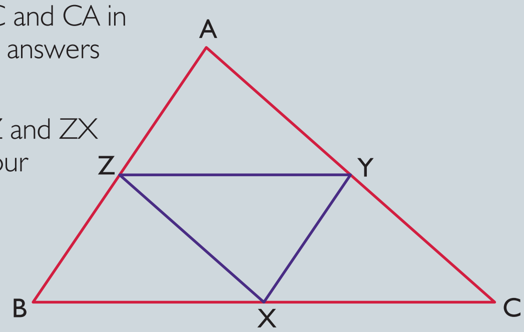
Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

➜ Length

a Measure the lengths AB, BC and CA in the triangle ABC. Give your answers in centimetres.

b Measure the lengths XY, YZ and ZX in the triangle XYZ. Give your answers in millimetres.

c Use your measurements to compare triangles ABC and XYZ.



➜ Mass

Match each object with the most appropriate mass.

a **b** **c** **d** **e** **f**

6 kg | 50 kg | 1 kg | 100 g | 25 kg | 450 g

➜ Time

Write these times in numbers. Use the 24-hour clock.

a half past 7 at night

b 20 minutes past 11 in the morning

c 5 minutes to 9 at night

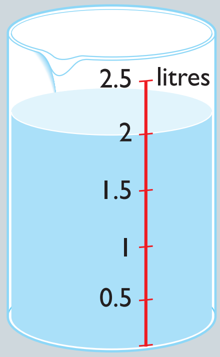
d 5 minutes past 2 in the afternoon

e 25 minutes to 6 in the morning

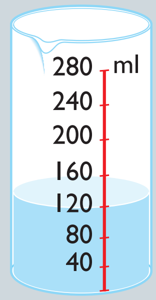
Volume

a How much liquid is in each of these beakers?

A



B

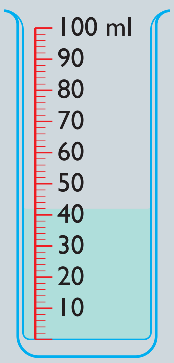


b Asaph pours as much liquid as he can from beaker B into beaker A. How much remains in beaker B?

Interpreting scales

How much liquid is in each of these measuring cylinders?

a



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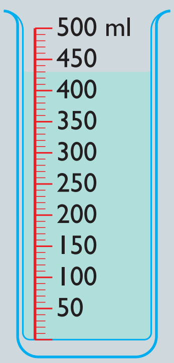
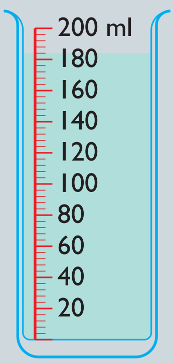


c



The metric system

Write down the smaller measurement in each pair.

a 17 mm and 3.2 cm

b $2\frac{1}{2}$ cm and 24 mm

c 3 m and 200 cm

d $\frac{1}{2}$ m and 48 cm

e 4000 m and 6 km

f $1\frac{1}{2}$ km and 1200 m

Metric-imperial conversions

a An elephant has teeth weighing up to 4.5 kg.
It has four teeth of this size.
What is their total mass?
Convert this into pounds.

b Each of these teeth is just over 25 cm long.
Is a tooth more than a foot long?
Use 2.2 pounds = 1 kg, 1 inch = 2.5 cm and 12 inches = 1 foot.

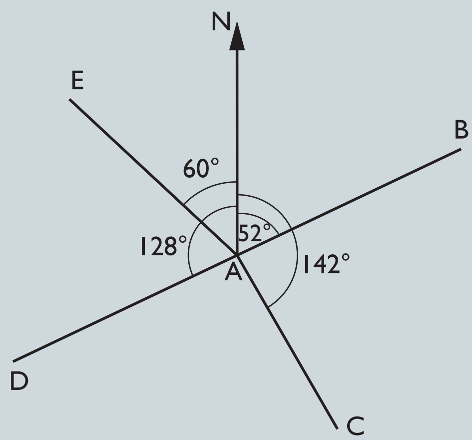
Bearings

a Work out the bearing of

i B from A

ii A from D.

b What can you say about the points D, A and B?



Applying the knowledge

① Harry is an angler.
His largest catch in the UK weighed 7 pounds 6 ounces.
His largest catch in France weighed 4.2 kg.
Which of these was the larger catch?
Use 16 ounces = 1 pound and 2.2 pounds = 1 kg.

② A boat sails from port A on a bearing of 045°.
The boat sails 5 miles every hour.
B is a point 7 miles due east of A.
A ship leaves B at the same time and sails on a bearing of 030°.
The ship sails 6 miles every hour.
Find, by measurement, the bearing of the boat from the ship after 30 minutes.

13.1 Scale drawing

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① An Ordnance Survey map has a scale of 1 : 50 000.

a Copy and complete these sentences.

1 : 50 000 on a map means 1 cm on the map represents a true length of ☐ cm on the ground.
Write this as 1 cm to ☐ km.

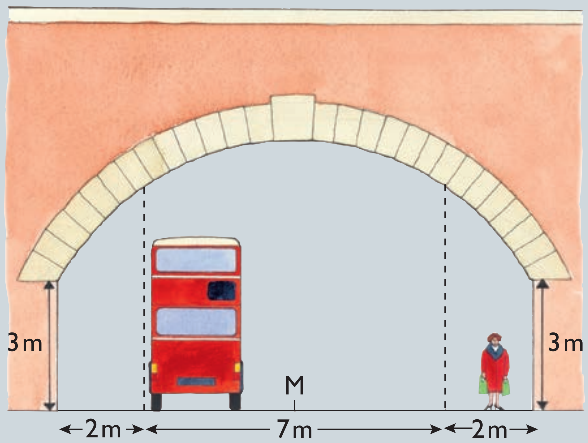
b A distance on the map is measured as 2.5 cm. How far is this?

c What distance on the map would represent 10 km?

If you can do the question above, try this one on problem solving.

② A road is 7 m wide and has a 2 m pavement along each side.

A bridge over the road has vertical sides 3 m high and its arch is part of a circle. The centre of the circle is at the midpoint M of the road.



a Make a drawing of the bridge on a scale of $\frac{1}{100}$.

b What is the greatest height of the arch?

c What height restriction should be placed on vehicles driving under the arch on the road (shown by the broken lines)?

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 173 (Problem solving exercise 13.1 Scale drawing).

What you need to know

Did you know?

Builders use scale drawings, produced by architects, when they build houses.

A **scale drawing** is the same shape as the original but a different size.

All the lengths are in the same **ratio**.

On a scale drawing where 1 cm on the scale drawing represents 2 m on the actual object, the scale can be written as

$\frac{1}{200}$ *or* 1 cm = 2 m *or* 1 : 200.

$\frac{1}{200}$ is sometimes referred to as the **scale factor**.

Be careful with areas and volumes.

$1\text{ cm} = 10\text{ mm}$ but $1\text{ cm}^2 = 10^2\text{ mm}^2 = 100\text{ mm}^2$

$1\text{ cm}^3 = 10^3\text{ mm}^3 = 1000\text{ mm}^3$

How to do it

Using a scale

Kitty has a toy car on a scale of $\frac{1}{50}$th.

The toy car is 6 cm long.

The real car is 1.7 m wide.

a How long is the real car?

b How wide is the toy car?

Solution

The toy car is $\frac{1}{50}$ the size of the real car.

So the real car is 50 times the size of the toy car.

a Length of real car = 6 × 50 cm ← To find the length of the real car, multiply by 50.

= 300 cm

The real car is 300 cm or 3 m long.

b 1.7 m = 170 cm ← It is easier to convert to centimetres first.

Width of toy car = 170 ÷ 50 cm ← To find the width of the toy car, divide by 50.

= 3.4 cm

The toy car is 3.4 cm wide.

Reading a map

Adebola is working out distances between places on a map.

The map has a scale of 1 : 50 000.

On the map, the bowling alley is 4.7 cm from the aqueduct.

a How far is the bowling alley from the aqueduct in kilometres?

Adebola's house is 3.8 km from the bowling alley.

b How far is her house from the bowling alley on the map?

Solution

a $4.7 \times 50\,000 = 235\,000$ cm ← To find a distance on the ground, multiply by the scale factor of 50 000.

$235\,000 \div 100\,000 = 2.35$ km ← There are 100 000 cm in 1 km.

b $3.8 \times 100\,000 = 380\,000$ cm ← Convert to centimetres first.

$380\,000 \div 50\,000 = 7.6$ cm ← To find a distance on the map, divide by the scale factor of 50 000.

Learning exercise

1. A map has a scale of 1 : 50 000.

   The distance between two villages on the map is 7 cm.

   a What is the actual distance between the villages?

   Littleton is 10.4 km from Avonford.

   b How far apart are the two towns on the map?

2. Write down the scale of these maps in the form 1 : n.

   a A street map where 3 cm represents 600 m.

   b An Ordnance Survey map where 20 cm represents 500 m.

   c A road map where 7 cm represents 17.5 km.

   d A map of the world where 8 cm represents 800 km.

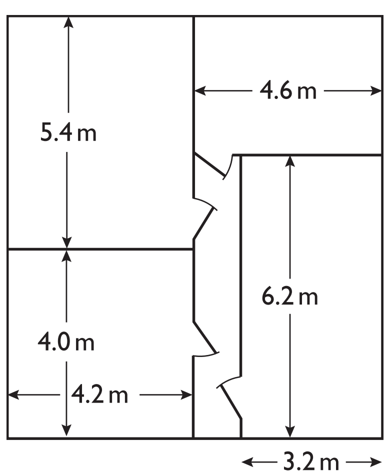
3. A town map is drawn to a scale of 1 : 20 000.

   Copy and complete this table to show the distances between the places listed.

Places	Distance on map (cm)	Distance in real life (km)
Library to sports centre		1.2
School to park	2.5	
Cinema to supermarket	7.5	
Café to cinema		2
Bowling alley to river		1.8

4. This is the plan of a flat. It is not drawn to scale.

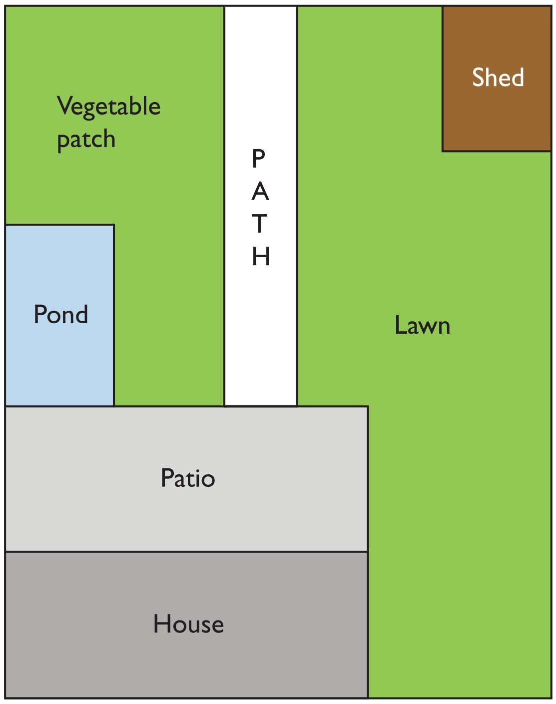
   Make an accurate scale drawing of the flat with the scale 1 cm = 2 m.



⑤ The diagram shows the plan of a garden.

The scale is 1 cm = 2 m.

Copy and complete the table to show the measurements of the garden.



Item	Plan measurement	True measurement
Length of patio		
Width of patio		
Length of lawn		
Length of vegetable patch		
Width of pond		
Length of pond		
Width of house		
Length of shed		
Width of shed		
Length of path		

⑥ Gemma is making a scale drawing of a swimming pool.

The actual pool is 30 metres long and 15 metres wide.

Gemma has a sheet of A4 paper and needs at least a 1 cm border around her diagram.

Gemma wants to make her drawing as large as possible.

What scale do you think she should use?

Problem solving exercise

1. Zac is orienteering.

   He starts at point A and then walks 6 km east followed by 7 km north-east to reach a lake at point B.

   a Make a scale drawing of Zac's journey.

   b How far is point B from Zac's starting point?

   c What is the bearing from the lake to point A?

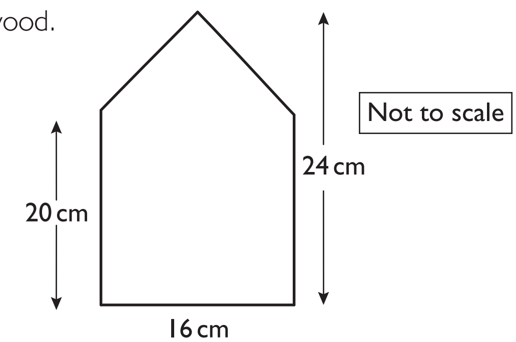
   d What is the bearing of the lake from point A?

2. Hannah wants to make a bird box from wood.

   She makes a rough sketch of the front of the bird box.

   Hannah wants to drill a circular hole of radius 2 cm in the front.

   The centre of the hole must be 12 cm from each of the bottom corners.



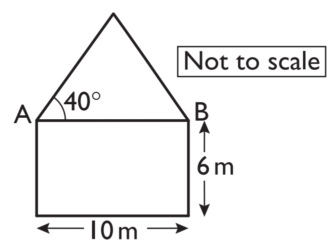
   a Make an accurate scale drawing of the front of the bird box.

   b What is the distance of the centre of the circular hole from the top corner?

3. The diagram is a sketch of the gable end of a house.

   The end of the house has a vertical line of symmetry.

   Building regulations state that the top of the house must be less than 4 m above the level AB.



   a Make an accurate scale drawing of the end of the house.

   b Is the top of the house less than 4 m from the level AB?

4. Rosie sails her boat from port A on a bearing of 070° for 20 miles to a point B.

   From B she sails on a bearing of 100° for 15 miles to a point C.

   a Make an accurate drawing of Rosie's journey.

   b **i** What is the bearing of the course that Rosie must set to get back to port A directly?

   ii What is the actual distance between point C and port A?

Do I know it now?

1. A model train engine is made to a scale of 1 : 50.

   The length of the model is 40 cm.

   a What is the length of the actual train engine?

   The height of the actual train engine is 4 m.

   b What is the height of the model?

Can I apply it now?

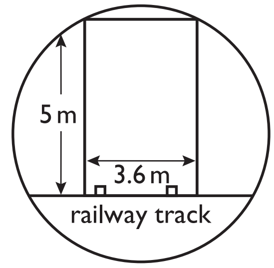
① The diagram shows a sketch of a tunnel for a railway. It is a circle of radius 4 m.

The diagram includes a horizontal chord. This shows where the railway track will be laid.

The diagram also includes a rectangle. It shows the space that must be allowed to let trains through.

a Make a scale drawing to show this.

b What is the height of the railway track above the lowest point of the tunnel?



13.2 Compound units

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① Gita is paid £22.50 for $2\frac{1}{2}$ hours work. What is her rate of pay

a in pounds per hour

b in pence per minute

c in pounds for a 40-hour week?

If you can do the question above, try this one on problem solving.

② An aeroplane flies at 1080 kilometres per hour.

a How far would it travel, at this speed, in

i 6 minutes **ii** 1 minute **iii** 15 seconds **iv** 5 seconds?

b Using the fact that 5 miles is approximately 8 kilometres, convert the aeroplane's speed into miles per hour.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 177 (Problem solving exercise 13.2 Compound units).

What you need to know

A **compound measure** is a measure involving two quantities.
An example is speed, which involves distance and time.

$$\text{speed} = \frac{\text{distance}}{\text{time}}$$

The unit of distance might be kilometres and the unit of time might be hours. In this case the **compound unit** for speed will be kilometres per hour (km/h or kmh^{-1}).

Other examples are grams per cubic centimetre for density and Newtons per square metre for pressure.

How to do it

Working with density

A metal block has a volume of 1000 cm^3.
It has a mass of 8.5 kg.
Work out the density of the block in kilograms per cm^3 using the formula

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

Solution

$\text{density} = \frac{\text{mass}}{\text{volume}}$

$= 8.5 \div 1000$

$= 0.0085$ kilograms per cm^3

This unit can also be written as kgcm^{-3} or kg/cm^3.

Working with speed

Jason leaves London on a motorbike at 06:00 and travels to his parents' house 432 km away.
He arrives at 10:00.

a Work out his average speed

i in kilometres per hour **ii** in kilometres per minute **iii** in metres per second.

b Write the unit 'metres per second' in two other ways.

Solution

a His journey takes from 6 o'clock to 10 o'clock so it lasts 4 hours.

Average speed $= \frac{\text{distance}}{\text{time}}$

i $\frac{\text{distance}}{\text{time}} = \frac{432 \text{ kilometres}}{4 \text{ hours}}$

$= 108$ kilometres per hour

ii $\frac{\text{distance}}{\text{time}} = \frac{432 \text{ kilometres}}{240 \text{ minutes}}$

4 hours $= 4 \times 60 = 240$ minutes

$= 1.8$ kilometres per minute

iii $\frac{\text{distance}}{\text{time}} = \frac{432\,000 \text{ metres}}{14\,400 \text{ seconds}}$

432 kilometres $= 432 \times 1000$
$= 432\,000$ metres

4 hours $= 4 \times 60 \times 60$
$= 14\,400$ seconds

$= 30$ metres per second

b Other ways of writing 'metres per second' include ms^{-1} and m/s.

Learning exercise

① Work out the average speed in kilometres per hour of each of these.

a An aeroplane travelling a distance of 900 km in 2 hours.

b A ferry travelling 8 km in 15 minutes.

c A car travelling 60 km in $1\frac{1}{2}$ hours.

② An athlete sprints 200 m in 20 seconds.

Work out his average speed in

a metres per second **b** metres per hour **c** kilometres per hour.

③ **a** Tom cycles for 3 hours. He travels a distance of 120 km.

Work out his average speed.

b A tortoise travels for 20 minutes. It travels 40 metres.

Work out its speed in

i metres per minute **ii** centimetres per second.

c Andrew swims for $1\frac{1}{2}$ hours. He covers 60 lengths, each of 20 metres.

What is his speed in metres per second?

④ Jamie and Sarah have part-time jobs.

In one week, Jamie gets £51 for 6 hours' work.

Sarah works 9 hours and gets £70.20.

Work out their rates of pay. Who is the better paid?

⑤ Bob's train departs at 12:38 and arrives at its destination 210 miles away at 14:18.

What is the train's average speed?

⑥ Jake and Amy go to different garages to fill their cars with petrol.

Jake pays £34.50 for 25 litres.

Amy pays £63.90 for 45 litres.

Who gets the cheaper petrol? Show all of your working.

⑦ Jo's lawn is rectangular.

It is 16 m long and 11.5 m wide.

She wants to feed it with fertiliser.

She buys a 2.5 kg box which covers 500 m^2.

How many grams should she use?

⑧ **a** Harry cycles at 8.5 km/h.

What is his speed in m/s?

b A cheetah can sprint at speeds of up to 30 m/s.

What is this speed in miles per hour?

Remember 5 miles is approximately 8 km.

⑨ Maya drives her car for 30 minutes at a steady speed of 70 mph.

She then drives for a further 48 miles taking $\frac{3}{4}$ hour.

a Work out the total distance she travels.

b Work out Maya's average speed for the whole journey.

10 A company makes metal files for use in engineering and metalwork.

The files have rows of sharp teeth for cleaning surfaces.

A smooth file has 55 teeth per inch. A rough file has 20 teeth per inch.

Both files are 12 cm long.

Work out the difference in the number of teeth on the two files.

You may use 2.5 cm = 1 inch.

Problem solving exercise

1 The speed limit on some main roads in France is 110 kilometres per hour.

The speed limit on motorways in the UK is 70 miles per hour.

Which speed limit is higher and by how much?

2 Hassan drives his car at a constant speed of 60 mph for 1 hour and then for 2 hours at a constant speed of 48 mph.

a How far does Hassan travel in this time?

b Work out Hassan's average speed for the whole journey.

3 A farmer wants to spread fertiliser on a field.

The field is rectangular and is 600 m long by 90 m wide.

a Work out the area of the field in hectares.

A hectare is 10 000 m^2

b The farmer puts on the fertiliser at 184 kg per hectare.

Show that 1000 kg of fertiliser will be enough.

Do I know it now?

1 Harry drives 130 km in 2 hours.

Work out his average speed in

a km/h **b** m/s.

Can I apply it now?

1 Jenny has an empty pool.

She is going to pump water into the pool at the rate of 20 litres per second. The pool is in the shape of a cuboid 10 m long by 5 m wide. Work out the length of time it will take to fill the pool to a depth of 150 cm. 1 m^3 = 1000 litres.

CHAPTER 14

ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Properties of shapes

JUST IN CASE

Line symmetry

When looking for **line symmetry**, imagine folding the object or shape.

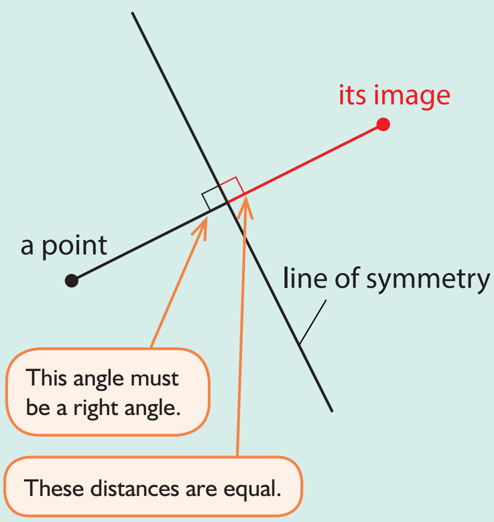
If the shape will fold precisely onto itself then that fold is a line of symmetry and the shape has line (or reflective) symmetry.

When completing a shape with a line (or lines) of symmetry, find where each vertex will go when it has been reflected in the line.

Once you have completed the vertices, just join them up to complete the shape.

You can find the image of each point by drawing a line from the point to the line of symmetry which is at a right angle.

Then continue the line the same distance the other side.



Angle facts

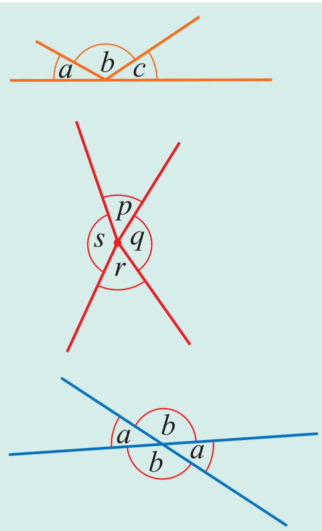
An **angle** is a turn. It is usually measured in degrees.

Angles on a straight line add up to 180°.

Angles around a point add up to 360°.

Vertically opposite angles are formed where two lines cross.

Vertically opposite angles are equal.



Rotational symmetry

The **order of rotational symmetry** is the number of times that a shape will fit on to itself in one complete turn.

The **centre of rotational symmetry** is the point about which the shape has to be rotated in order for it to fit on to itself.

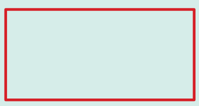
This is usually just called the centre of rotation.

If a shape fits on to itself only once in one turn then it has rotational symmetry of **order 1**. This means it has **no** rotational symmetry.

When identifying the order of rotational symmetry it can be helpful to use tracing paper. Trace the outline of the shape and then rotate the tracing paper over the original image.

What is the order of rotational symmetry of each of these shapes?

a



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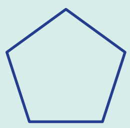


Solution

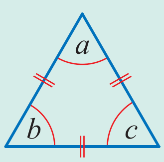
a The rectangle will fit on to itself two times in one turn, so the order is 2.
b The pentagon will fit on to itself five times in one turn, so the order is 5.

Angles in triangles and quadrilaterals

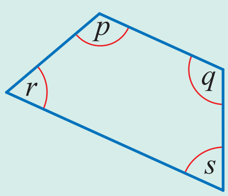
The three interior angles at the vertices in a triangle add up to 180°.

The four interior angles at the vertices in a quadrilateral add up to 360°.

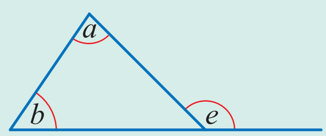
The exterior angle of a triangle equals the sum of the two opposite interior angles.



$a + b + c = 180°$



$p + q + r + s = 360°$



$e = a + b$

An **equilateral triangle** has all of its sides equal, and so all of the angles are also equal by symmetry. They are all 60°.

An **isosceles triangle** has two equal sides, and so, by symmetry, has two equal angles as well.

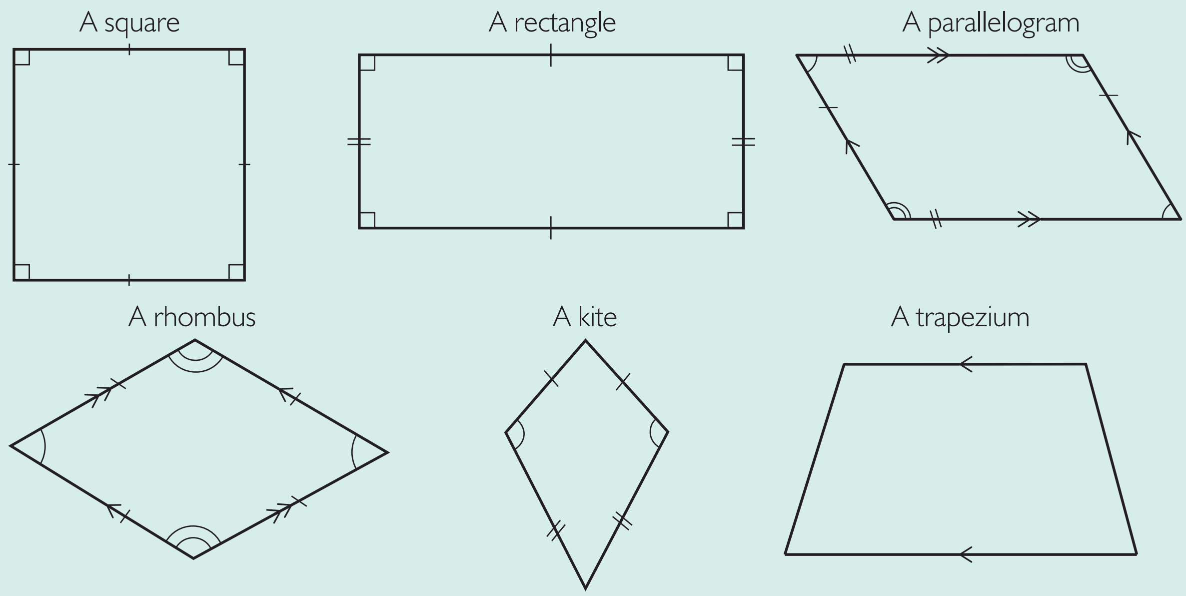
A triangle with all of its sides of different lengths is called **scalene**.

Types of quadrilateral

When classifying shapes the key things to look for are:

- the number of sides
- the lengths of the sides
- the lines of symmetry
- whether opposite sides are parallel
- the sizes of the angles.

There are six special types of quadrilateral to remember.



Equal sides are marked with the same number of dashes.
Parallel sides are marked with the same number of arrows.
Equal angles are marked with the same number of arcs.

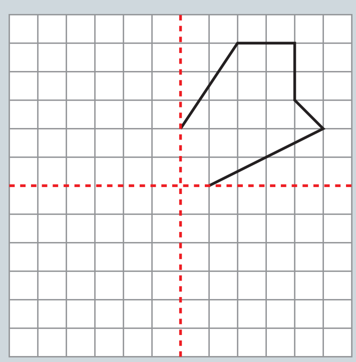
SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

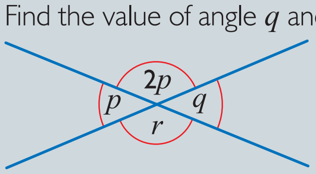
Line symmetry

Copy this shape onto squared paper.
Draw the reflection in the lines of symmetry.



Angle facts

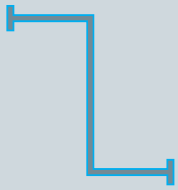
Find the value of angle q and angle r in this diagram.



Rotational symmetry

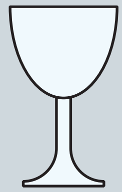
Write down the order of rotational symmetry of each pattern.

a

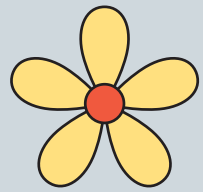


b

c



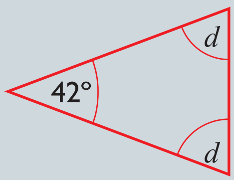
d



Angles in triangles and quadrilaterals

Work out the size of the lettered angles in these diagrams.

a



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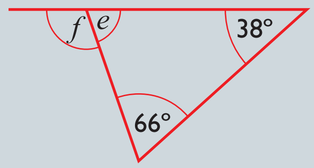
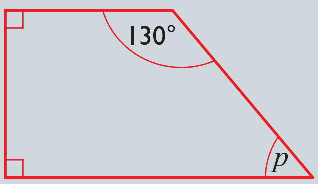
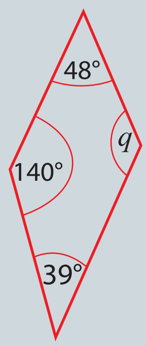


c



d



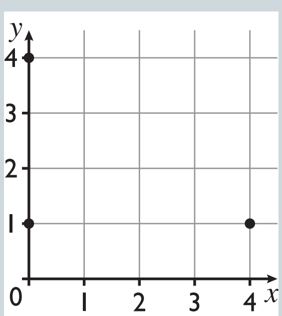
Types of quadrilateral

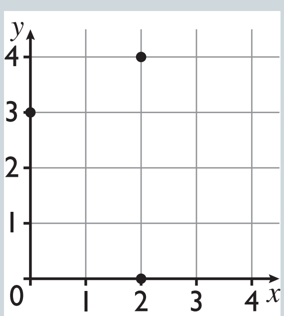
Look at these diagrams.

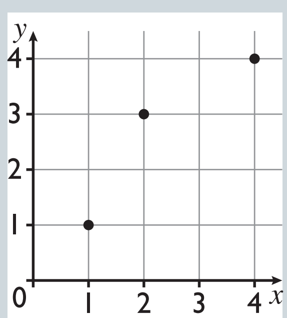
In each case, one vertex of the quadrilateral is missing.

Write down the co-ordinates of the missing vertex to make

a a rectangle **b** a kite **c** a rhombus.

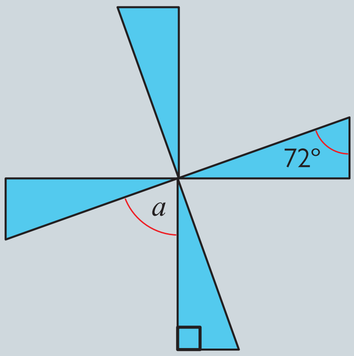






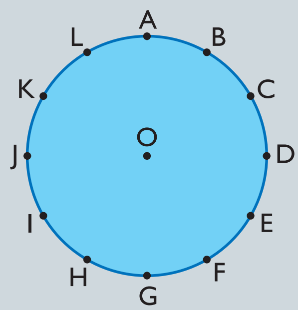
→ Applying the knowledge

① This figure has rotational symmetry of order 4.



Work out the size of angle a.

② The 12 points A to L are equally spaced round a circle.



a What type of quadrilaterals can you draw by joining some of the points?

Give an example of each.

b Which quadrilaterals can't you draw in this way?

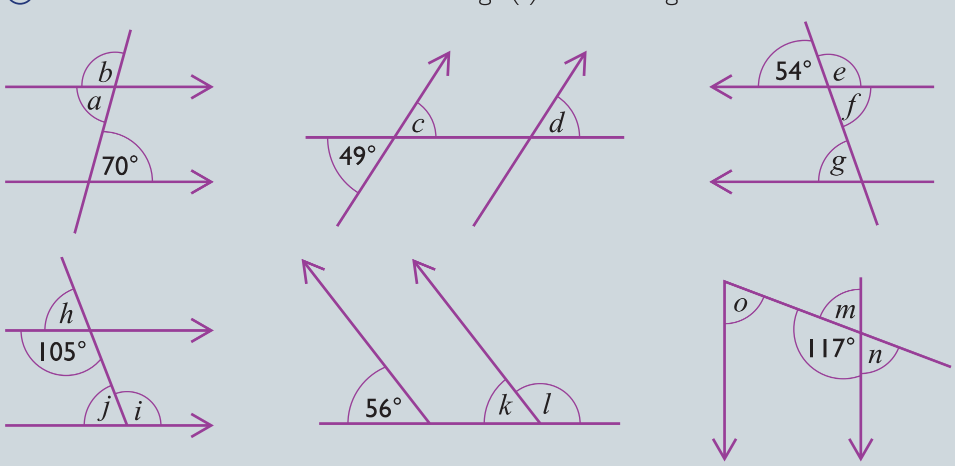
14.1 Angles in parallel lines

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① Write down the size of the lettered angle(s) in each diagram.



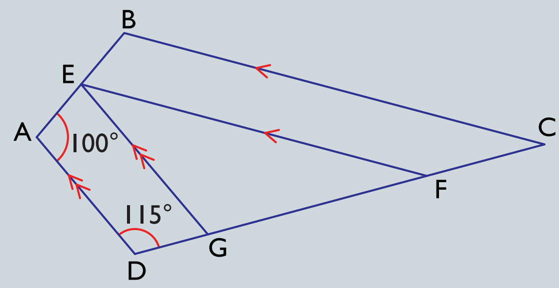
If you can do the question above, try this one on problem solving.

② ABCD is a kite.

EF is parallel to BC.

AD is parallel to EG.

Calculate the size of ∠GEF.



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 187 (Problem solving exercise 14.1 Angles in parallel lines).

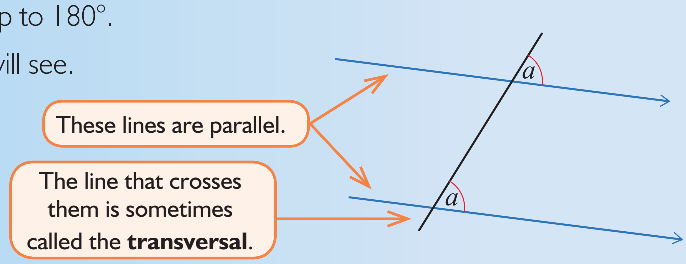
What you need to know

Parallel lines go in the same direction. They never meet, no matter how far they are extended.

When a third line crosses a pair of parallel lines, it creates a number of angles.

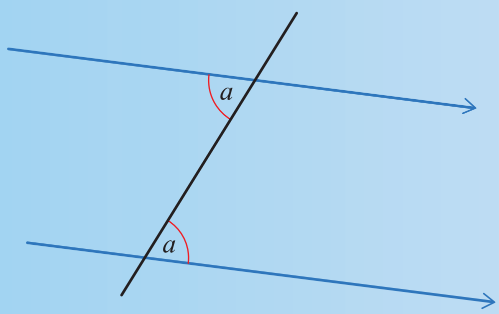
Some of these are equal; others add up to 180°.

The three diagrams show things you will see.



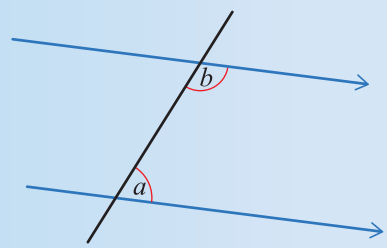
The two angles marked a are exactly the same.

They are called **corresponding angles**.



The two angles marked a are also the same.

Angles in this position are called **alternate** angles.



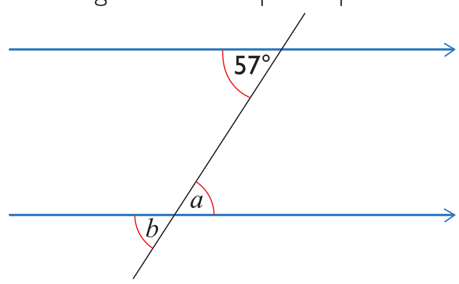
Angles a and b add up to 180°.

They are called **allied** and **co-interior** angles.

How to do it

Using alternate angles

The diagram shows a pair of parallel lines and an intersecting line.



Work out the size of the lettered angles. Give a reason for each of your answers.

Solution

Alternate angles are equal.

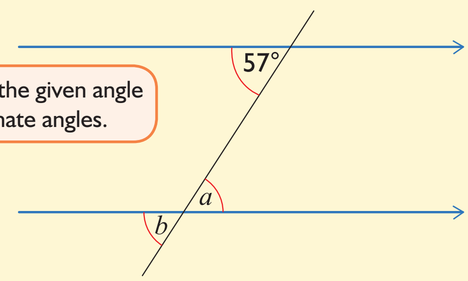
$a = 57°$

Opposite angles are equal.

$b = 57°$

Angle a and the given angle are alternate angles.

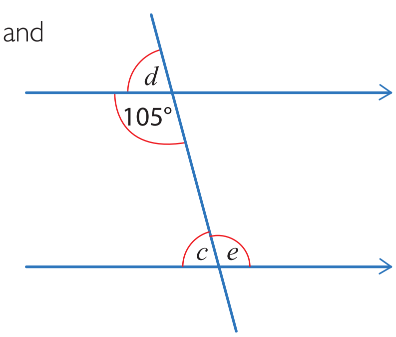
Angle b is vertically opposite angle a.



Using allied and corresponding angles

The diagram shows a pair of parallel lines and an intersecting line.

Work out the size of the lettered angles. Give a reason for each of your answers.



Solution

Allied angles add up to 180°.

$c = 180 - 105 = 75°$

c and d are corresponding angles.

Corresponding angles are equal.

$d = 75°$

Alternate angles are equal.

e is an alternate angle to the given angle.

$e = 105°$

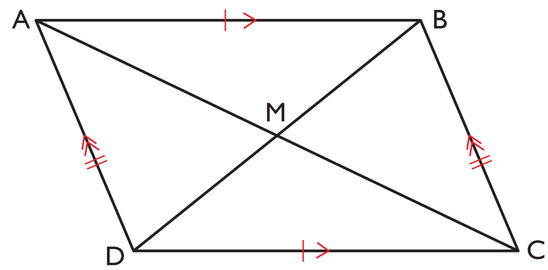
➤ Finding angles in a parallelogram

ABCD is a parallelogram.
Its diagonals meet at M.
On a copy of the diagram:

a Mark two pairs of alternate angles between the parallel lines AB and DC.

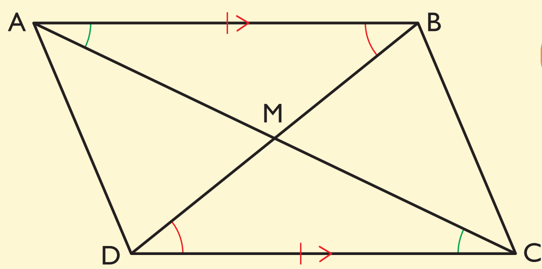
b Mark two pairs of vertically opposite angles.

c Show a pair of congruent triangles.



Solution

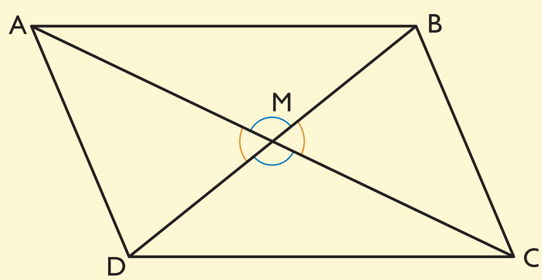
a Alternate angles



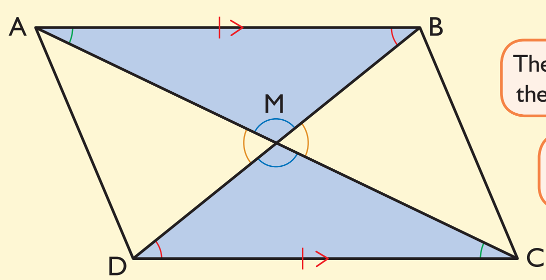
The red angles are equal.

The green angles are equal.

b Vertically opposite angles



c Congruent triangles



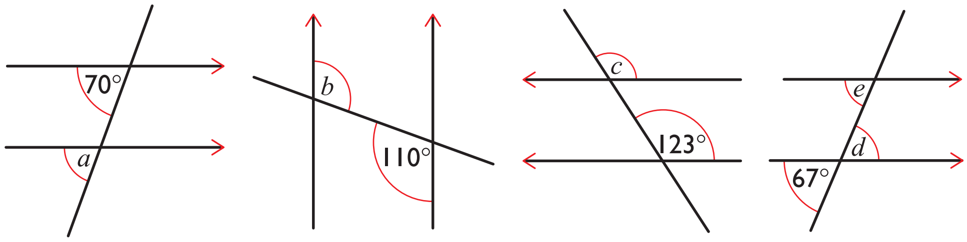
The three angles of these triangles are the same so they are the same shape.

Also AB and CD are the same length so they are the same size.

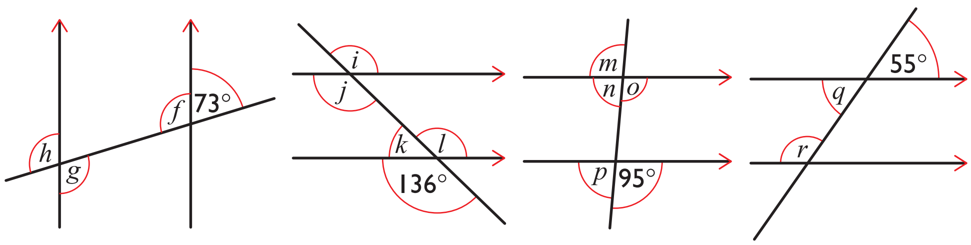
Triangles AMD and CMB are also congruent.

Learning exercise

① Work out the sizes of the lettered angles in these diagrams.



② Work out the sizes of the lettered angles in these diagrams.

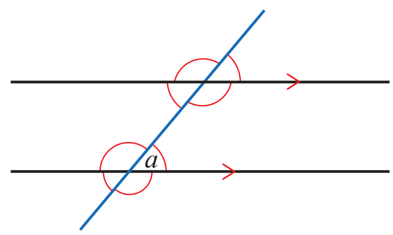


③ Joe says there are only two different sizes of angle marked in this diagram.

Angle a is 50°.

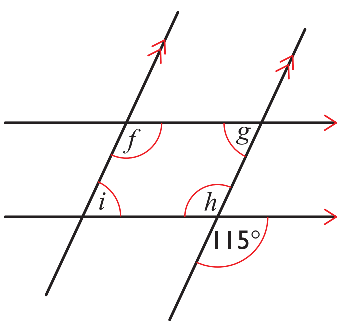
Work out the values of the other angles in the diagram.

Is Joe right?



④ Write down the sizes of the lettered angles in this diagram.

Give a reason for each answer.



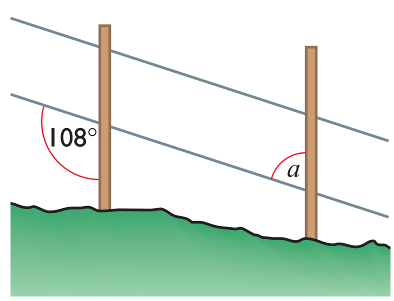
⑤ This fence is going down a hill.

The posts are vertical.

The wires are parallel.

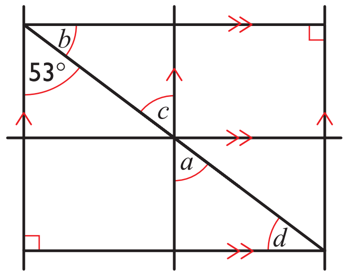
a What is the size of angle a?

b What angle do the wires make with the horizontal?



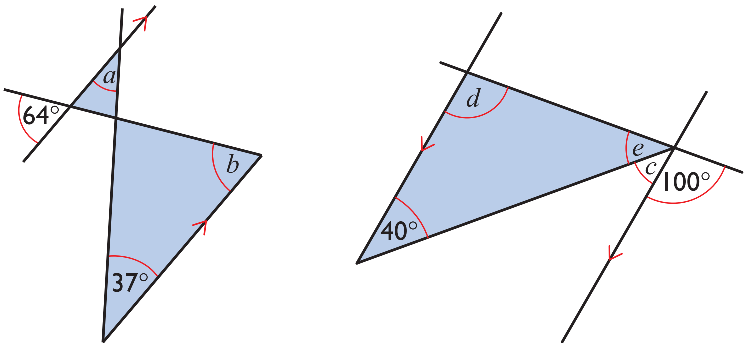
⑥ The diagram shows part of a fence panel.

Work out the sizes of angles a, b, c and d.

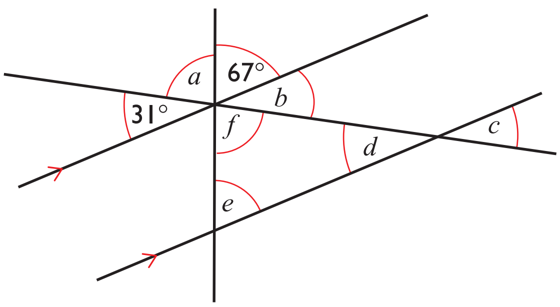


⑦ Write down the sizes of the lettered angles in these diagrams.

Give a reason for each answer.



⑧



a Write down the size of each lettered angle.

Give a reason for each answer.

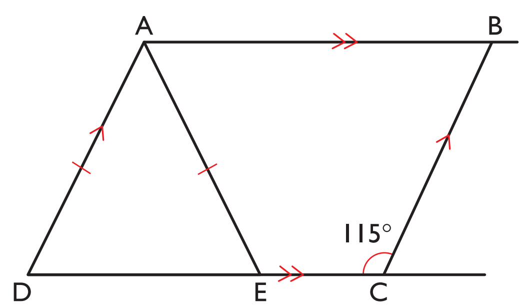
b What is $f + d + e$?

Problem solving exercise

① AB is parallel to DC. AD is parallel to BC.

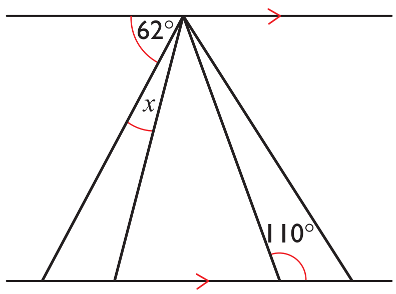
E is a point on DC such that ADE is an isosceles triangle.

Work out the size of angle DAE. Give your reasons.



② This diagram has a single line of symmetry and a pair of parallel lines.

Work out the size of angle x. Give your reasons.

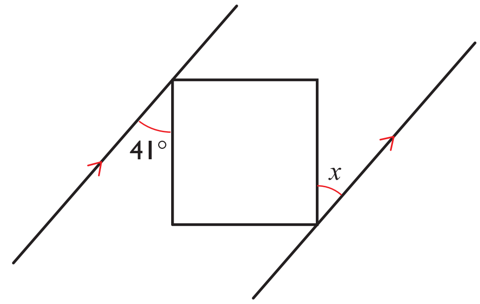


③ The diagram shows a square and a pair of parallel lines through two of its vertices.

Jim thinks that $x = 41°$.

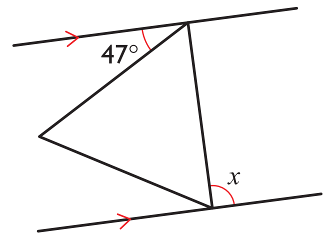
Is Jim correct?

Give a reason for your answer.



④ The diagram shows an equilateral triangle and a pair of parallel lines at two of its vertices.

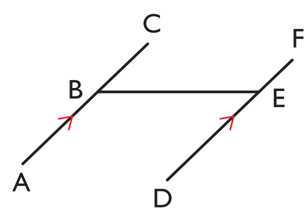
Work out the value of x. Give your reasons.



⑤ ABC and DEF are parallel lines.

Angle ABE is three times the size of angle BED.

Work out the size of angle ABE.

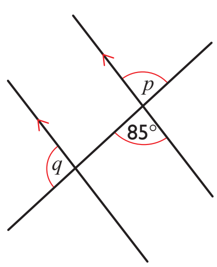


Do I know it now?

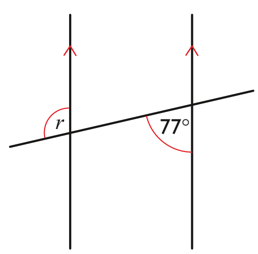
① Work out the size of the lettered angles.

Write down the angle fact(s) that you use for each one.

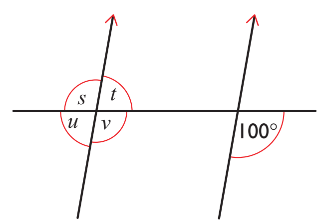
a



b



c

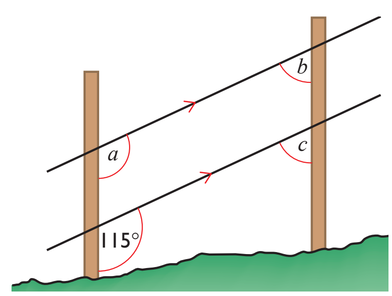


② Jack makes a wire fence.

The posts are vertical.

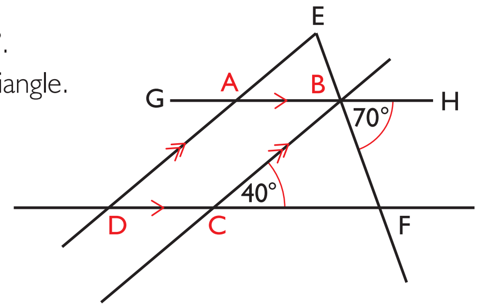
The wires are parallel.

Work out the size of the angles a, b and c.



Can I apply it now?

① ABCD is a parallelogram.
Angle BCF = 40° and angle HBF = 70°.
Show that triangle EDF is an isosceles triangle.



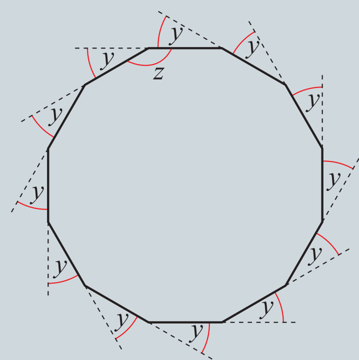
14.2 Angles in a polygon

SKILLS CHECK

➜ Do I need to do this section?

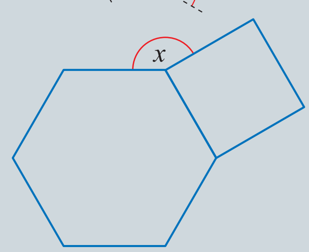
Complete this section if you need help with the questions below.

① Calculate the size of each lettered angle.
② Calculate the size of the interior angle of a regular polygon with 120 sides.
③ A regular polygon has an interior angle of 144°.
How many sides does the polygon have?



If you can do the questions above, try this one on problem solving.

④ The diagram shows a regular hexagon and a square.
Work out the size of angle x.



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 192 (Problem solving exercise 14.2 Angles in a polygon).

What you need to know

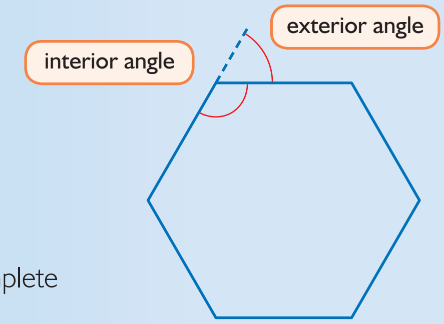
An **interior angle** is the angle inside the corner of a shape.

An **exterior angle** is the angle that has to be turned through to move from one side to the next.

At each vertex, **interior angle + exterior angle = 180°**.

In a regular polygon, all of the interior angles are equal and all of the exterior angles are equal.

The exterior angles of a polygon together always make one complete turn and so add up to 360°.



The **sum of the interior angles of a polygon = 180(n − 2)°**, where n is the number of sides.

For example, the sum of the interior angles of a hexagon is $180 \times (6 - 2) = 720°$.

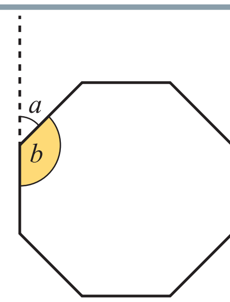
How to do it

Finding angles of regular polygons

The diagram shows a regular octagon.

a Work out the size of an exterior angle of a regular octagon.

b Hence find the size of an interior angle of a regular octagon.



Solution

a The exterior angles of a polygon add up to 360°.
For a regular octagon, each exterior angle $= 360° \div 8$
$= 45°$

This is angle a in the diagram.

b Interior angle + exterior angle = 180°
For a regular octagon, each interior angle $= 180° - 45°$
$= 135°$

This is angle b in the diagram.

Finding the number of sides of a regular polygon

A regular polygon has an interior angle of 162°.
How many sides does the polygon have?

Solution

First find the exterior angle.

interior angle + exterior angle = 180°

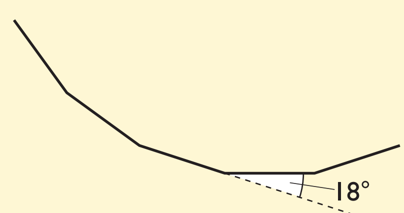
162° + exterior angle = 180°

exterior angle = 180° − 162° = 18°

The exterior angles add up to 360°.

number of exterior angles = 360 ÷ 18 = 20

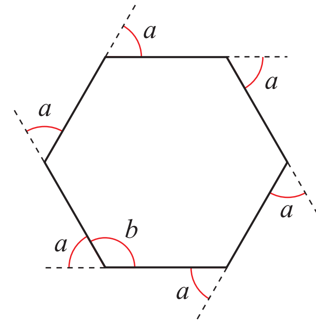
The polygon has 20 sides.



The number of sides is the same as the number of exterior angles.

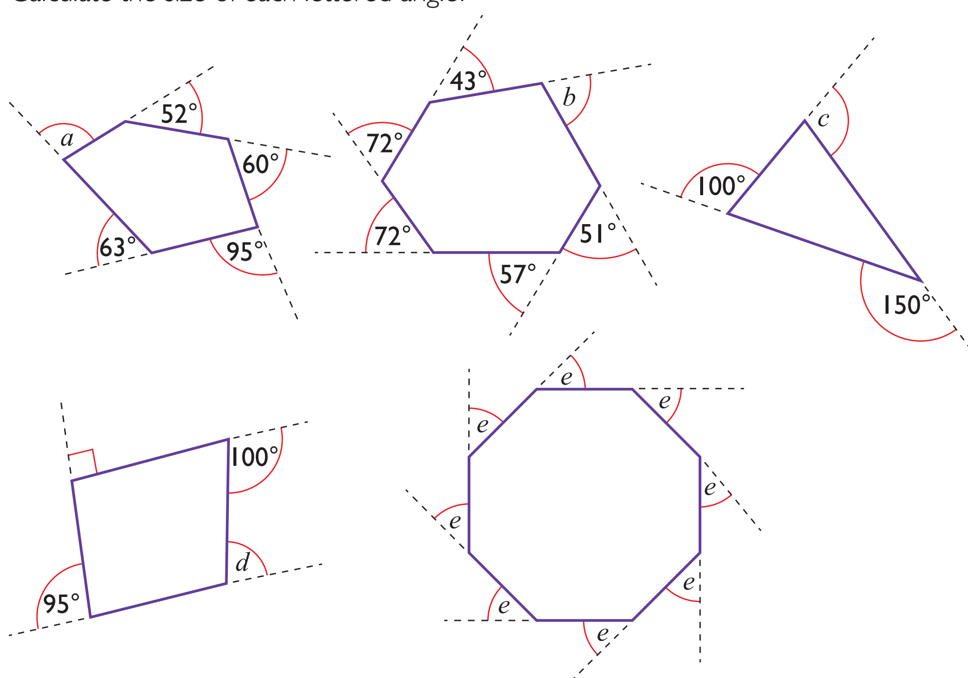
Learning exercise

① **a** Calculate the exterior angle, a, of a regular hexagon.
b Calculate the interior angle, b, of a regular hexagon.



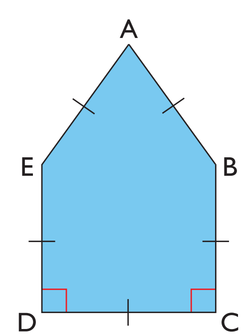
② **a** Calculate the exterior angle of a regular octagon.
b Calculate the interior angle of a regular octagon.

③ Calculate the size of each lettered angle.

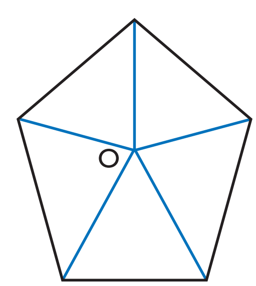


④ The external angle of a regular polygon is 10°.
a How many sides does the polygon have?
b What is the size of one interior angle?
c What is the sum of the interior angles of the polygon?

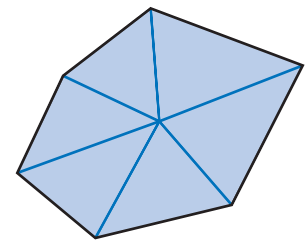
⑤ Seb draws a pentagon.
a Is Seb's pentagon regular?
Give a reason for your answer.
b What is the sum of the interior angles of a pentagon?
c Work out the size of each of the interior angles in Seb's pentagon.



⑥ The diagram shows a pentagon.
The vertices are all joined to a point O in the middle, making five triangles.
a Work out the total of all the angles in the five triangles.
b What is the total of the angles at the point O?
c Using your answers to parts **a** and **b**, work out the total of the interior angles of the pentagon.
d Calculate the size of each interior angle if the pentagon is regular.



7 The diagram shows a hexagon split into six triangles.



a What is the sum of

i all the angles in all the triangles

ii the angles at the centre

iii the interior angles of the hexagon?

b Explain why the sum of the interior angles of a polygon with n sides is $n \times 180° - 360°$. Refer to the diagram in your explanation.

c Show that the formula in part **b** is the same as $(n - 2) \times 180°$.

8 A regular polygon has exterior angles of 30°.

a What is the sum of the exterior angles?

b How many sides does the polygon have?

c What is the size of each interior angle?

d What is the sum of the interior angles of the polygon?

9 **a** A regular polygon has an interior angle of 174°. How many sides does the polygon have?

b The sum of the interior angles of a regular polygon is 1980°. How many sides does the polygon have?

c A regular polygon has an exterior angle of 10°. What is the sum of its interior angles?

10 Mark is tiling a floor using polygon-shaped tiles.

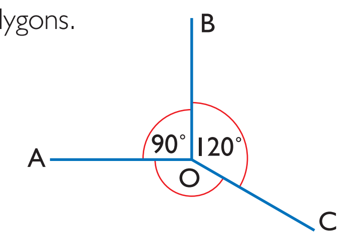
a Mark tiles part of the floor using equilateral triangles. How many tiles meet at each vertex?

b Can Mark tile the floor using regular hexagons? Explain your answer fully.

c Can Mark tile the floor using regular pentagons? Explain your answer fully.

11 **a** In the diagram, the lines AO, BO and CO are all sides of regular polygons. How many sides does each of the three polygons have? What are their names?

b The line CO is rotated by 15° clockwise about O. How would you answer part **a** when it is in this position?

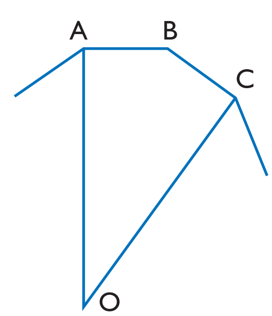


Problem solving exercise

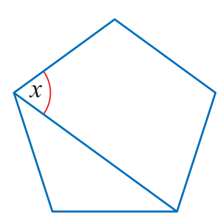
1 A, B and C are three vertices of a regular 10-sided polygon.
O is the centre of the polygon.

a Calculate the size of angle AOC.

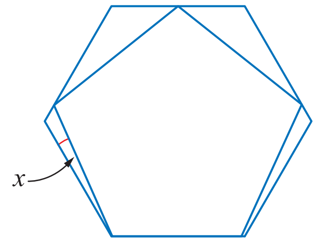
b Show that OABC is a kite.



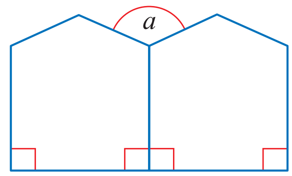
2 The diagram shows a regular pentagon. Work out the size of the angle marked x.



③ The diagram shows a regular hexagon and a regular pentagon on the same base.
Work out the size of angle x.



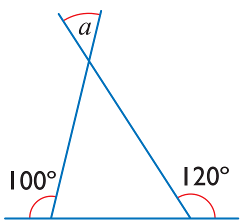
④ The diagram shows two identical pentagons.
For each pentagon, two of the angles are right angles and the other three angles are the same.
Work out the size of angle a.



Do I know it now?

① Calculate the sizes of the lettered angles.

a



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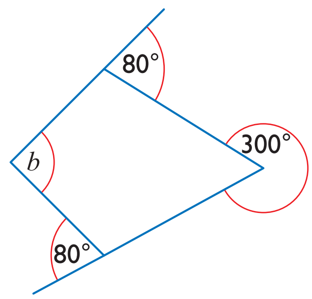

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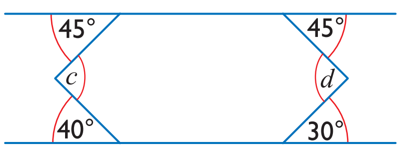
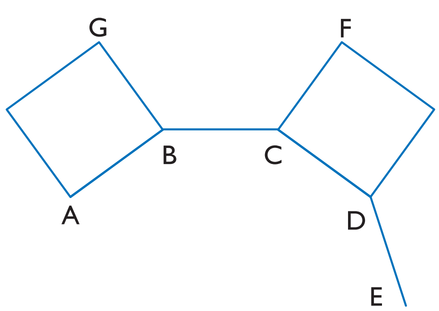


② **a** The exterior angle of a regular polygon is 5°.
How many sides does the polygon have?

b Is there a regular polygon with an exterior angle of 7°?
Explain your answer.

Can I apply it now?

① **a** Show that the interior angle of a regular dodecagon (12 sides) is 150°.
ABCDE is part of a regular dodecagon.
The two quadrilaterals are squares with sides the same length as AB.



b Find the size of the angle GBC.

c Show that angles GBC and FCB are each equal to the interior angle of a regular hexagon.

CHAPTER 15

ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Measuring shapes

JUST IN CASE

Area and perimeter

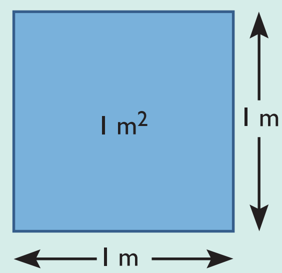
Area is the amount of space inside a two-dimensional shape.

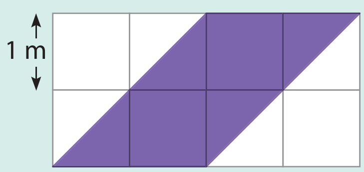
It is measured in square units such as cm^2 or m^2.

One way to find the area of a shape is to count the number of squares inside it.

$1\ m^2$ is the space inside a square of side 1 m.

An area of $4\ m^2$ means an area equivalent to $4 \times 1\ m^2$.





This parallelogram covers two full squares and four half squares.

So in total it covers 4 squares.

Its area is $4\ m^2$.

There are some formulae for working out the **area** of standard shapes.

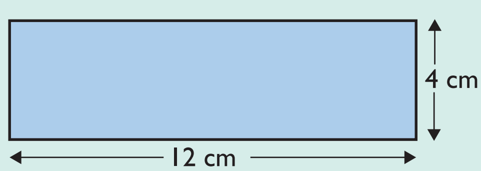
Rectangle: Area $=$ length $\times$ width

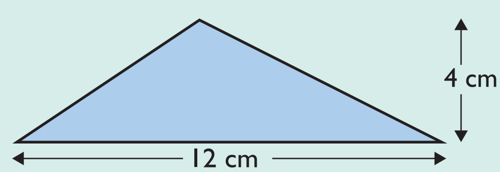
Triangle: Area $= \frac{1}{2} \times$ base $\times$ height

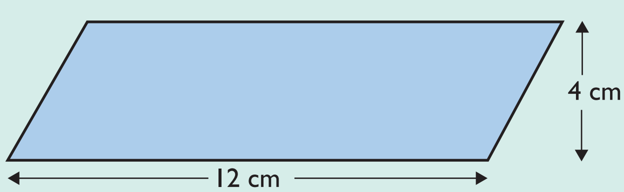
Parallelogram: Area $=$ base $\times$ height

Choose one of the sides as the base.

Height is the distance between the base and its parallel side.

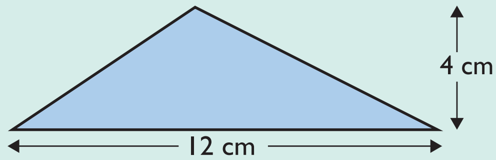






Perimeter is the distance around the edges of a shape.
It is measured in linear units such as cm or m.

Find the area of this triangle.



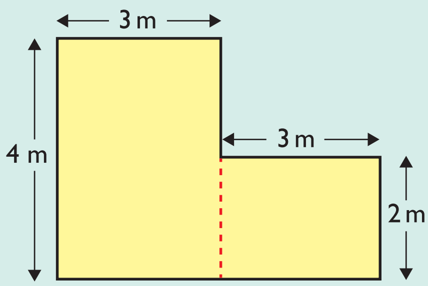
Solution

Area of a triangle $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 12 \times 4$

$= 24\,\text{cm}^2$

Find the area and perimeter of this shape.



Solution

Perimeter $= 4\,\text{m} + 3\,\text{m} + 2\,\text{m} + 3\,\text{m} + 2\,\text{m} + 6\,\text{m}$

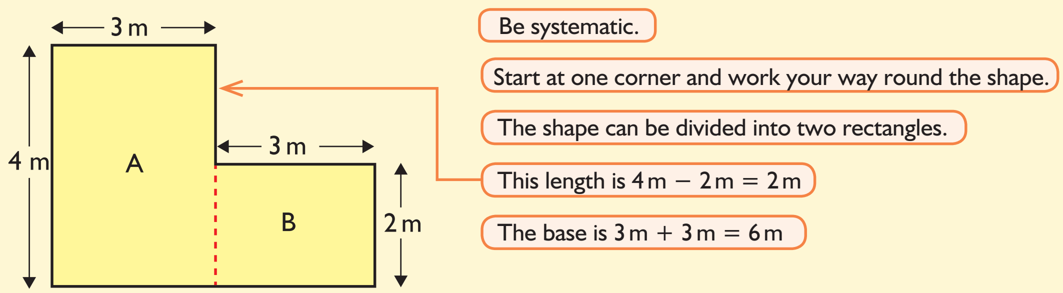
$= 20\,\text{m}$

Area of whole shape $=$ Area A $+$ Area B

$= (3\,\text{m} \times 4\,\text{m}) + (3\,\text{m} \times 2\,\text{m})$

$= 12\,\text{m}^2 + 6\,\text{m}^2$

$= 18\,\text{m}^2$



Circumference

The **circumference** is the distance around the edge of a circle.
The **radius** of a circle is the distance from the centre to the circumference.
The **diameter** is the distance across the circle through the centre.
The circumference, C, of a circle of diameter d is
$C = \pi d$.

It is also given by

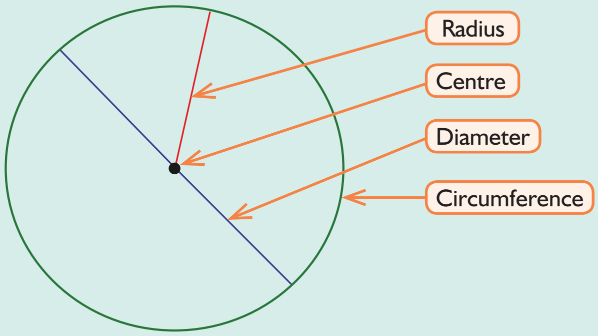
$C = 2\pi r$

where r is the radius.

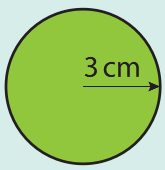
π (pi) is a Greek letter which represents the number value 3.141 592 654… .

An approximate value of π is 3.14.

To be more accurate use the π key on your calculator.



Find the circumference of this circle.



Solution

$C = 2\pi r$	The radius is given so use the formula $C = 2\pi r$.
$= 2 \times \pi \times 3$	$r = 3$. Use the value of π stored in your calculator or 3.14.
$= 18.8$ to 1 d.p.	Round your answer.

SKILLS CHECK

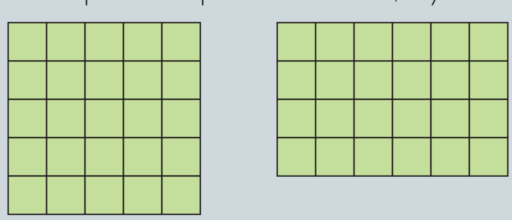
➜ Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

➜ Understanding area

Look at each pair of shapes. In each case, say which has the larger area.

a



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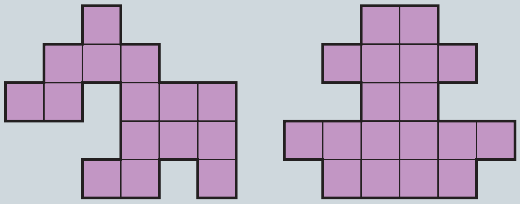
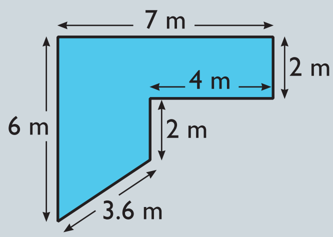


Finding area and perimeter

Work out the perimeter and area of each shape.

a



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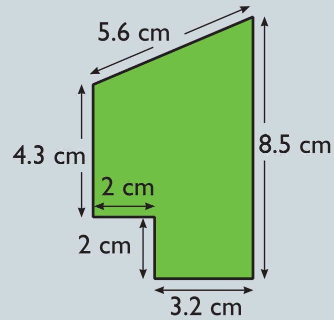
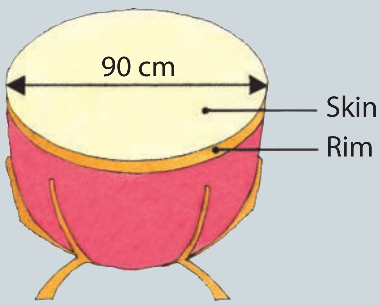


Circumference

Find the circumference of the rim of each timpani drum.

a



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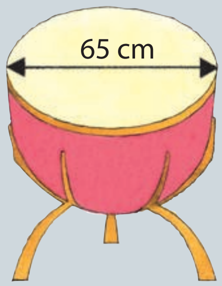

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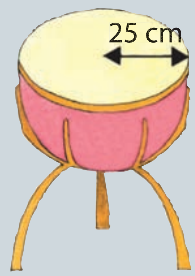


Applying the knowledge

① Alan is going to varnish the floor of the village hall.

The hall floor is rectangular, 12 m by 8 m.

He will need to apply two coats of varnish.

He can buy the varnish in 2 litre tins.

One tin is enough for 15 m^2.

How many tins of varnish will Alan need?

② A bin has a circumference of 100 cm. What is its diameter?

15.1 Area of circles

SKILLS CHECK

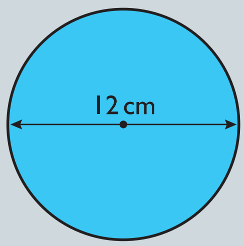
Do I need to do this section?

Complete this section if you need help with the question below.

① Work out the area of each circle.

Give your answers to one decimal place.

a



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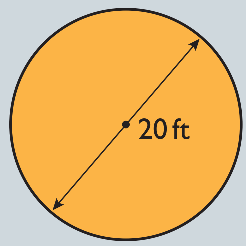

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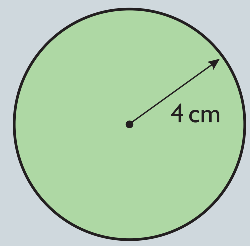
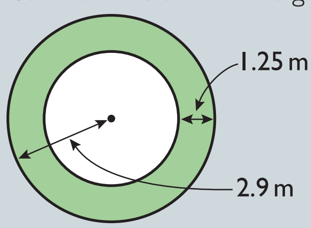


If you can do the question above, try this one on problem solving.

② Calculate the area of the green region.



These circles are concentric; they have the same centre.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 201 (Problem solving exercise 15.1 Area of circles).

What you need to know

The area, A, of a circle of radius r is given by

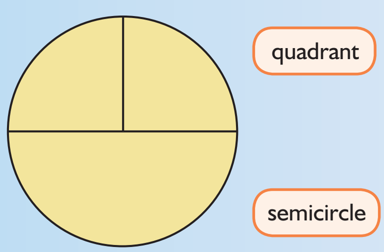
$A = \pi r^2$.

A semicircle is half a circle.

Its area is $\frac{1}{2}\pi r^2$.

A quadrant is a quarter of a circle.

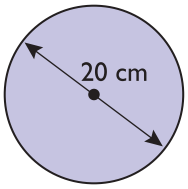
Its area is $\frac{1}{4}\pi r^2$.



How to do it

Finding the area of a circle given the diameter

Work out the area of this circle.



Solution

First find the radius.

$r = \frac{d}{2}$

$= \frac{20}{2}$

$= 10$ cm

$A = \pi r^2$

$= \pi \times 10^2$

$= \pi \times 100$

$= 314.2\,\text{cm}^2$ (to 1 d.p.)

Finding the radius and diameter of a circle given the area

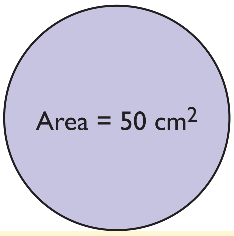
The area of this circle is $50\,\text{cm}^2$.

Calculate

a the radius

b the diameter.

Give your answers to the nearest centimetre.



Solution

a $A = \pi r^2$

$50 = \pi r^2$ Divide both sides by π.

$\frac{50}{\pi} = r^2$ Take the square root of both sides.

$\sqrt{\frac{50}{\pi}} = r$

$r = \sqrt{\frac{50}{\pi}}$

$r = 3.989\ldots$ Your calculator often gives you a long number. Write it like this, using … for the later digits. That is better than writing them all down.

The radius is 4 cm (to the nearest centimetre).

b Diameter $= 2 \times$ radius

$= 2 \times 3.989\ldots$ Use the most accurate value.

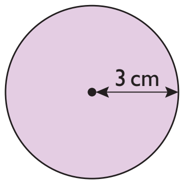
$= 7.978\ldots$ Keep the number on your calculator until you get your final answer.

The diameter is 8 cm (to the nearest centimetre).

Learning exercise

1. Calculate the area of each circle. Give your answers to one decimal place.

a



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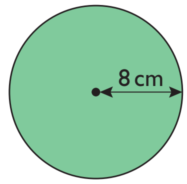

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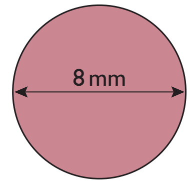

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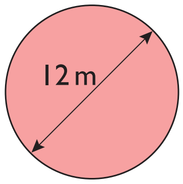

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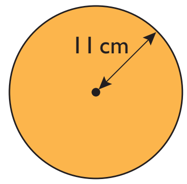

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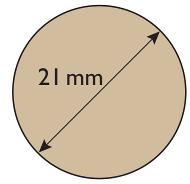
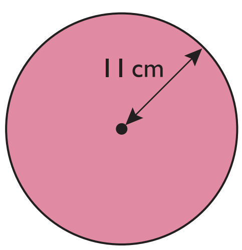


2. A circle has an area of 35 cm^2.
   Using the formula $A = \pi r^2$, calculate its radius to one decimal place.
3. A circle has an area of 60 cm^2.
   a Calculate its radius to one decimal place.
   b Write down its diameter.
   c Calculate the circle's circumference.
4. The table below gives some measurements for circles. Copy and complete the table.

	Radius	Diameter	Area	Circumference
a	7 cm			44.0 cm
b		17 cm		
c			80 cm^2	
d			250 cm^2	56.0 cm

5. Calculate the area of each shape.

a



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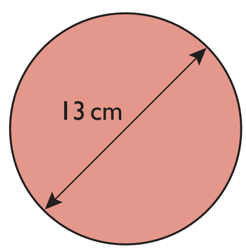

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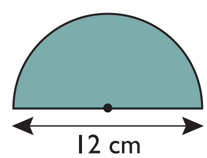

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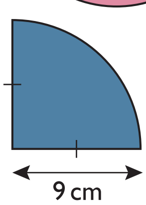

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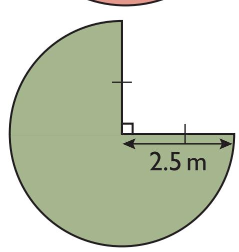

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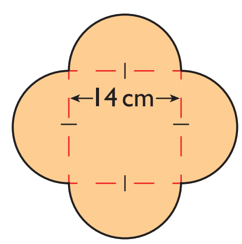
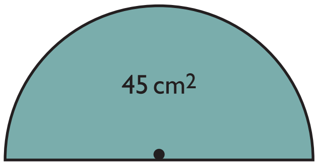


6. A circle has area 200 cm^2. What is its circumference?
7. A circle has circumference 30 cm. What is its area?

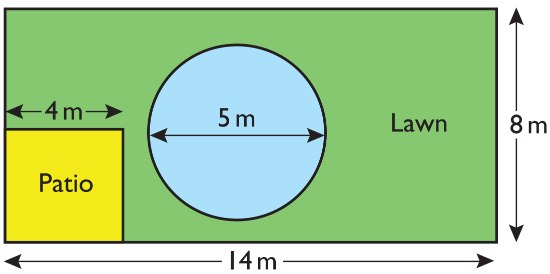
8. This semicircle has area $45\,cm^2$.

Work out the perimeter of the semicircle.



Problem solving exercise

1. Here is a plan of Sophie's garden.



The patio is a square 4 m by 4 m.

The pond is a circle of diameter 5 m.

The rest of the garden is lawn.

Sophie wants to put new turf on the lawn.

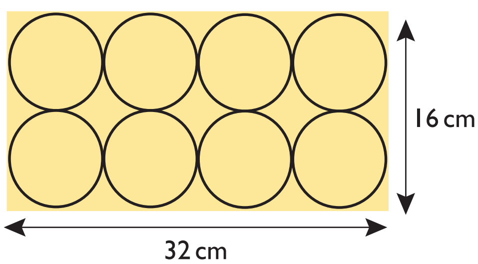
Work out the area of the lawn.

2. Candice is making biscuits. She has a rectangular sheet of biscuit mixture measuring 32 cm by 16 cm.

She cuts out eight circular biscuits of diameter 8 cm.

She uses the remainder of the mixture to make one last biscuit.

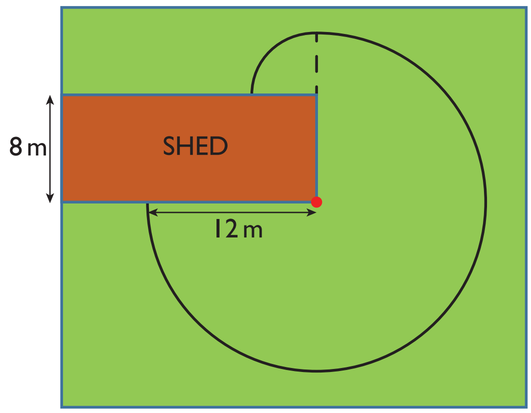
What is its diameter, assuming all the biscuits have the same thickness?



③ Adam ties his goat to the corner of his garden shed.

The shed is 19 m by 8 m. The rope is 12 m long.

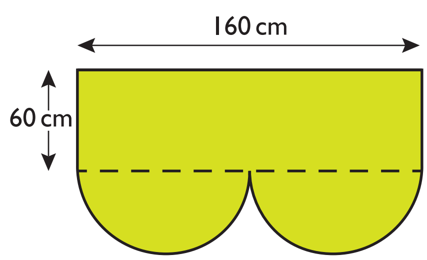
Calculate the total area that Adam's goat can graze.



④ Betsy is designing a display board for the entrance to her company.

The board is in the shape of a rectangle and two semicircles.

The board has a single line of symmetry.



a Work out the area of the display board.

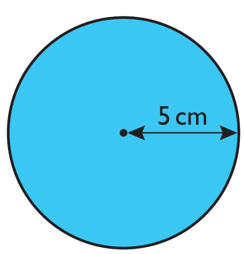
b She uses very expensive paint. It costs her £73 (to the nearest £1).

What is the cost of the paint per square centimetre?

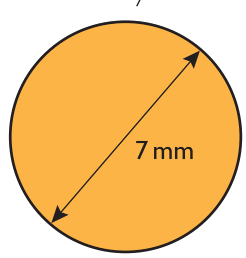
Do I know it now?

① Calculate the area of each of these circles. Give your answers to one decimal place.

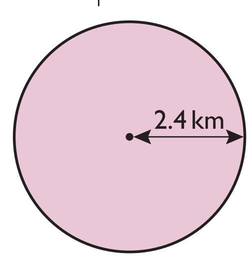
a



b



c



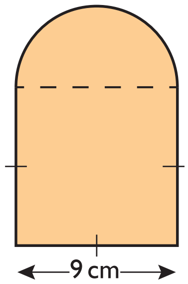
② A circle has an area of 52 cm^2. Calculate to one decimal place

a the radius of the circle

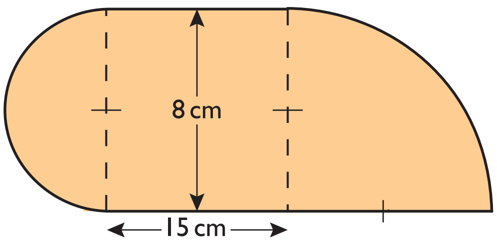
b the diameter of the circle.

③ Calculate the area of each shape.

a



b



Can I apply it now?

① The diagram shows part of the cross-section of a building.

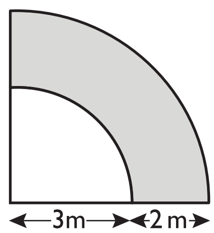
The shaded part of the diagram shows a support for the building.

The edges of the support are quarter-circle arcs with the same centre.

The width of the support is 2 metres.

The radius of the inner quarter circle is 3 metres.

Work out the area of the support's cross-section.



15.2 Pythagoras' theorem

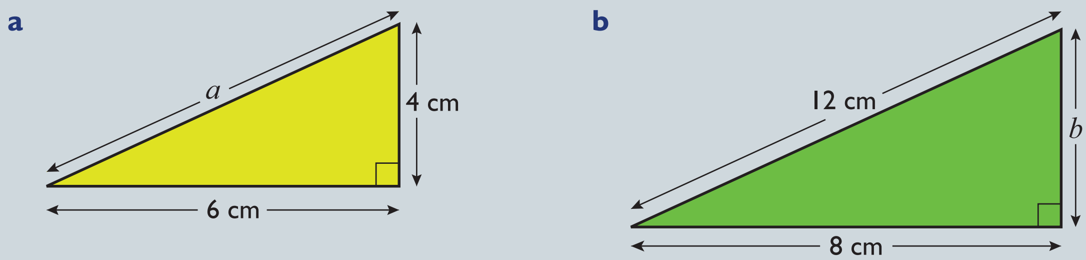
SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Work out the lengths of the lettered sides in these right-angled triangles.

Give your answers correct to one decimal place.



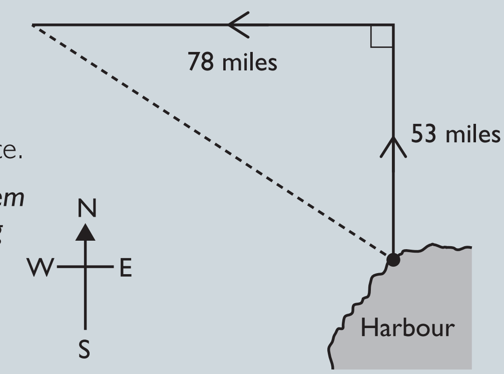
If you can do the question above, try this one on problem solving.

② A ship leaves harbour and sails 53 miles north then 78 miles west.

Calculate the shortest distance back to the harbour.

Give your answer in miles correct to one decimal place.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 209 (Problem solving exercise 15.2 Pythagoras' theorem).



What you need to know

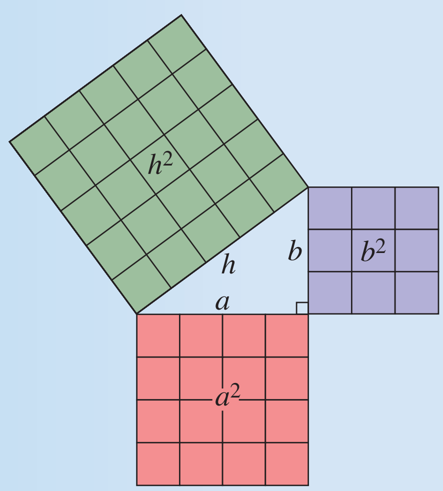
The longest side of a right-angled triangle (the side opposite the right angle) is called the **hypotenuse**, the side marked h on the diagram.

Pythagoras' theorem says that, for a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

Pythagoras' theorem allows the third side of a right-angled triangle to be calculated when the other two sides are known.

Pythagoras' theorem:

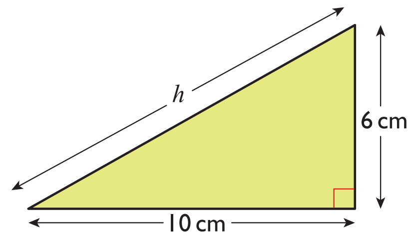
$$a^2 + b^2 = h^2$$



How to do it

Finding the length of the hypotenuse

What is the length of the side labelled h in this triangle?



Solution

Let $a = 10$ and $b = 6$. It could be $a = 6$ and $b = 10$.

$a^2 + b^2 = h^2$ Pythagoras' theorem

$10^2 + 6^2 = h^2$ Substitute the lengths.

$100 + 36 = h^2$

$136 = h^2$

$h = 11.661\ldots$

$h = 11.7$ cm (to 1 d.p.)

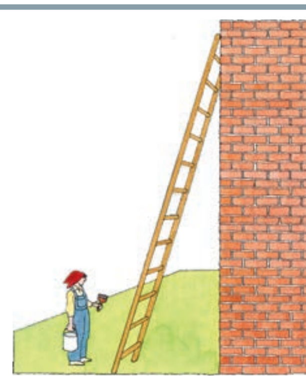
Solving a problem involving a right-angled triangle

Gill is a decorator.

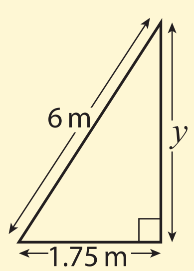
She uses a ladder 6 m long.

The safety instructions on the ladder say that the foot of the ladder must be at least 1.75 m from the base of the wall on horizontal ground.

What is the maximum height her ladder will reach up a vertical wall?



Solution



Draw a sketch and label it. y represents the height reached by the ladder.

$1.75^2 + y^2 = 6^2$ Pythagoras' theorem rewritten for this problem.

$3.0625 + y^2 = 36$

$y^2 = 36 - 3.0625$

$y^2 = 32.9375$

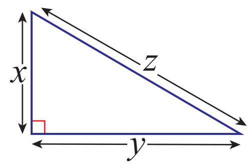
$y = \sqrt{32.9375}$ Square root both sides.

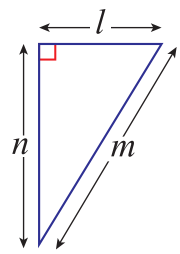
$y = 5.739\ldots$

Maximum height of ladder $= 5.74$ m (to the nearest centimetre)

Learning exercise

1. For each triangle, write down which is the longest side (the hypotenuse). Then write Pythagoras' theorem in terms of the letters given.

a 

b 

2. Work out the length of the hypotenuse of each triangle. Copy and complete each calculation.

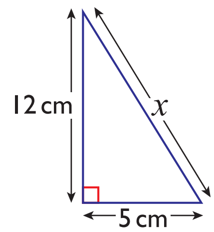
a $x^2 = 5^2 + 12^2$

$x^2 = 25 + \square$

$x^2 = \square$

$x = \sqrt{\square}$

The hypotenuse is $\square$ cm.



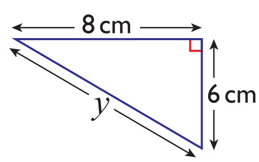
b $y^2 = 6^2 + 8^2$

$y^2 = \square + \square$

$y^2 = \square$

$y = \sqrt{\square}$

The hypotenuse is $\square$ cm.



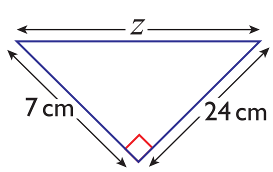
c $z^2 = \square + \square$

$z^2 = \square + \square$

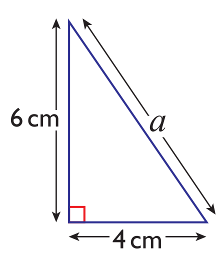
$z^2 = \square$

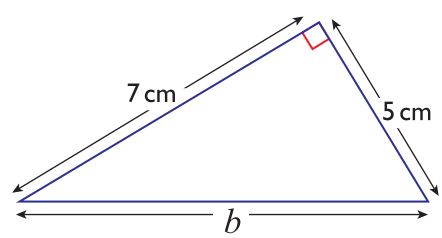
$z = \sqrt{\square}$

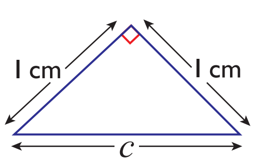
The hypotenuse is $\square$ cm.



3. Work out the length of the unknown side in each triangle. Give each answer correct to 1 decimal place.

a 

b 

c 

④ Work out the length of the unknown side of each triangle. Copy and complete each calculation.

a $x^2 + 4^2 = 5^2$

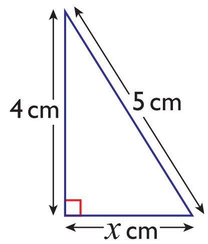
$x^2 + 16 = 25$

$x^2 = 25 - 16$

$x^2 = \square$

$x = \sqrt{\square}$

The base of the triangle is $\square$ cm.



b $y^2 + 15^2 = 17^2$

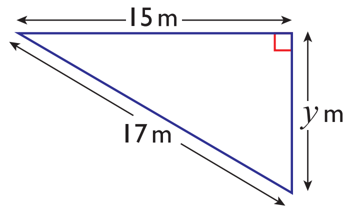
$y^2 + \square = \square$

$y^2 = \square$

$\square$

$\square$

The base of the triangle is $\square$ m.



c $z^2 + \square = \square$

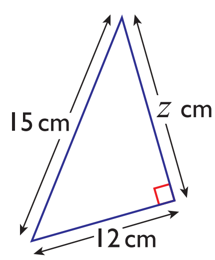
$\square$

$\square$

$\square$

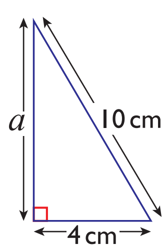
$\square$

The base of the triangle is $\square$ cm.

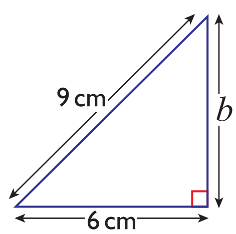


⑤ Work out the length of the unknown side in each triangle. Give each answer correct to 1 decimal place.

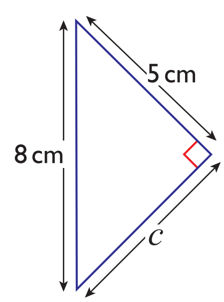
a



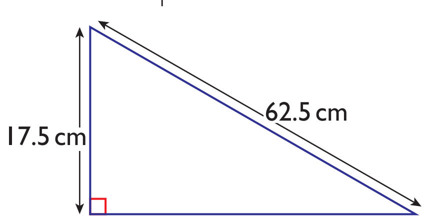
b



c



⑥ Calculate the perimeter and the area of this triangle.



⑦ The sides of triangle A are 7 cm, 9.4 cm and 12.2 cm.

The sides of triangle B are 7.2 cm, 9.6 cm and 12 cm.

The sides of triangle C are 7.4 cm, 9 cm and 12.4 cm.

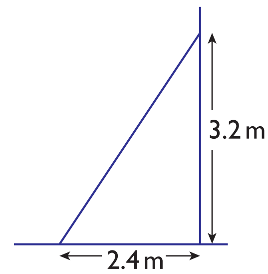
Which of the three triangles are right-angled? Explain your answer.

8 A ladder is placed on horizontal ground and leans against a vertical wall.

It reaches a height of 3.2 m up the wall.

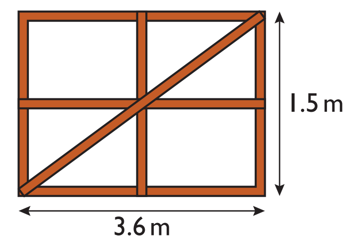
The foot of the ladder is 2.4 m from the base of the wall.

Calculate the length of the ladder.

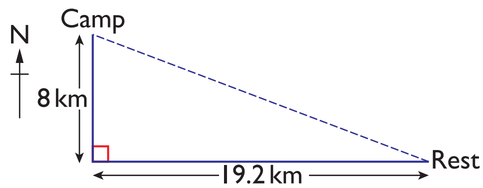


9 Harry is making a wooden gate with three horizontal lengths, three vertical lengths and one diagonal length, as shown in the diagram.

Work out the total length of timber he needs to make the gate.

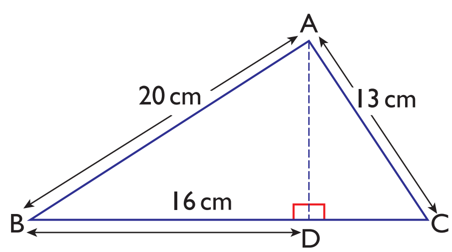


10 A hiker leaves camp and walks south for 8 km, east for 19.2 km and then stops for a rest. She then walks directly back to camp. How much shorter is her return journey?

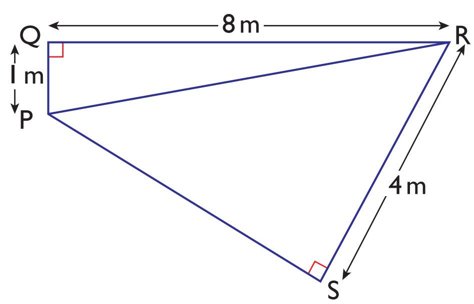


11 Work out

a the length CD **b** the perimeter of triangle ABC **c** the area of triangle ABC.

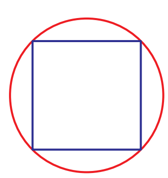


12 Work out the perimeter and area of the quadrilateral PQRS.



13 A square is drawn with its vertices on the circumference of a circle of radius 5 cm.

What is the length of each side of the square?



Problem solving exercise

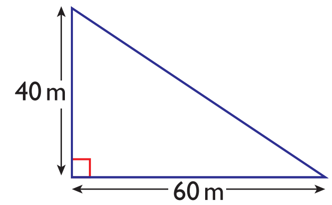
① A field is in the shape of a right-angled triangle, as shown in the diagram.

Fencing is going to be put around the field.

The fencing costs £17.50 for a 2-metre length.

Work out the cost to put fencing round the field.

Give your answer correct to the nearest £10.

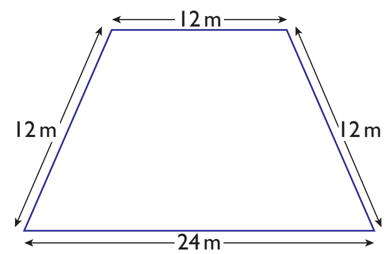


② The diagram shows a garden in the shape of a trapezium.

Turf is going to be put down in the garden.

Turf costs £2.50 per square metre.

Work out the total cost of the turf for the garden.



③ Two boats leave a port at the same time.

The Sirius sails due north at 10 km/h.

The Polaris sails due east at 24 km/h.

What is the distance between the boats after one hour?

④ This diagram shows a field in the shape of a rectangle ABCD.

The rectangle has a width of 57 m and a length of 76 m.

Ali starts from A and runs once around the field.

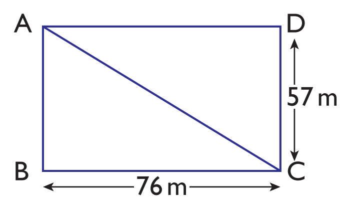
He goes from A to B to C to D and back to A.

Beth starts from A and runs along AC and back again.

They both run at the same speed.

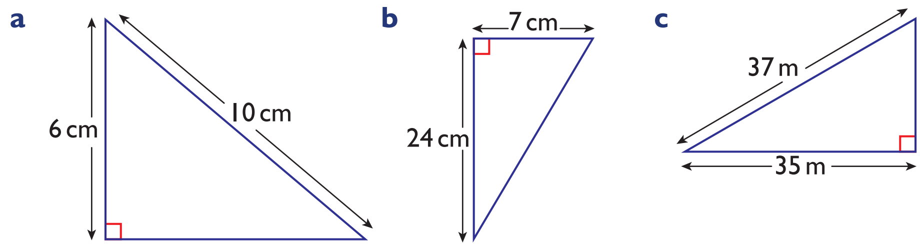
They start at the same time.

Show that when Beth gets back to A, Ali is at D.

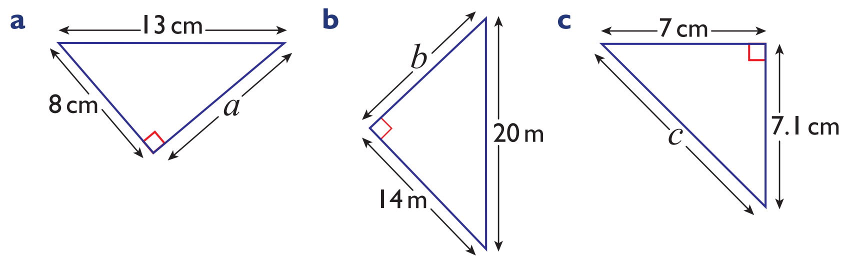


Do I know it now?

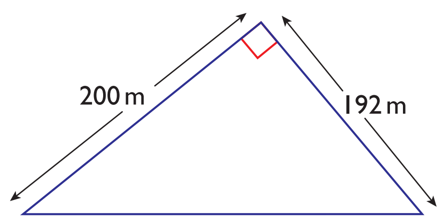
① Calculate the length of the unknown side in each triangle.



② Calculate the length of the unknown side in each triangle. Give your answers correct to one decimal place.

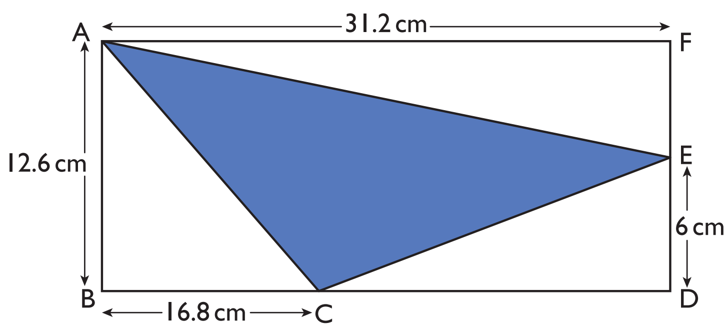


③ Calculate the perimeter and area of this field.



Can I apply it now?

① ABCDEF is a rectangle. Calculate the perimeter of triangle ACE.



CHAPTER 16

ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Construction

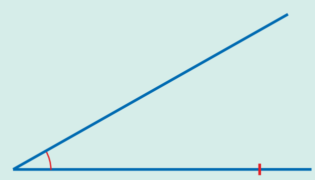
JUST IN CASE

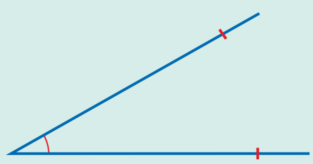
Constructions with a ruler and a protractor

Constructing a triangle when two sides and the included angle are given

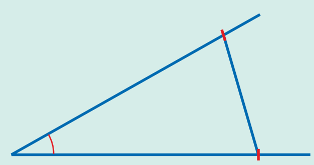
- Use a ruler to draw one side of the correct length.

- Use a protractor to draw the correct angle at one end of the side.
- Use a ruler to mark the correct length of the second side.





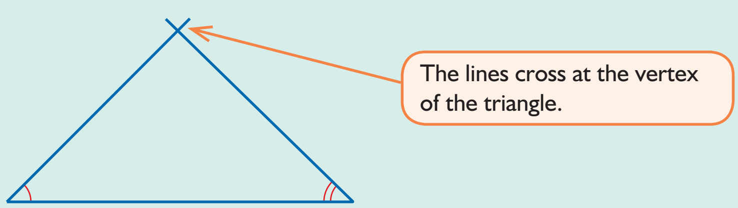
- Draw the third side to form the triangle.



Constructing a triangle when two angles and the included side are given

- Use a ruler to draw one side of the correct length.

- Use a protractor to draw the correct angles at both ends of the side.



SKILLS CHECK

Warming up

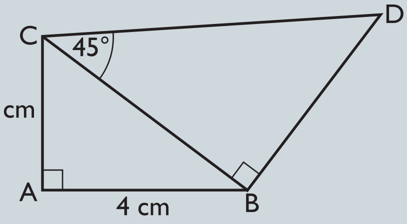
Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Construction with a ruler and protractor

a Use a ruler and protractor to make an accurate drawing of ABDC.

b Measure

i the angle ABC **ii** the length BC **iii** the length CD.



16.1 Constructions with a pair of compasses

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

1

a Make an accurate drawing of this triangle.

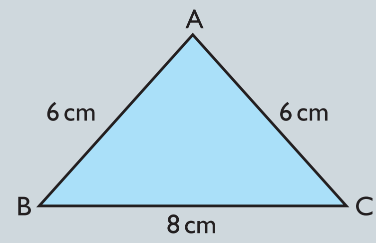
b What type of triangle is it?

c Bisect angle A.

The angle bisector meets BC at D. Mark point D on your triangle.

d Measure BD and DC.

e What do you notice?



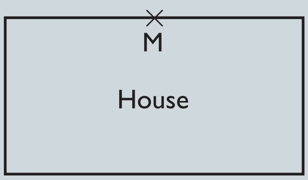
If you can do the question above, try this one on problem solving.

2 Look at this plan of a house.

A garden wall is built from M.

It is perpendicular to the house.

Copy the diagram and construct the line of the wall.



If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 220 (Problem solving exercise 16.1 Constructions with a pair of compasses).

What you need to know

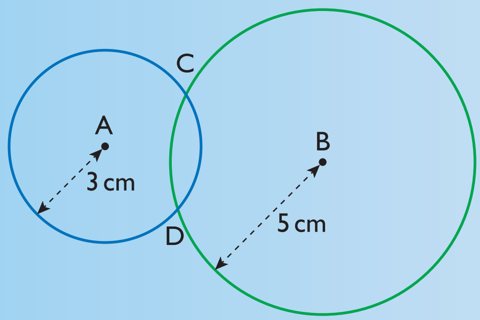
Did you know?

The plans for a house need to be accurate, with right angles and the lengths of walls carefully constructed. This accuracy needs to be replicated on the ground as foundations are dug and walls built.

You use a pair of compasses to draw a circle.

All the points on the circumference of a circle are the same distance from the centre. This is a very useful property.

The intersection of two circles will locate a point at given distances from two other points.



The points C and D are both 3 cm from A and 5 cm from B.

This property means you can use circles to construct many other things, including

- triangles, given their sides
- line bisectors
- angle bisectors
- perpendicular from a point to a line
- perpendicular from a point on a line.

Bisectors are lines that cut other lines and angles into two equal halves.

A **line bisector** cuts a line into two equal halves and is perpendicular to the line.

A **perpendicular** is a line at right angles to another line.

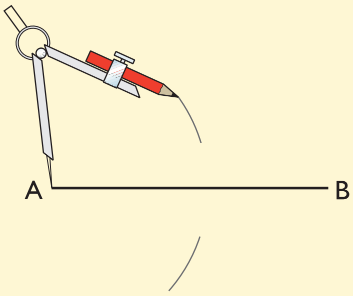
How to do it

Bisecting a line

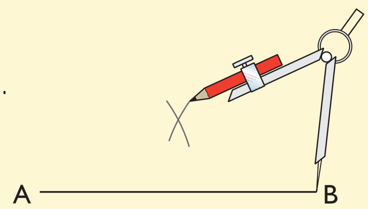
Bisect the line AB.

Solution

- Open your compasses.
  Put the point on A.
  Draw arcs above and below the line.

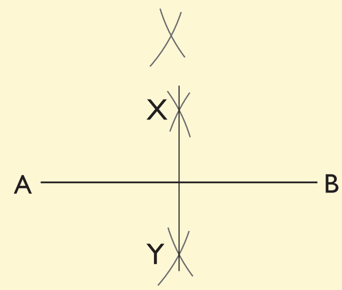


- Do not adjust the compasses.
  Put the point on B.
  Draw two more arcs to cut the first two.



- The arcs meet at X and Y.
  Draw the line XY.

The bisector of a line cuts the line in half.



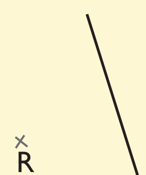
The bisector is a line of symmetry for the line AB.

➤ Constructing a perpendicular from a point to a line

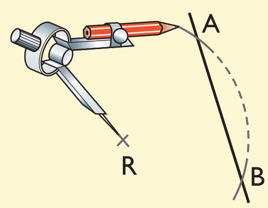
Construct the perpendicular from the point R to the line.

Solution

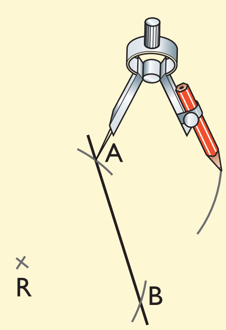
- Draw a line.
  Mark point R away from the line.



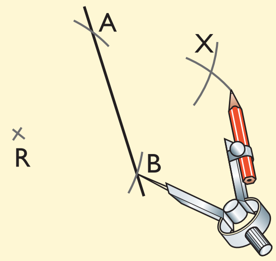
- Put the point of the compasses on R.
  Draw an arc.
  It intersects the line at A and B.



- With the point on A, draw an arc on the opposite side of the line from R.

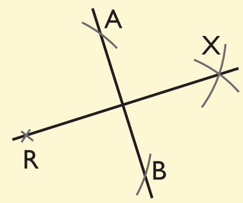


- Do not adjust the compasses.
  With the point on B, draw another arc.
  The arcs intersect at X.



- Draw a line through R and X.

The line RX is perpendicular to the line AB.

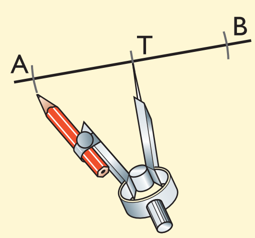


➤ Constructing a perpendicular from a point on a line

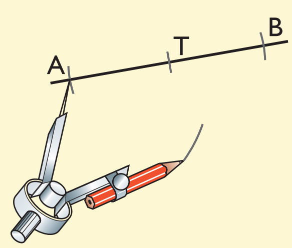
Construct a perpendicular from the point T on this line.

Solution

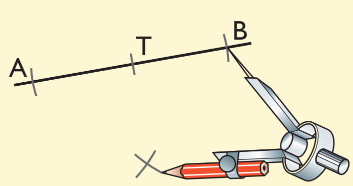
- Put the point of the compasses on T.
- Mark two points, A and B, on the line.
  Each point is the same distance from T.



- Open the compasses wider.
  With the point on A, draw an arc.

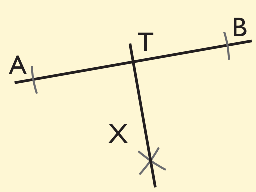


- Do not adjust the compasses.
  With the point on B, draw an intersecting arc.



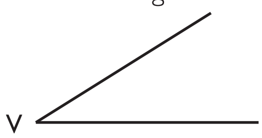
- Draw a line through T and the intersection X.

Line TX is perpendicular to AB from point T.



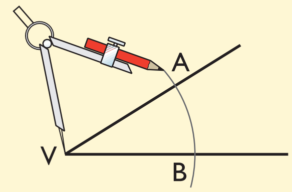
Bisecting an angle

Bisect this angle.

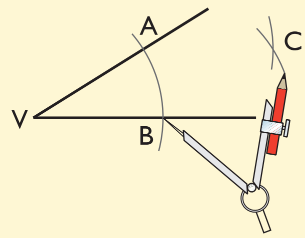


Solution

- Put the point of the compasses on V.
  Draw an arc cutting both lines at points A and B.

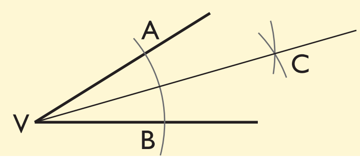


- Put the point of the compasses on A.
  Draw an arc.
- Do not adjust the compasses.
  Put the point of the compasses at B.
  Draw an arc.
- These two arcs meet at C.



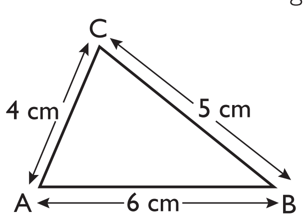
- Join V to C.

The line VC is the angle bisector.



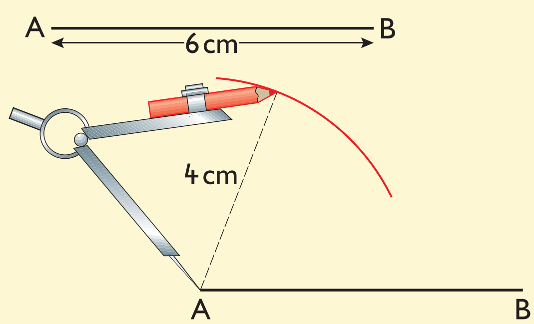
Constructing a triangle

Make an accurate drawing of this triangle.



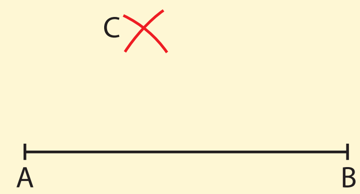
Solution

- Draw a line 6 cm long.
- Draw an arc of radius 4 cm and centre A.
  The point C is on this arc.



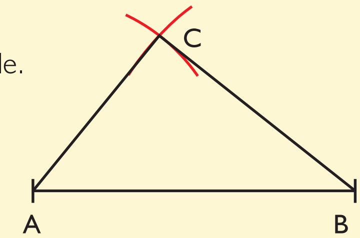
- Now draw an arc of radius 5 cm and centre B.

  C is also on this arc.



- The two arcs meet at C.

  Join AC and BC to form the triangle.

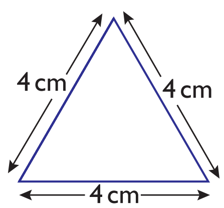


Learning exercise

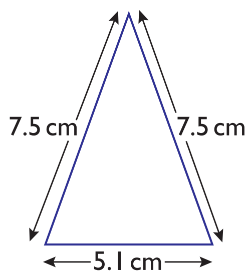
① Use a ruler and a pair of compasses to construct each triangle accurately.

Then measure its angles.

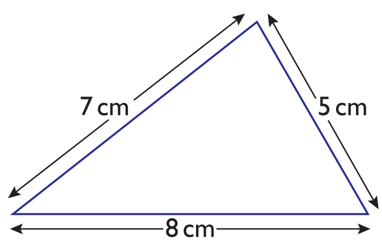
a



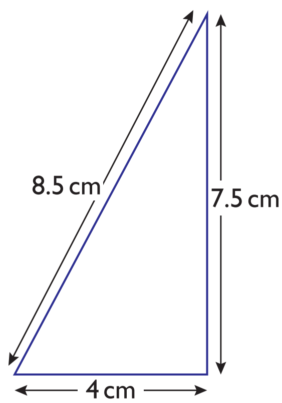
b



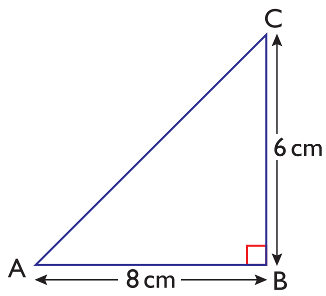
c



d



② **a** **i** Draw this triangle accurately. (You may use a protractor for the right angle.)



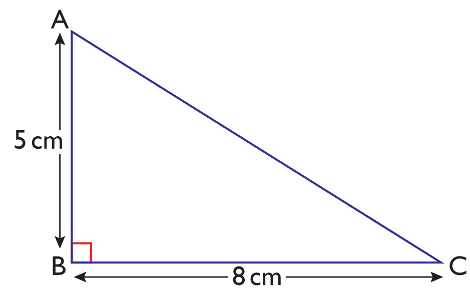
ii Draw the perpendicular bisectors of AB and BC. Remember to use a ruler and a pair of compasses.

iii What do you notice about the point where they meet?

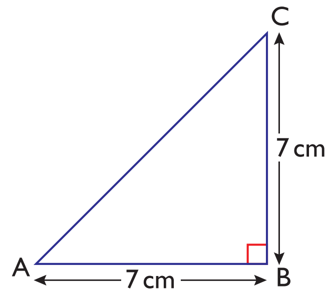
b Repeat part **a** for each of the following right-angled triangles.

Do you always notice the same thing?

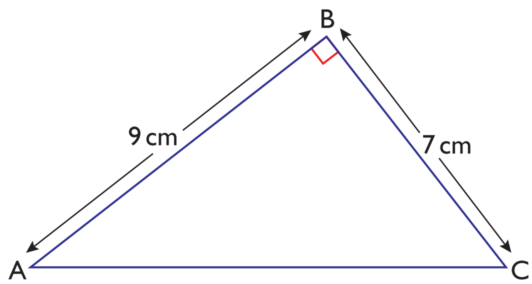
i



ii



iii



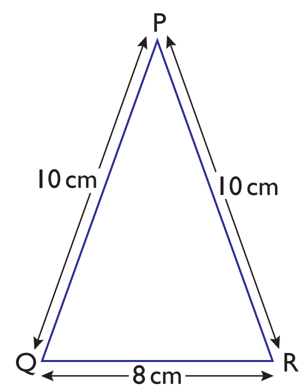
③ **a** **i** Make an accurate drawing of the triangle shown.

ii Draw the perpendicular bisector of each of the three sides. Remember to use a ruler and a pair of compasses.

You should find that the three lines you have just drawn all go through a common point. (If your lines do not go through a point, start again. You have not done the drawing accurately!)

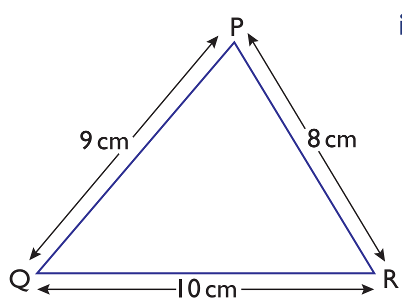
iii Label this common point O. Draw a circle with centre O and radius OP.

iv What do you notice?

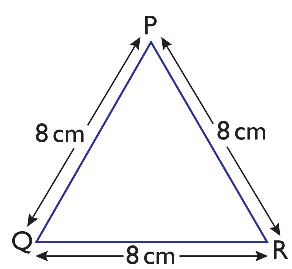


b Repeat part **a** for each of the following triangles. Do you always notice the same thing?

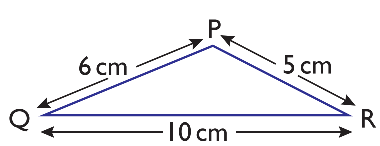
i



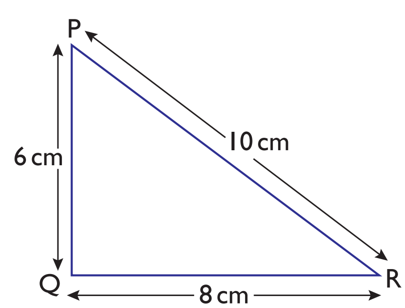
ii



iii



iv



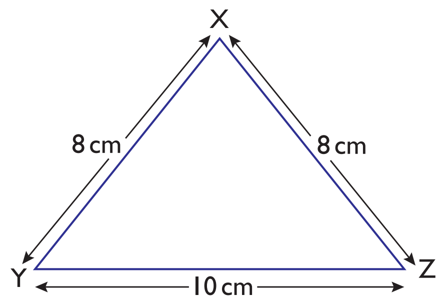
④ **a** **i** Make an accurate drawing of the triangle shown.

ii Construct the bisectors of the three angles. You can use a ruler and a pair of compasses, but not a protractor.

You should find that the three lines you have just drawn all go through a common point. (If your lines do not go through a point, start again. You have not done the drawing accurately!)

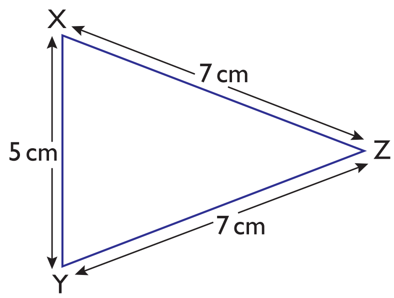
iii Label this common point C. Draw a circle with centre C. Set your compasses so that the circle just touches the line XY.

iv What do you notice?

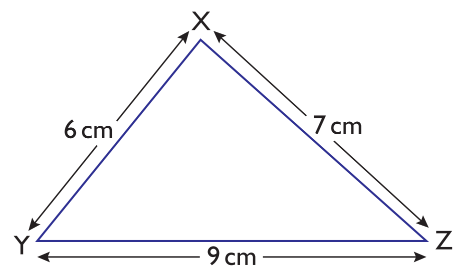


b Repeat part **a** for each of the following triangles. Do you always notice the same thing?

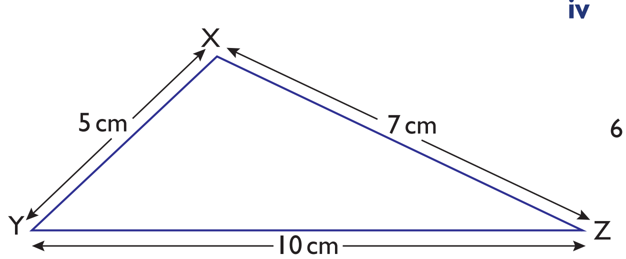
i



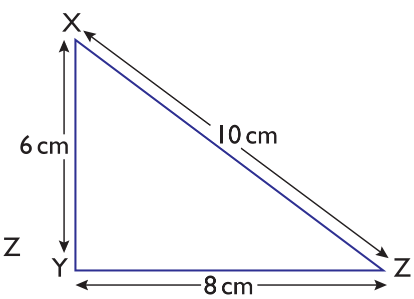
ii



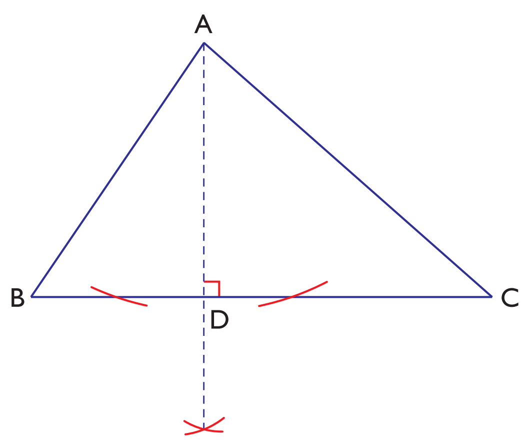
iii



iv



⑤ **a** Copy and complete the working to calculate the area of triangle ABC.



By measurement

BC = $\square$ cm

AD = $\square$ cm

So the area of triangle ABC = $\frac{1}{2} \times \square \times \square\ \text{cm}^2$

= $\square\ \text{cm}^2$ (to 1 decimal place)

b For each part, first draw the triangle and then use the method from part **a** to calculate its area. Use only a ruler and a pair of compasses.

i Triangle XYZ where XY = 8 cm, YZ = 10 cm and ZX = 7 cm

ii Triangle LMN where LM = 5 cm, MN = 6 cm and NL = 8 cm

iii Triangle PQR where PQ = 7 cm, QR = 10 cm and RP = 5 cm

6 Draw a horizontal line AC 10 cm long. Mark point T on the line 3 cm from A.
Construct a line perpendicular to AC that passes through T.
On one side of this line, mark a point B such that BT = 4 cm.
On the other side, mark a point D such that DT = 4 cm.
Join ABCD to make a quadrilateral.
Describe this quadrilateral.

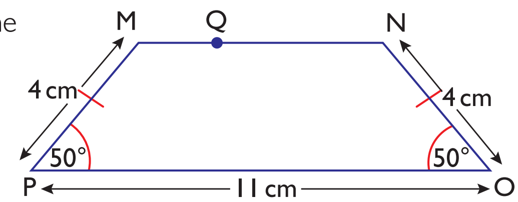
7 **a** Draw a vertical line PR 8 cm long.
Construct the perpendicular bisector of PR. Mark the mid-point of the line PR as M.
Mark points Q and S on the perpendicular bisector of PR, on either side of the line, where QM = SM = 6 cm.
Join PQRS to make a quadrilateral.
Describe this quadrilateral.

b Repeat part **a** but this time let QM = SM = 4 cm.
Describe this quadrilateral.

8 **a** Construct the trapezium MNOP accurately. Mark the point Q 2 cm from point M.

b Construct a line perpendicular to PO that passes through Q.

c Make suitable measurements and work out the area of the trapezium.



Problem solving exercise

1 Here is a line AB.

a Using only a straight edge (ruler) and a pair of compasses, draw

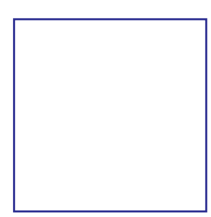
i a triangle ABC where the angles at A and B are both 45°

ii an equilateral triangle ABD.

b Name the two possible shapes for the quadrilateral ACBD.

2 Jim has a triangular garden. The lengths of the sides of his garden are 20 m, 16 m and 14 m.
Draw a suitable scale diagram of the garden, using only a pencil, a ruler and a pair of compasses.
Construct a line through one vertex of the triangle perpendicular to the opposite side.
Make a suitable measurement and then calculate the area of Jim's garden.

3 Here is a square.



Using only a pencil, a straight edge and a pair of compasses, construct

a a triangle that has the same area as the square

b a rectangle that has the same area as the square, but half the height

c a rhombus with the same area as the square.

④ P and Q are two lighthouses, 10 km apart. P is due north of Q.

At 08:00 the Whinlatter is at point R, 14 km from P and 6 km from Q.

a Show the information on a scale drawing.

At 08:30 the Whinlatter is at point S, 8 km from P and 9 km from Q.

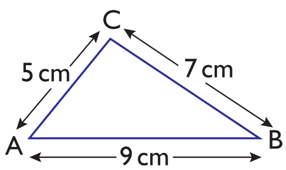
b Add this to your scale drawing.

c What is the distance between R and S?

d At what speed is the Whinlatter sailing?

Do I know it now?

① **a** Use a pencil, a ruler and a pair of compasses to make an accurate drawing of triangle ABC.

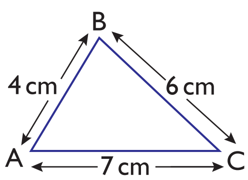


b Measure the angles of the triangle.

c Construct a line through C perpendicular to AB.

d Make a suitable measurement and then work out the area of the triangle.

② **a** Use a pencil, a ruler and a pair of compasses to construct triangle ABC.



b Construct the perpendicular bisector of each of the three sides.

c Mark the point M where the three perpendicular bisectors meet.

d Draw a circle with centre M which passes through the points A, B and C.

e Measure the radius of the circle.

Can I apply it now?

① Alice is designing a logo for her company. It is the shaded region in this sketch.

The circle has radius 4 cm.

The logo must have rotational symmetry of order 6.

a Using only a pencil, a ruler and a pair of compasses, construct an accurate drawing of the logo.

b Measure the lengths of one side of the large equilateral triangles.

c Without doing any further measurements, work out the area of the logo.

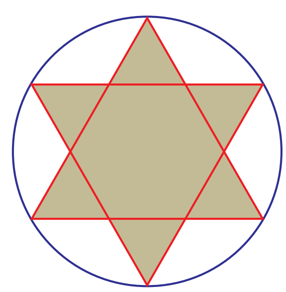


Diagram **not** accurately drawn.

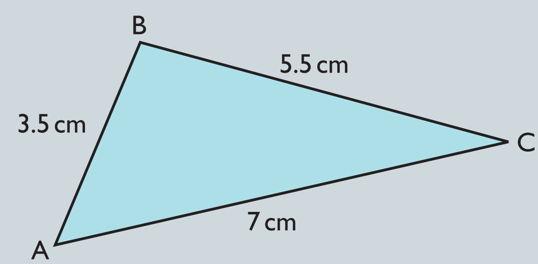
16.2 Loci

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Make two accurate constructions of this triangle.



a On the first triangle, draw the three loci that show points the same distance from

i A and B

ii B and C

iii C and A.

How do these three loci help you to find a point which is the same distance from A, B and C?

b On the second triangle, draw the three loci that show the points the same distance from

i AB and AC **ii** BA and BC **iii** CA and CB.

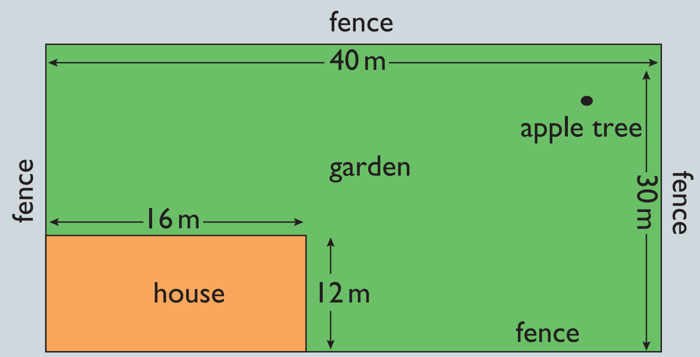
How do these three loci help you to find a point which is the same distance from all three sides?

If you can do the question above, try this one on problem solving.

② The diagram shows the plan of a house and a garden.

A tree is to be planted which must be at least

- 10 metres from the house
- 6 metres from the apple tree located in the corner of the garden
- 4 metres from the fences.



Make a scale drawing of the garden and shade the area in which the tree can be planted.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 227 (Problem solving exercise 16.2 Loci).

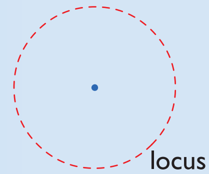
What you need to know

Did you know?

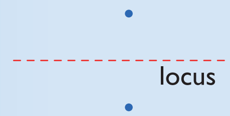
The Earth's orbit around the Sun is an ellipse. An ellipse is the locus of points such that the sum of the distances from its two centres, called foci, remains constant. The Sun is at one of the foci of the ellipse.

A **locus** is the path of all the points which obey a rule.
In two dimensions:

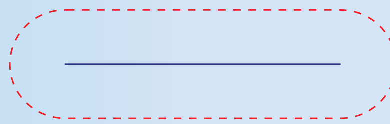
- The locus of all the points which are **the same distance from one point** is a circle.



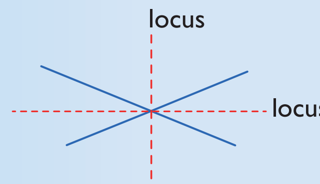
- The locus of all the points which are **the same distance from two points** is the perpendicular bisector of the line which joins those two points.



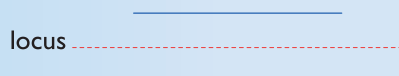
- The locus of all the points which are **the same distance from a line segment** is two parallel lines with semicircles joining the ends.



- The locus of all the points which are **the same distance from two lines that cross** is the angle bisector of the angles formed by the two lines.



- The locus of all the points which are **the same distance from two lines that do not cross** is a line parallel to them, and midway between them.



Note that the distance of a point from a line is always the shortest distance – the one that is perpendicular to the line.

Note:

- *Construct* means do it accurately using instruments.
- *Sketch* means do it by eye illustrating the important features.
- *Draw* means somewhere between constructing and sketching. You should use your judgement to decide what is required.

How to do it

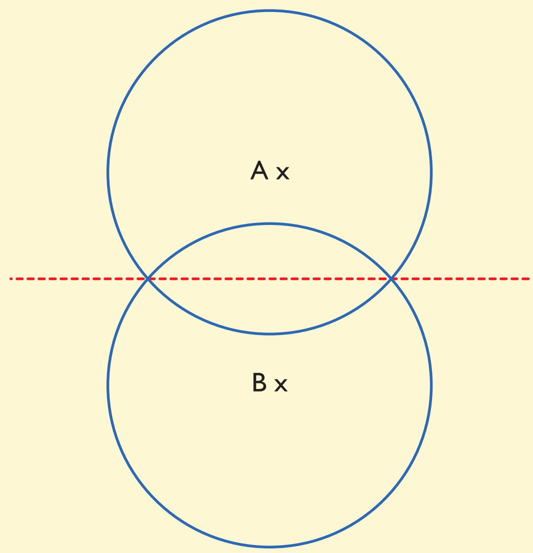
Loci equidistant from two points

A and B are the positions of two radio beacons.
An aeroplane flies so that it is always the same distance from each beacon.
Construct a line to represent the aeroplane's path.

A x

B x

Solution



The aeroplane's path is the perpendicular bisector of the line AB.

Learning exercise

① Mark a point P on your page.

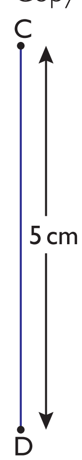
a Make an accurate drawing of the locus of points that are 4 cm from the point P.

b What name is given to this locus?

② Mark a point P on your page.

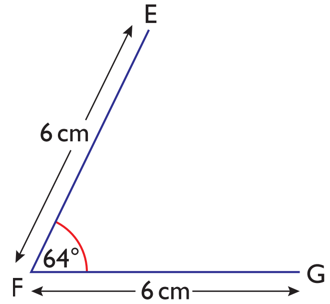
Shade the locus of the points that are more than 4.5 cm from P but less than 6.5 cm from P.

③ **a** Copy this diagram.



b Draw the locus of points that are the same distance from C as they are from D.

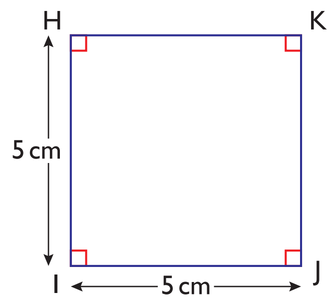
4 a Make a full-size copy of this diagram.



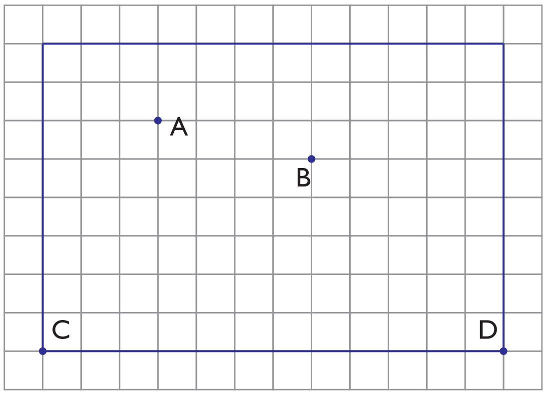
b Make an accurate drawing of the locus of points that are the same distance from EF and FG.

c What is the name given to this locus?

5 a Draw the square HIJK. Its sides are 5 cm long.
b Draw the locus of points that are the same distance from HK and HI.



6 a Copy this diagram on to centimetre-squared paper.



b Shade these regions.

i Points that are at most 2 cm from A

ii Points that are less than $2\frac{1}{2}$ cm from point B

iii Points that are less than 1 cm from the line CD. (Remember the shortest distance to CD is measured by drawing a line that is at right angles to CD)

c Are there any points in all three regions?

7 Hassan designs a logo.

He starts with an equilateral triangle with sides of length 6 cm.

He splits the triangle into six regions by drawing three lines. Each line is the locus of points inside the triangle that are the same distance from two of its sides.

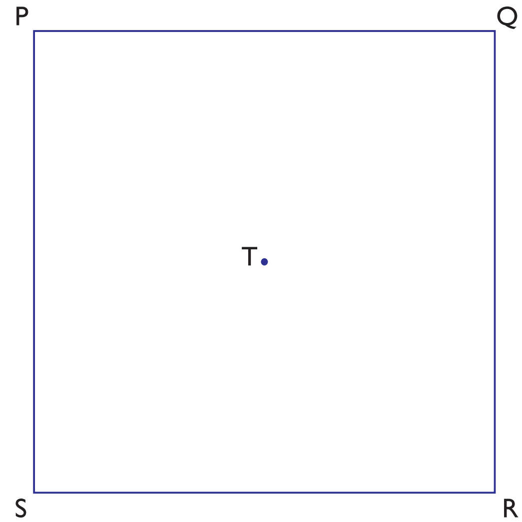
Make an accurate full-size drawing of the logo.

8 PQRS is a square. Its sides are 6 cm long. T is the centre point.

a Draw the square accurately.

b Shade the locus of points on the diagram that are

- inside the square
- and more than 3 cm from T
- and closer to PQ than any of the other sides of the square
- and closer to QR than PS.

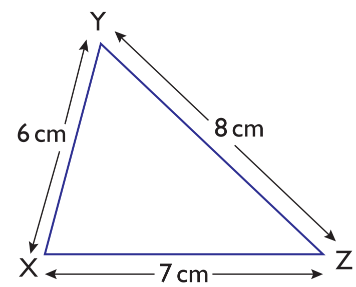


9 **a** Construct triangle XYZ accurately.

b Draw the locus of points that are the same distance from X and Z.

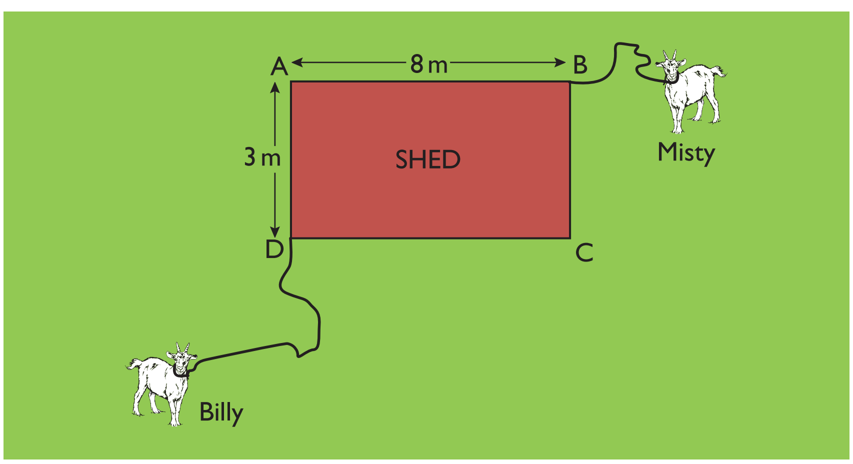
c Draw the locus of points that are the same distance from X and Y.

d Mark the point Q that is the same distance from X, Y and Z, then measure QX.



10 The diagram shows two goats tied to a rectangular shed. One goat, Misty, is attached to point B by a rope that is 4 m long. The other goat, Billy, is attached to point D by a rope that is 8 m long.

The shed measures 8 m by 3 m and is located in the centre of a large field.



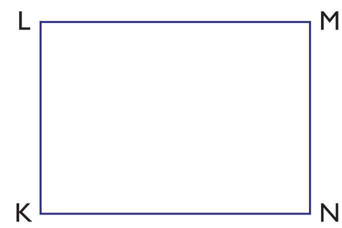
a Make an accurate scale drawing using the information given. Use a scale of 1 cm = 1 m.

b Using different colours, shade the areas where

i only Misty can graze

ii only Billy can graze

iii both goats can graze.



⑪ **a** Draw a rectangle KLMN with side LM longer than side KL.
Then draw the locus of points in the rectangle that are the same distance from KL and KN.

b Draw the locus of points that are the same distance from LM and LK.

c How many points are the same distance from the three lines LM, LK and KN?

d Explain why there are no points that are the same distance from all four edges of the rectangle.

e In a different rectangle, PQRS, there is a point O that is the same distance from all four edges. What can you say about the rectangle PQRS?

Problem solving exercise

① PQRS is a field in the shape of a rectangle.

PQ = 100 m and QR = 80 m.

There is a path crossing the field. All points on the path are the same distance from PS and PQ.

There is a second path that also crosses the field. All points on this path are the same distance from PS and QR.

The two paths cross at the point X.

Make a suitable drawing and work out the distance of X from S.

② A, B and C are three towns.

C is 20 miles due east of B.

A is 16 miles due north of B.

A mobile phone tower, T, is to be built such that it is the same distance from A and B, and 10 miles from C.

a Draw a scale diagram. Use it to locate the two possible positions of the tower.

b Find the shorter distance of the tower from A.

③ ABCD is the plan of a house.

AB = DC = 8 m

BC = 6 m

A path of width 2 m is to be made round three sides of the house.

The path starts at A, goes along AB, then BC and then CD.

a Make an accurate scale drawing of the path.

b What is the total area of the path?

c Jake says, 'The path is the locus of the points that are 2 m away from the house.'

Give two reasons why Jake's statement is not correct.

④ A pitch for five-a-side football is 32 m long and 16 m wide.

a Draw a scale diagram of the pitch. Use the scale 1 cm = 4 m.

Lights are placed at the four corners of the pitch and at the mid-points of the two longer sides.

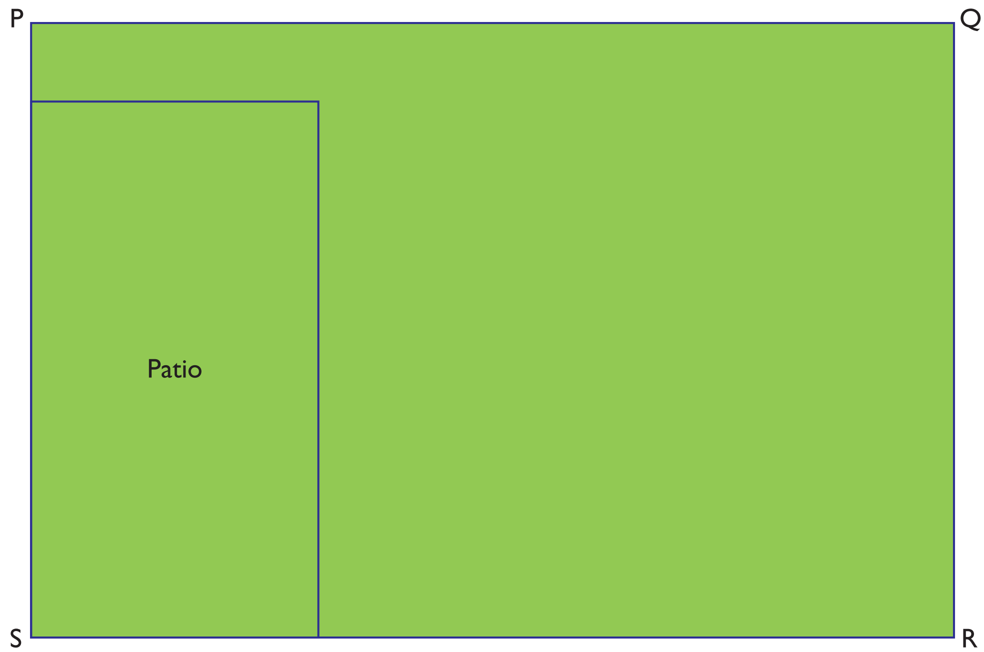
A light can light all points up to 10 m away.

b Show that the six lights are not enough to light the whole football pitch.

c Show how some of the lights could be moved to light the whole pitch.

⑤ Here is a scale drawing of a rectangular garden PQRS.

Scale: 1 cm represents 2 m

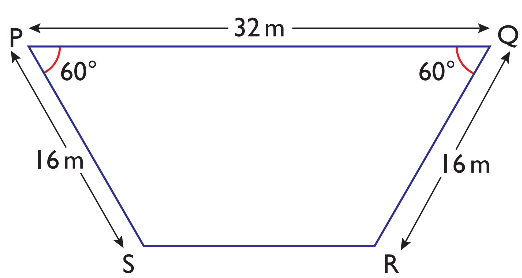


Jane wants to plant a tree in the garden at least 10 m from the corner R, less than 6 m from SR and nearer to PQ than to PS.

The tree cannot be planted on the patio.

On a copy of the diagram, shade the region where Jane can plant the tree.

⑥ PQRS is the plan of an enclosure at a zoo.
Its shape is an isosceles trapezium.



a Make an accurate scale drawing of the enclosure.

The enclosure is divided up into four regions – p, q, r and s – by wire netting.

Region p is those points that are nearer to vertex P than to any other vertices, region q is those points that are nearer to vertex Q than to any other vertices, and so on.

b Draw lines on your diagram to show the positions of the wire netting.

c Make suitable measurements and work out the total length of wire netting. Give your answer to the nearest metre.

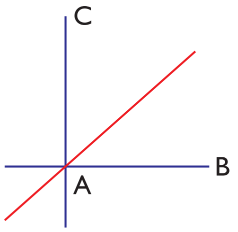
d Work out the area of each region to the nearest square metre.

e Show that, with another 32 m of wire netting, the enclosure can be divided into six regions of equal area.

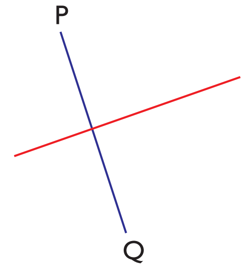
Do I know it now?

① Describe the loci represented by the red lines or curves in each of the following diagrams.

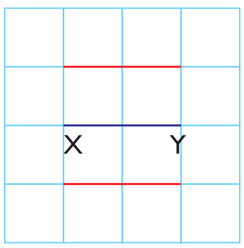
a



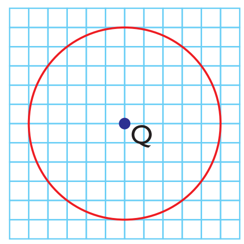
b



c

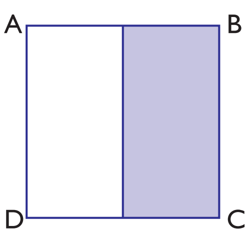


d

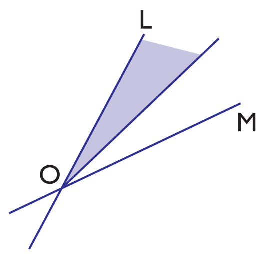


② Describe the region(s) shaded in each diagram.

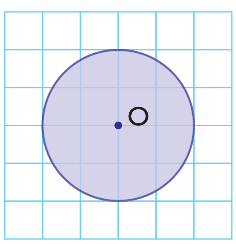
a



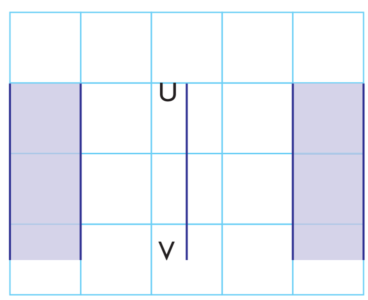
b



c



d



Can I apply it now?

① A watering pipe is laid in the shape of a circle of radius 40 m.

The region of the garden within 10 m of the pipe can be watered from the pipe.

a Draw a scale diagram to show this region.

b What is the area of the garden that can be watered from the pipe?

CHAPTER 17

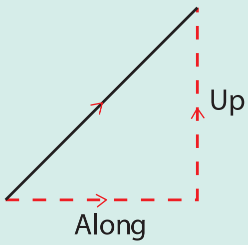
ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Transformations

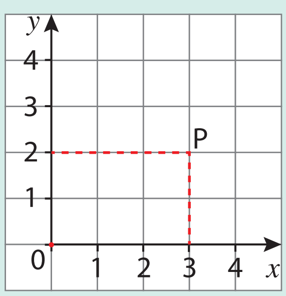
JUST IN CASE

Co-ordinates

To describe a position in two dimensions you need two pieces of information.
These are often the distances, along and up.



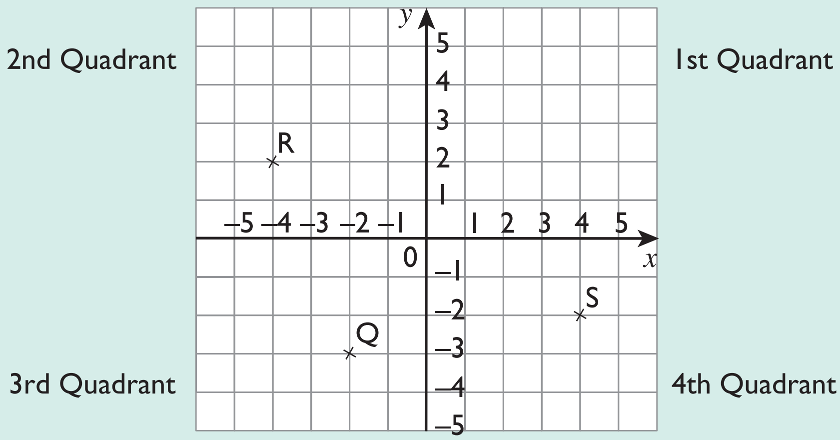
You may want to give the position of a point or place.
Cartesian co-ordinates are used for the position of points.
For example, the point P, marked on the grid below, has co-ordinates (3, 2).
Remember the x co-ordinate comes before the y co-ordinate.



The y axis is vertical.
The origin is the point (0, 0).
The x axis is horizontal.

Co-ordinates in four quadrants

Cartesian co-ordinates can be extended to a larger grid.
The x axis is extended to the left and the y axis is extended downwards.
This gives four quadrants.
Q has co-ordinates (−2, −3).
R has co-ordinates (−4, 2).
S has co-ordinates (4, −2).

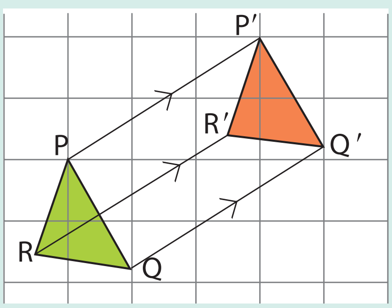


Translation

A **transformation** moves an object according to a rule.

One transformation is a **translation**.

In the diagram, object PQR is translated to the image P′Q′R′.



The object slides without turning.

The translation is described as 3 units right and 2 units up.

It can also be written as

$\begin{pmatrix} 3 \\ 2 \end{pmatrix}$ ← This way of writing it is called a **vector**.

The translation from A to B is sometimes written as $\overrightarrow{AB}$.

The image and the original object are **congruent**.

This means they are the same shape and the same size.

To fully describe a translation, you need to give the vector or an equivalent description.

Reflection

Reflection is a transformation.

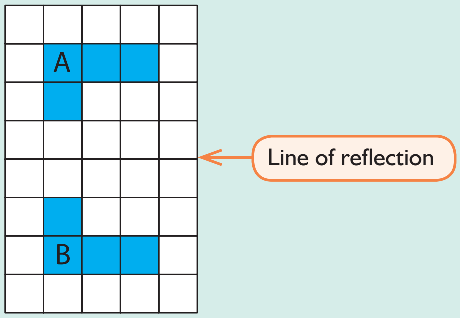
When an **object** is reflected in a line, its **image** is formed on the other side of the line. Each point is at the same distance from the line.

The line is called the **mirror line** or the **line of reflection**.

The image and the original object are **congruent**.

They are the same shape and the same size.

The line of reflection does not have to be horizontal or vertical; it can be in any direction.



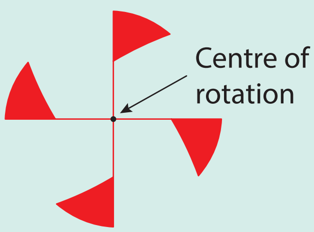
To fully describe a reflection, you need to give the mirror line.

Rotation

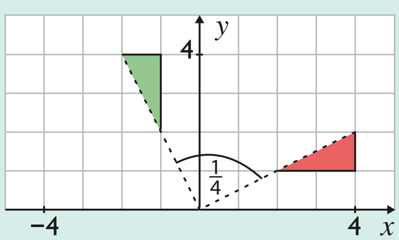
Another transformation is a **rotation**.
Rotation is a movement made by **turning** an object.
The flag here has been rotated four times, each time by a quarter turn.
A full turn is 360°, so

- a quarter turn is 90° (a right angle)
- a half turn is 180°
- a three-quarters turn is 270°.

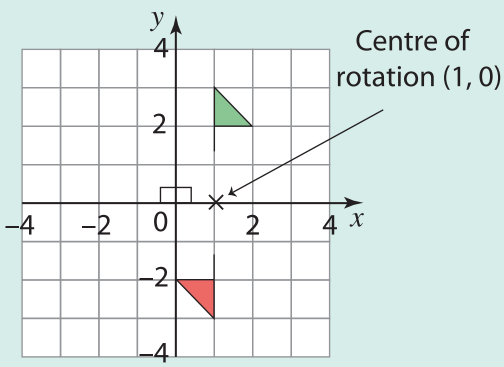
The flag has been rotated by holding the end of the stick still.
This is the **centre of rotation**.



In this diagram the red triangle is the image of the green triangle when it is rotated through 90° clockwise about the origin.



In this diagram the red flag is the image of the green one when it is rotated through 180° about the point (1, 0).



A rotation of 180° can be **clockwise** or **anticlockwise**.

Objects and their images as a result of a rotation are always congruent.

To fully describe a rotation, you need to give its centre, angle and direction.

Enlargement

Enlargement is a transformation that changes the size of an object.

One shape is an enlargement of another if **all the angles in the shape are the same** and the **lengths of the sides** have all been increased by the **same scale factor**.

In this diagram the pentagon ABCDE is enlarged by a scale factor of 3 to A′B′C′D′E′.

The position of the image depends on the centre of enlargement, X.

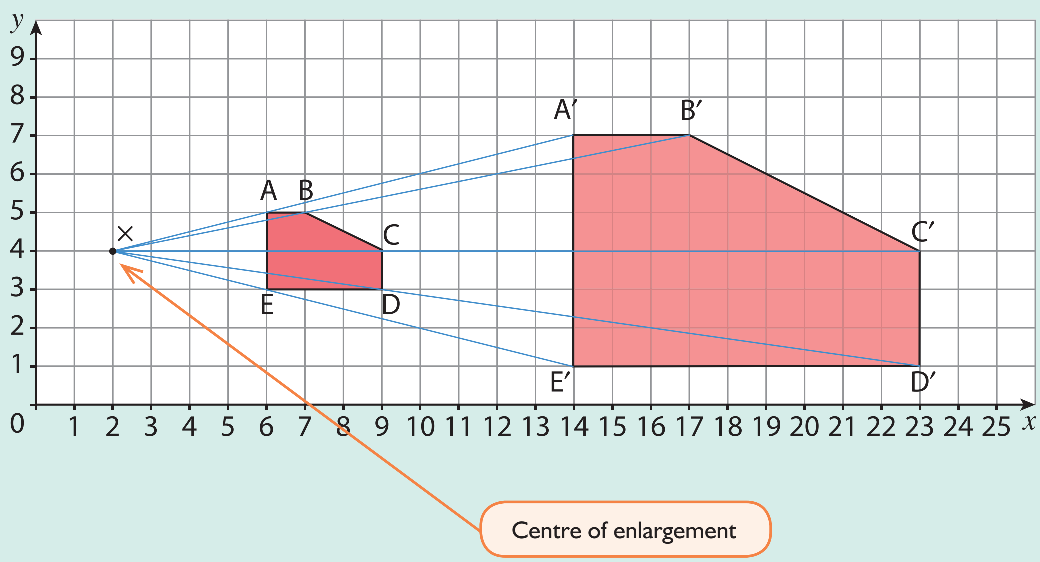
Because the scale factor of enlargement is 3:

- side A′B′ is three times as long as AB
- side E′D′ is three times as long as ED, etc.
- the distance XB′ is three times the distance XB
- the distance XD′ is three times the distance XD, etc.

The term enlargement is also used in situations where the shape is made smaller.

In these cases the scale factor is a fraction, such as $\frac{1}{2}$ or $\frac{1}{3}$.

In the diagram the scale factor for the enlargement of A′B′C′D′E′ to ABCDE is $\frac{1}{3}$.



To fully describe an enlargement, you need to give its centre and scale factor.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Position and Cartesian co-ordinates

Draw a pair of axes from 0 to 6.

a Plot the points (1, 2), (4, 2) and (4, 5).

b Mark a fourth point to make a square. What are the co-ordinates of this point?

Cartesian co-ordinates in four quadrants

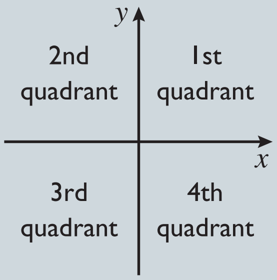
The point (1, 3) is in the first quadrant. In which quadrants are these points?

a (−3, −6)

b (5, −10)

c (−3, 8)

d (100, 100)



Translation

Look at the grid.

a Describe each translation.

i A to D

ii B to A

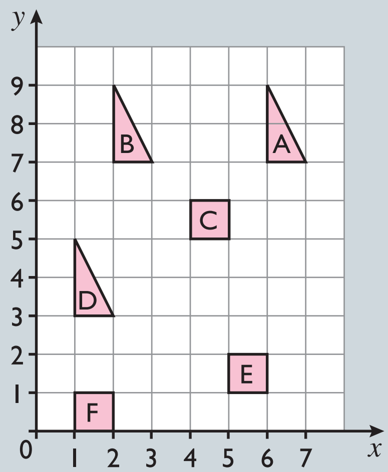
iii E to C

iv C to E

v F to E

vi D to B

b Triangle D is translated by 5 units to the right and 4 units up. Which triangle does it land on? (Note: 5 units along and 4 units up can be described using the vector $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$. See page 480).



Reflection

Sam is designing a window. The diagram shows his plan so far.

He draws shape A and then uses it to make the rest of the pattern using reflections.

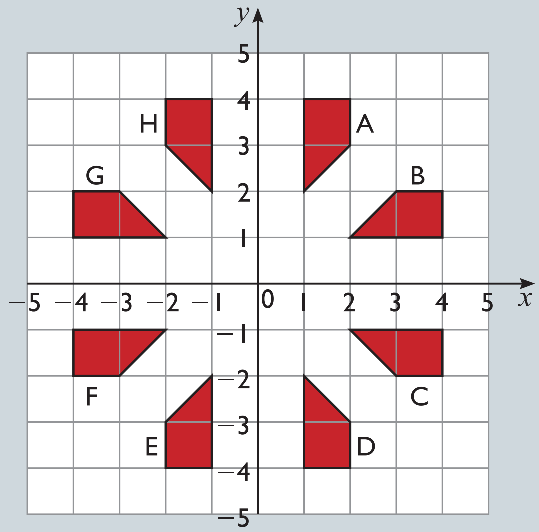
a In what line does he reflect A to get B?

b In what line does he reflect B to get C?

c In what line does he reflect C to get D?

d What single line of reflection could he now use to complete his design?

e What information do you need to fully describe a reflection?



Rotation

① Copy this flag.

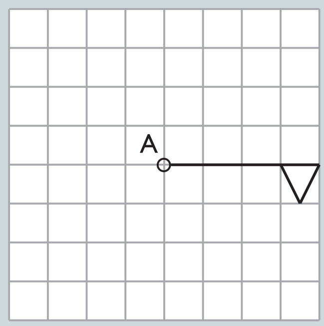
Rotate the flag about point A

a a quarter turn anticlockwise

b a half turn clockwise

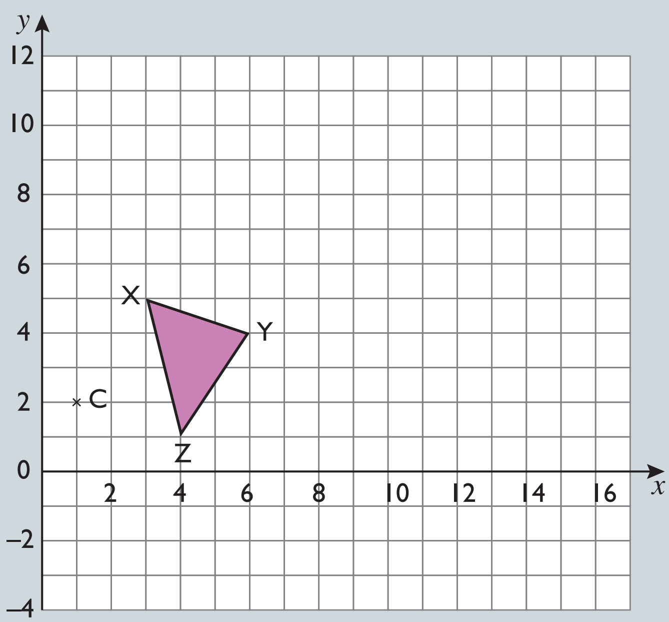
c a three-quarter turn clockwise.

② What information do you need to fully describe a rotation?



Enlargement

Copy this diagram.



a Draw an enlargement of XYZ, scale factor 2, centre C. Label the image X′ Y′ Z′.

b Draw an enlargement of XYZ, scale factor 3, centre C. Label the image X″ Y″ Z″.

c What transformation maps X′ Y′ Z′ to X″ Y″ Z″?

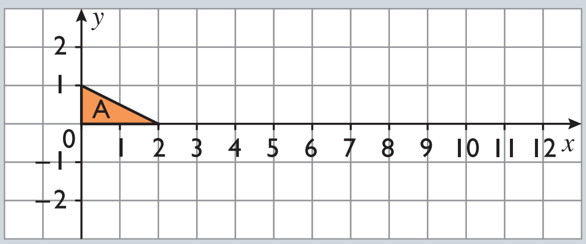
➜ Applying the knowledge

① **a** P and Q are points which are images of each other under a reflection.
P has co-ordinates (5, 8).
Q has co-ordinates (5, 12).
What is the equation of the line of reflection?

b S and T are points which are images of each other under a different reflection.
S has co-ordinates (2, 4).
T has co-ordinates (−2, 4).
Write down the equation of the line of reflection.

② Donna wants to make a wallpaper pattern.

She practises first with a simple shape.



She rotates triangle A by 180° about the point (2, 0) to give triangle B.

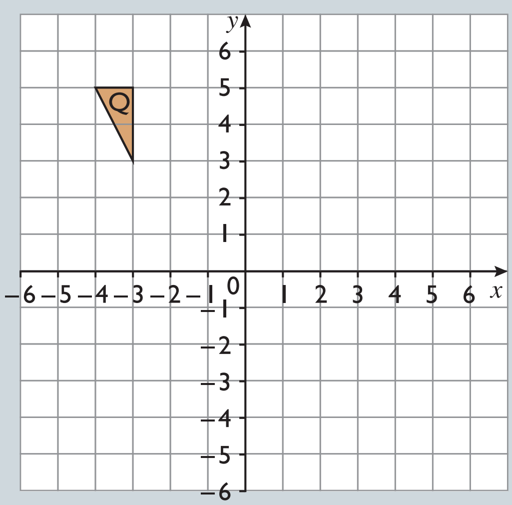
She then rotates triangle B by 180° about the point (4, 0) to give triangle C.

a Describe the single transformation that maps triangle A to triangle C.

For a different design, Donna rotates triangle A by 180° about the point (4, 0) to give triangle B and then rotates triangle B by 180° about the point (8, 0) to give triangle C.

b Describe fully this transformation.

③ Copy this diagram.



a Enlarge shape Q by a scale factor of 2 using (−5, 6) as the centre of enlargement.

Label the image A.

b Enlarge shape Q by a scale factor of 4 using (−5, 6) as the centre of enlargement.

Label the image B.

c Find the centre of enlargement and the scale factor of the enlargement that maps A to B.

d Find the centre of enlargement and the scale factor of the enlargement that maps B to A.

17.1 Similarity

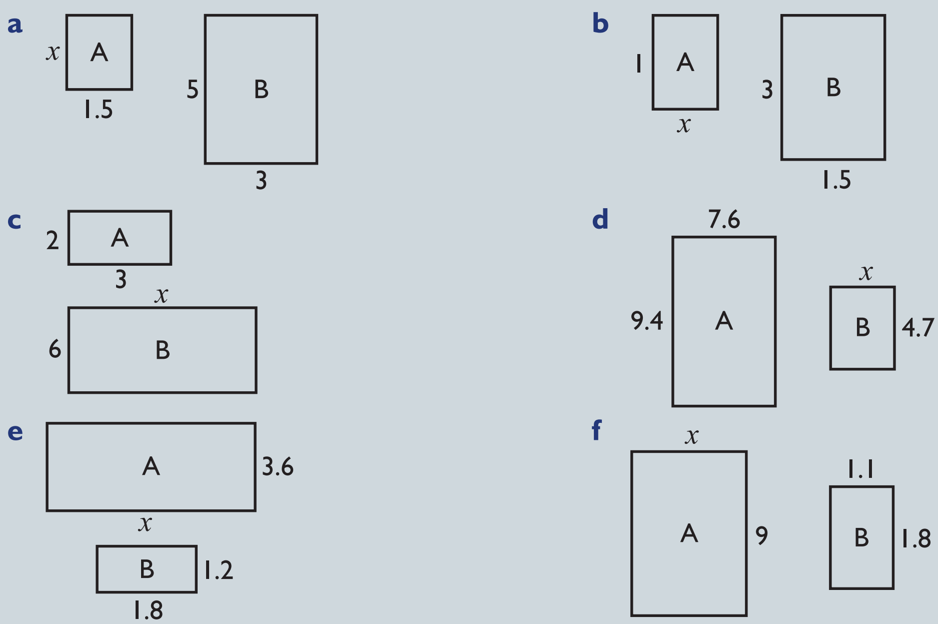
SKILLS CHECK

➜ Do I need to do this section?

Complete this section if you need help with the question below.

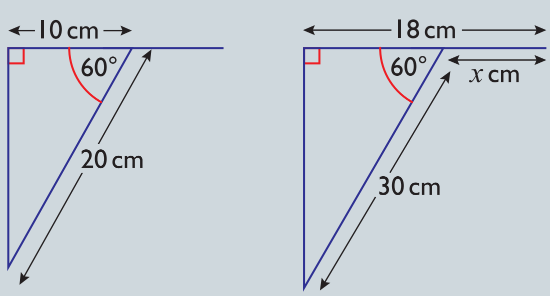
① These pairs of rectangles are similar.

Work out the length of x for each pair.



If you can do the question above, try this one on problem solving.

② The diagram shows a set of supports of two sizes made to support shelves.



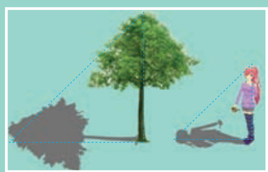
a Explain how you know that the triangular parts of the supports are similar.

b Work out the value of x.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 242 (Problem solving exercise 17.1 Similarity).

What you need to know

Did you know?



On a sunny day, you can work out the height of a tree using the length of its shadow.

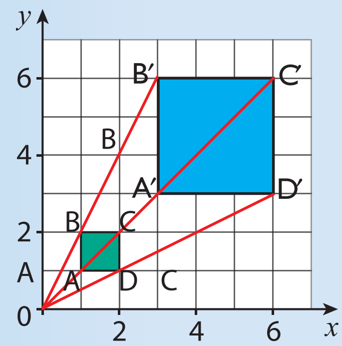
When you enlarge a figure with a scale factor of 3, all the lengths are made three times longer but the corresponding angles remain the same.

They are the same shape but different sizes.

The figures are **similar**.

The angles in the two figures are the same.

The lengths of the sides are also all in the same ratio.



- The corresponding sides between the shapes are in the same ratio, 1 : 3 in each case.
- The corresponding sides within the shapes are in the same ratio, 1 : 1 : 1 : 1 in each case.

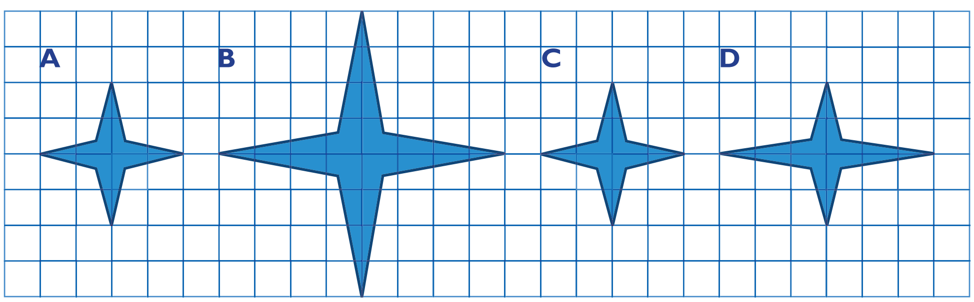
How to do it

Similar and congruent figures

Sarah has drawn the stars on this grid.

a Which stars are congruent to A?

b Which stars are similar to A?



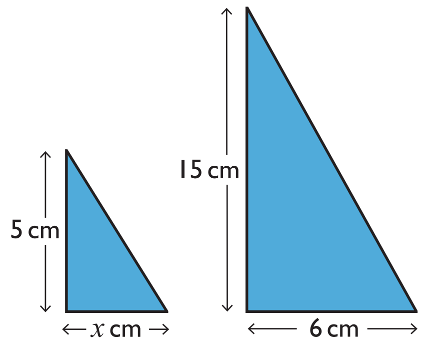
Solution

a Congruent figures are the same shape and size.
Only A and C are congruent.

b Similar figures are the same shape but they may be different sizes.
A, B and C are all similar.
Notice that congruent figures are always similar but similar figures need not be congruent.

➤ Using ratios of similar figures

These triangles are similar.
What is the value of x?



Solution

Method 1

The ratio of the vertical sides small : large is $5 : 15 = 1 : 3$.

So the sides of the large triangle are three times as long as those of the small triangle.

For the bases

$$3x = 6$$
$$x = 2$$

The base of the small triangle is 2 cm long.

Method 2

In the large triangle, the ratio of the sides base : height is $6 : 15 = 1 : 2.5$.

So the height of each triangle is 2.5 times the base.

For the small triangle

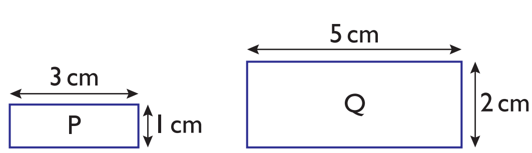
$$2.5x = 5$$
$$x = 2$$

The base of the small triangle is 2 cm long.

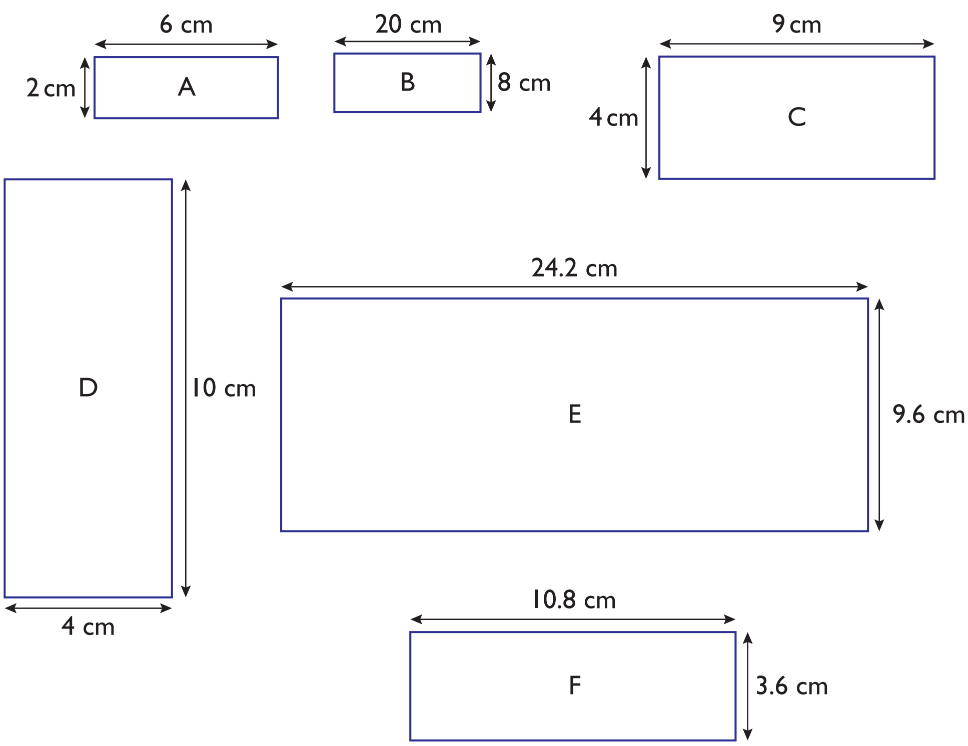
Learning exercise

① P is a rectangle 3 cm by 1 cm.

Q is a rectangle 5 cm by 2 cm.



Say whether the following rectangles are similar to P, similar to Q or similar to neither.



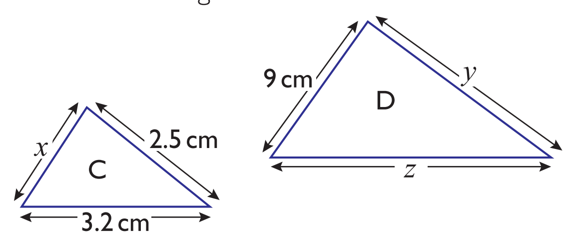
② Triangle C is enlarged to give triangle D. The scale factor of the enlargement is 3.

a Find the value of y.

b Find the value of z.

c Find the value of x.

d Are triangles C and D congruent or similar?



③ Triangle A is enlarged to give triangle B.
The scale factor is 4.

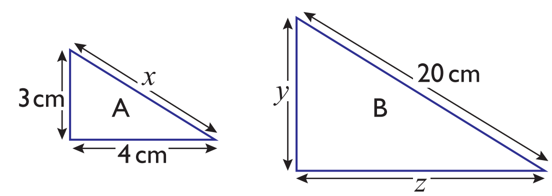
a Find the value of y.

b Find the value of z.

c Find the value of x.

d Are triangles A and B congruent or similar?

e What is special about both triangles?



④ Triangle E is enlarged to give triangle F.

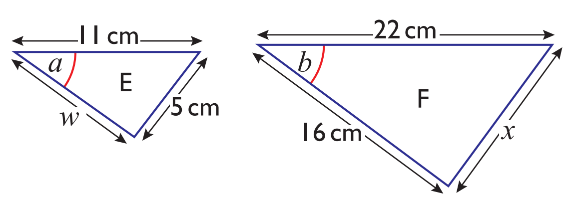
a What is the scale factor of the enlargement?

b Find the value of x.

c Find the value of w.

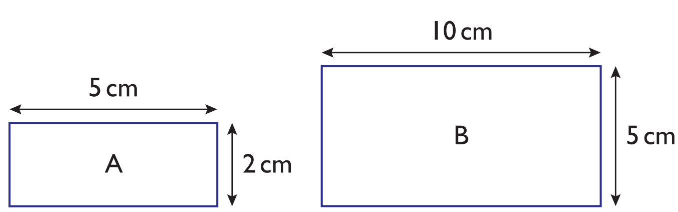
d What do you know about the size of the angles labelled a and b?

e What is the relationship between triangles E and F?



⑤ Decide if shapes A and B are similar or not similar. Give a reason for each answer.

a



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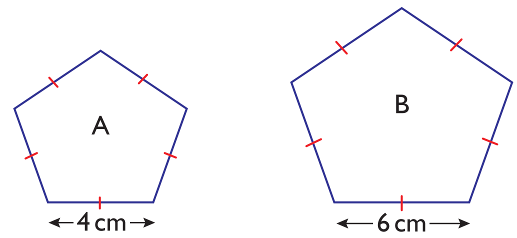

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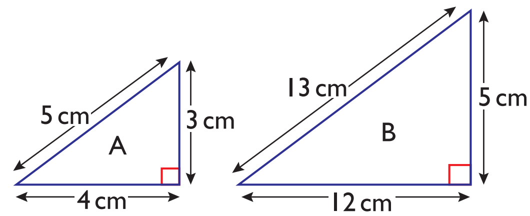

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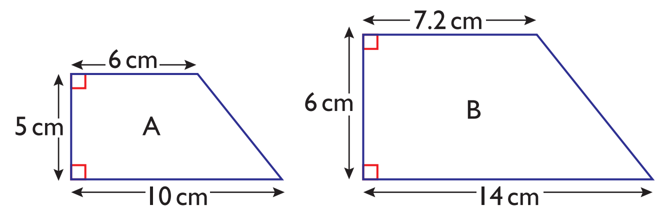
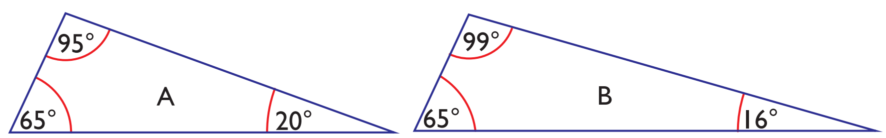
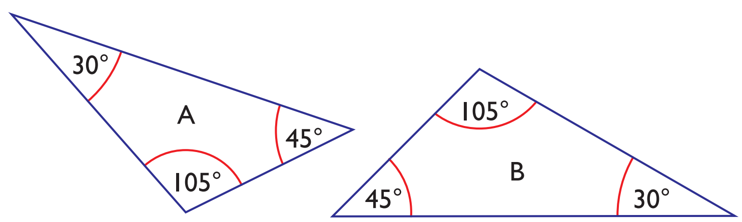


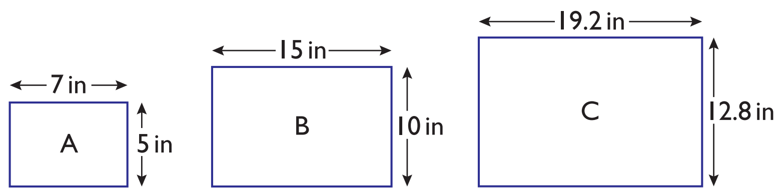
e



f



⑥ A photograph is 6 inches by 4 inches. Which of these are similar to it?



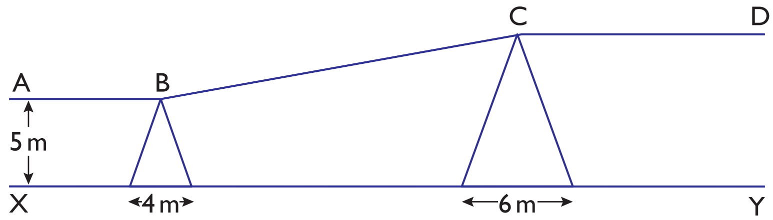
⑦ The diagram shows the cross-section of part of a landing stage for a ferry. The two triangles are similar.

AB, CD and XY are horizontal.

AB is 5 m above the sea.

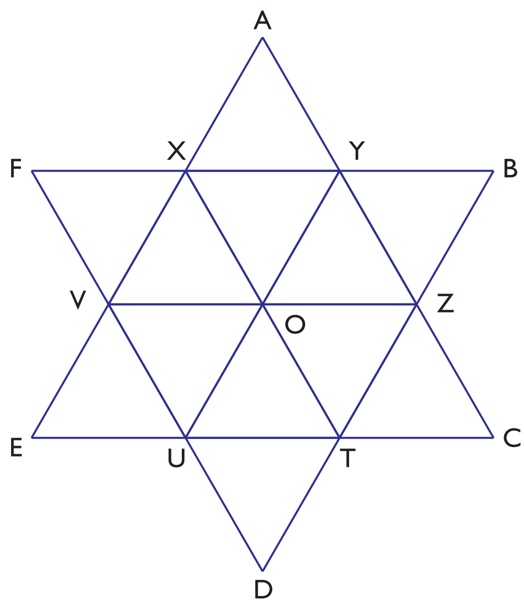
The bases of the triangles are 4 m and 6 m.

How much higher above sea level is CD than AB?



8 In the diagram, ABCDEF is a 6-pointed star with rotational symmetry of order 6.

AXY is an equilateral triangle.



a How many triangles in the diagram are congruent to AXY?

b Which triangles are similar to AXY but not congruent to it? Place them in groups that are congruent to each other.

9 Shape P is reflected to give shape A, rotated to give shape B, translated to give shape C and enlarged to give shape D.

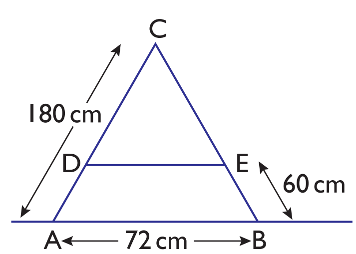
a Which of the shapes are congruent?

b Which of the shapes are similar?

Problem solving exercise

1 The diagram shows a step ladder on horizontal ground. DE is a horizontal bar inserted for stability.

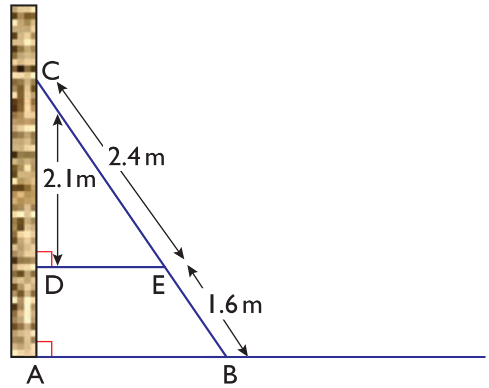
What is the length of DE?



② The diagram shows a support for a vertical wall. The ground is horizontal and the strut DE is perpendicular to AC.

a Show that triangles CDE and CAB are similar.

b Work out the height of the point D above the floor AB.

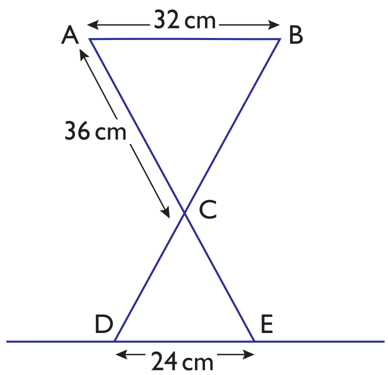


③ The diagram shows a design for a fold-up seat.

AB = 32 cm, AC = 36 cm and DE = 24 cm.

The design has a vertical line of symmetry.

Work out the length of the leg AE.

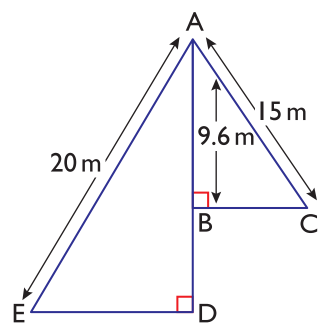


④ In the diagram, the straight line DBA bisects ∠EAC.

BC and DE are perpendicular to DBA.

AC = 15 m, AE = 20 m and AB = 9.6 m.

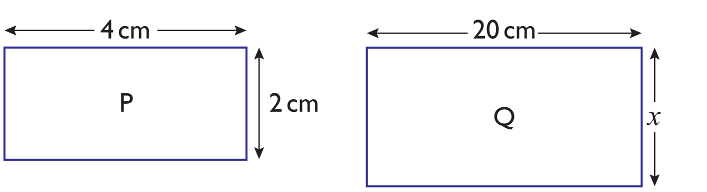
Calculate the length of BD.



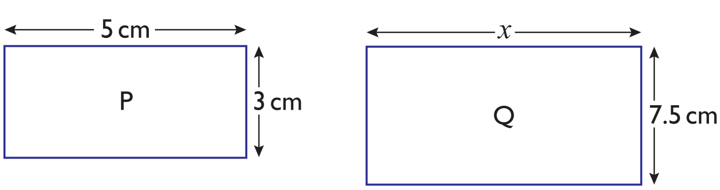
Do I know it now?

① Rectangles P and Q are similar. Find the value of x in each case.

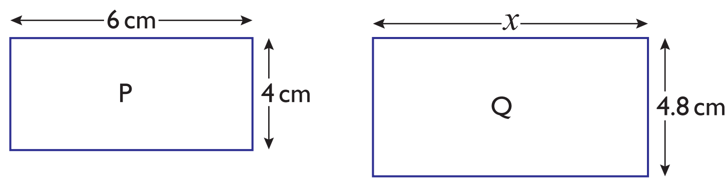
a



b



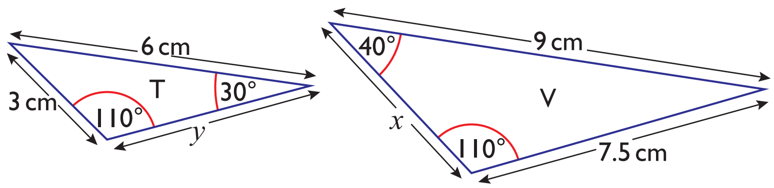
c



② **a** Prove triangles T and V are similar.

b Find the value of x.

c Find the value of y.



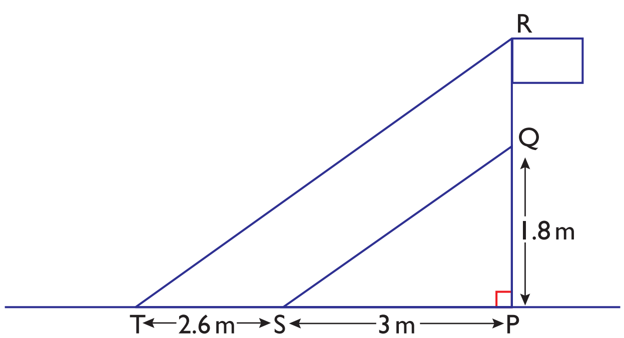
Can I apply it now?

① The diagram shows a flagpole PR standing on horizontal ground PST.

PQ is a vertical rod of length 1.8 m.

PS and PT are the shadows of the rod and the flagpole.

Work out the height of the flagpole.



CHAPTER 18

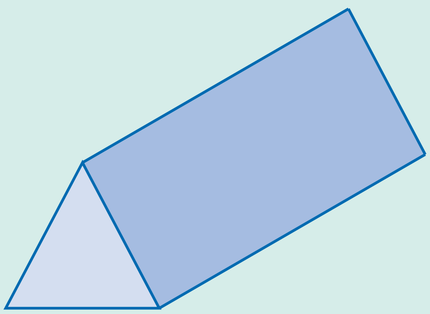
ESSENTIAL TOPICS – GEOMETRY AND MEASURES

Three-dimensional shapes

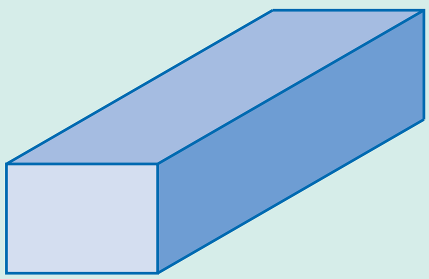
JUST IN CASE

Properties of 3-D shapes

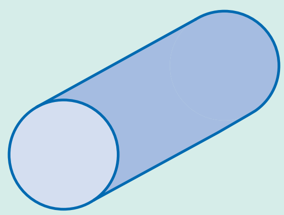
A prism is a three-dimensional (3-D) shape with a constant cross-section.



A triangular prism.

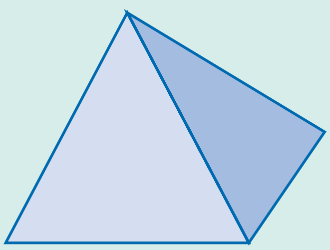


This cuboid is a rectangular prism.

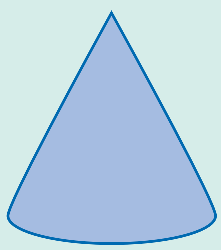


A cylinder is a circular prism.

Not all 3-D shapes are prisms.



A pyramid.



A cone.

Understanding nets

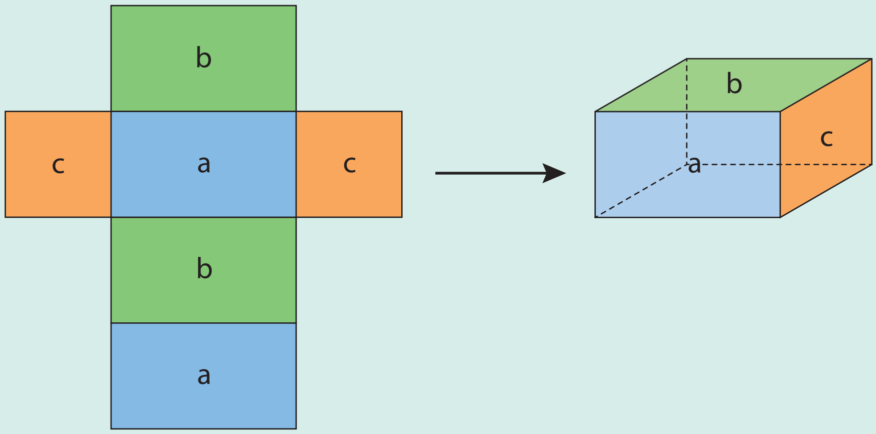
2-D nets are designed to be folded to make 3-D objects.

The same object may have different nets.

Whatever their arrangement, the sections of the 2-D design must match the faces of the 3-D object that is to be made.

Nets need flaps to enable the 3-D shape to become rigid.

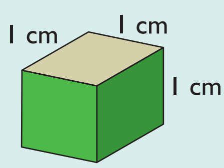
Clever designers can design the flaps to make the box rigid without glue or staples.



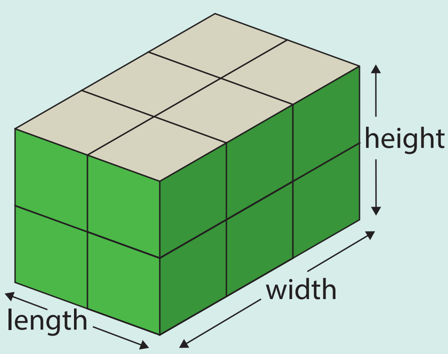
Nets can be used to find the surface area of shapes.

Volume and surface area of cuboids

The **volume** of a solid shape is the space inside it.
It is measured in cubic units such as cm^3 and m^3.
This is a centimetre cube.



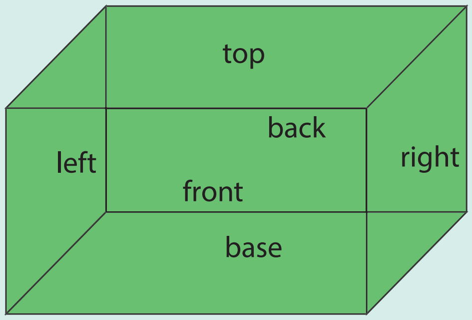
It has a volume of 1 cubic centimetre (1 cm^3).
You can sometimes find the volume of a cuboid by counting the number of cubes that fit into it.
This cuboid is made of 12 centimetre cubes.



So its volume is 12 cm^3.
Another way to find the volume of a cuboid is to use the formula
Volume of a cuboid = length × width × height.

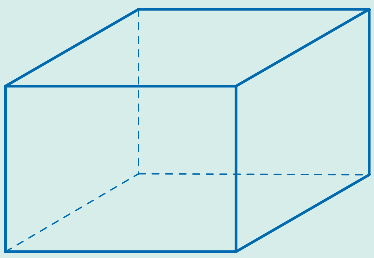
The surface area of a solid is the total area of all its faces.

In a cuboid, the front and back are the same; so are the top and bottom, and the two sides.
So, the **surface area** of a cuboid
= 2 × (area of base + area of front + area of one side)



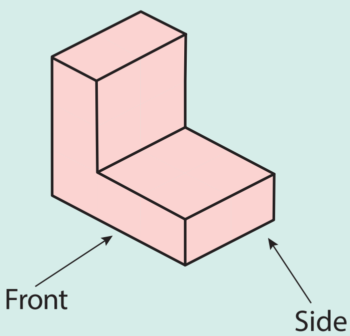
2-D representation of 3-D shapes

You can represent a 3-D object in two dimensions by drawing its plan and elevations separately.

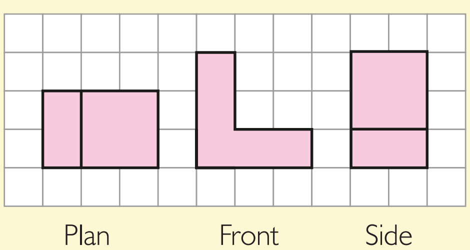


Plan view
Side elevation
Front elevation

You can also use isometric paper to draw a representation of a 3-D object.
Draw the plan, front elevation and side elevation of this solid.



Solution



SKILLS CHECK

Warming up

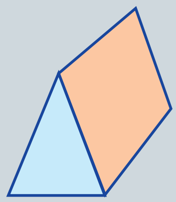
Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Properties of 3-D shapes

Match these names to the shapes below.

cuboid | cylinder | hexagonal prism | irregular prism | equilateral triangular prism | isosceles triangular prism

a



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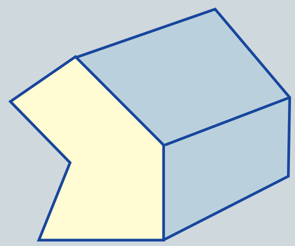

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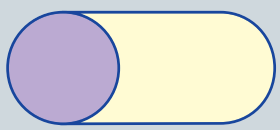

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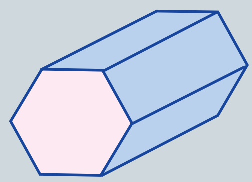

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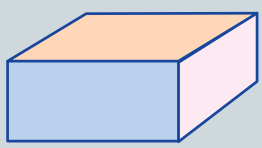

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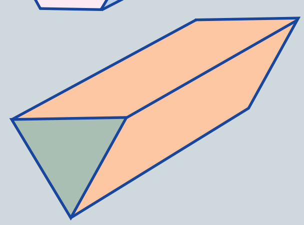


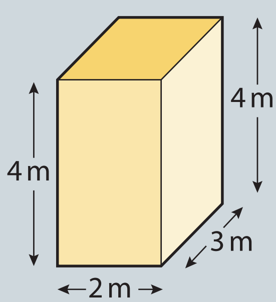
Understanding nets

The ends of a prism are equilateral triangles with sides of 3 cm. The prism is 7 cm long. Draw a net of the prism.

Add the minimum number of flaps.

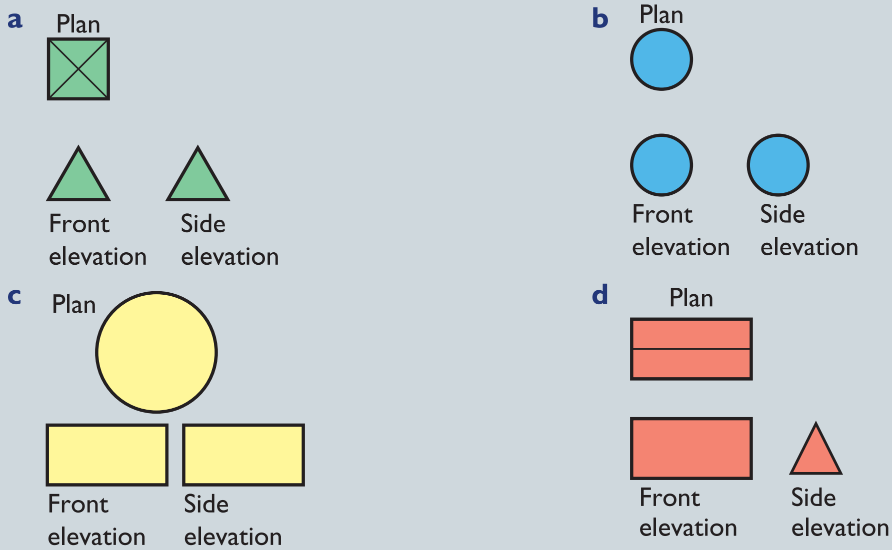
Volume and surface area of cuboids

Work out the volume and surface area of this cuboid. Give your answers in suitable units.

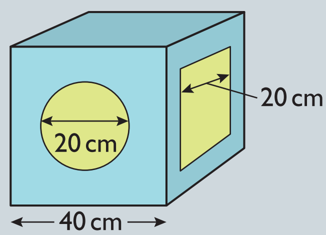


2-D representations of 3-D shapes

Sketch the solids that match these sets of plans and elevations.



Applying the knowledge



① Jeff is designing a box for children's toys.

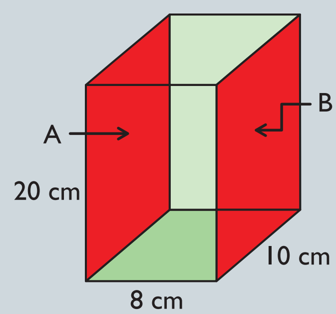
The box is a cube with sides of length 40 cm.

Two opposite faces of the box each have a circle on them.

Two opposite faces of the box each have a square on them.

The other two faces are left blank.

Draw a scale diagram of the net, including the circles and squares.



② The diagram shows a box on a table.

A and B are opposite faces.

The box has length 10 cm, width 8 cm and height 20 cm.

Lawrence pours sand into the box until the level of the sand is 2 cm below the top.

Then he puts the lid on and turns the box over so that face A is on the table.

Work out how far the level of the sand is below face B.

18.1 Prisms

SKILLS CHECK

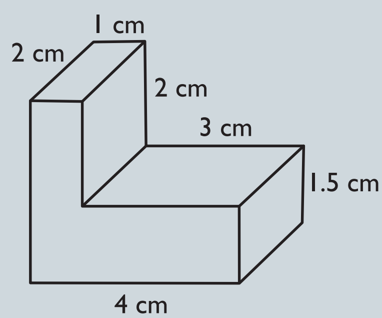
Do I need to do this section?

Complete this section if you need help with the question below.

① Work out the volume and surface area of each solid. All lengths are in centimetres.

Remember to give the units for each of your answers.

a



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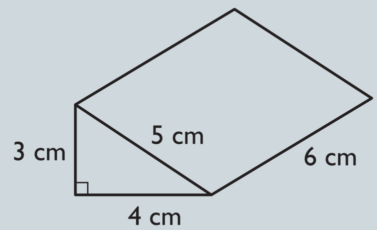


If you can do the question above, try this one on problem solving.

② A copper pipe has inner radius 2.6 cm and outer radius 3 cm.

It is 500 cm long.

What is the volume of copper in the pipe?

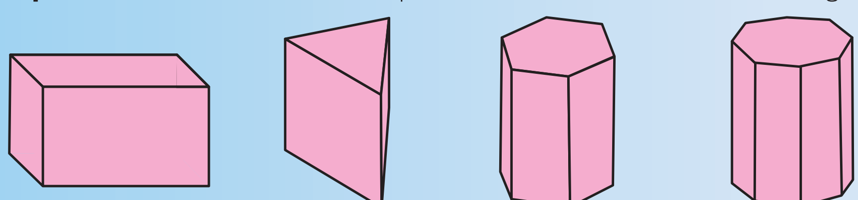
If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 255 (Problem solving exercise 18.1 Prisms).

What you need to know

Did you know?

The word cylinder comes from the Greek *kulindros*, meaning roller or tumbler.

A **prism** is a three-dimensional shape with the same cross-section all along its length.



Cuboid — Triangular prism — Hexagonal prism — Octagonal prism

The **surface area** of a prism is the total area of all the faces.

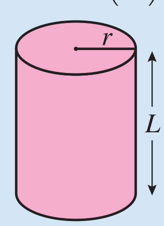
Volume is measured in cubic units such as cubic centimetres (cm^3) or cubic metres (m^3).

A cylinder is a prism with a circular base.

Its volume is $\pi r^2 L$.

Its surface area is $2\pi rL + \pi r^2 + \pi r^2 = 2\pi rL + 2\pi r^2$

To work out the **volume of a prism**, calculate the area of the cross-section and multiply by the height (or length).

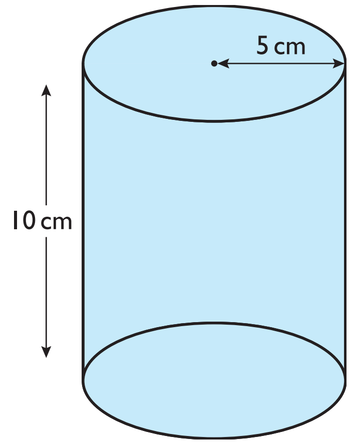


Cylinder

How to do it

Finding the volume of a prism

Work out the volume of this cylinder.



Solution

Area of circle $= \pi r^2$

$A = \pi \times 5^2$

$= 78.539\ldots$

The cross-section is a circle with a radius of 5 cm.

Volume of cylinder = area of cross-section × height

$= 78.539\ldots \times 10$

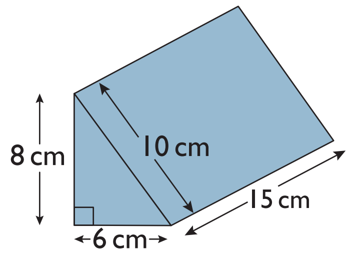
The height of the cylinder is 10 cm.

$= 785.398\ldots$

The volume of the cylinder is $785.4\,\text{cm}^3$ (to 1 d.p.).

Finding the surface area of a prism

Calculate the surface area of this prism.



Solution

Work out the area of each face individually.

Area of one triangular face $= \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times 6 \times 8$

$= 24\,\text{cm}^2$

Area of other triangular face: $24\,\text{cm}^2$

Area of rectangular base: $6 \times 15 = 90\,\text{cm}^2$

Area of rectangular face: $8 \times 15 = 120\,\text{cm}^2$

Area of sloping rectangular face: $10 \times 15 = 150\,\text{cm}^2$

Total surface area $= 24 + 24 + 90 + 120 + 150$

$= 408\,\text{cm}^2$

The two triangular faces are identical.

The rectangular faces all have a length of 15 cm.

Add the areas of all the faces to find the total surface area.

Learning exercise

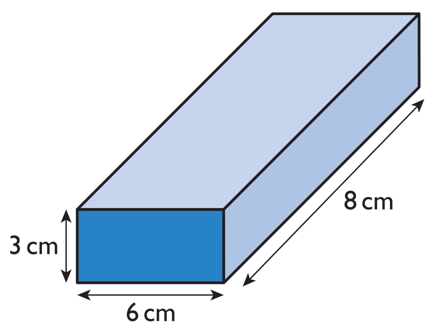
① Here are some prisms.

For each prism, work out

i the area of the cross-section

ii the volume.

a



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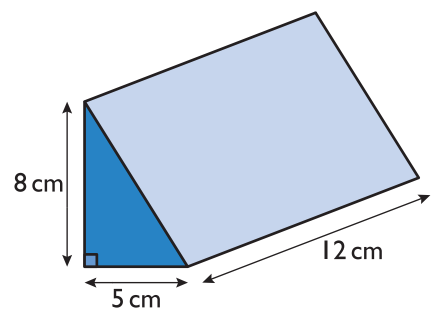

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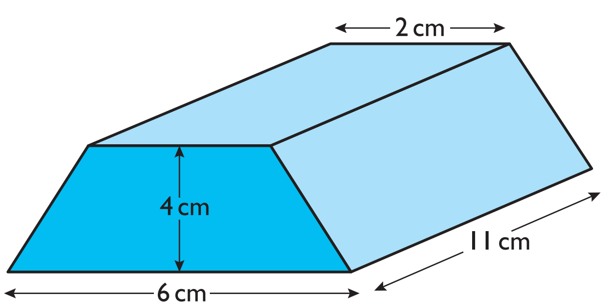
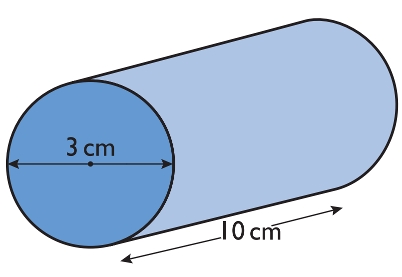


② A cylinder is a prism. The cross-section is a circle. For each cylinder

i work out the area of the circle using the formula area $= \pi r^2$

ii work out its volume.

a



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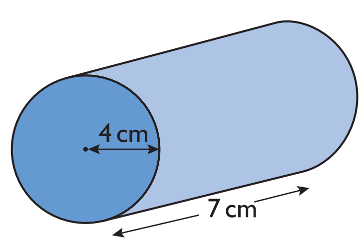

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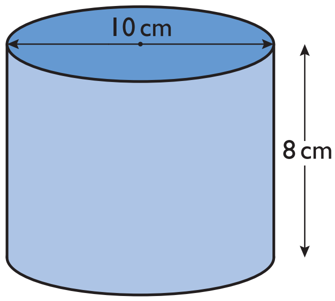
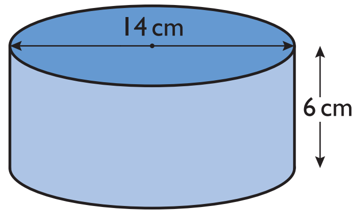


d



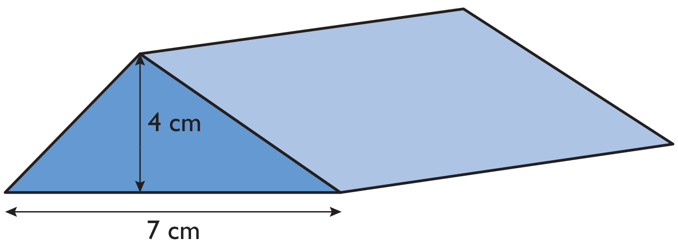
③ A prism has cross-sectional area $18\,\text{cm}^2$ and length 3 cm.

Work out its volume.

④ A prism has cross-sectional area $15\,\text{cm}^2$ and volume $120\,\text{cm}^3$.

Work out its length.

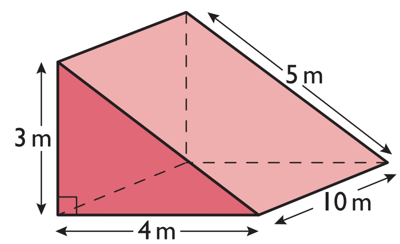
⑤ The volume of this triangular prism is $280\,\text{cm}^3$.



a Work out the area of the triangular face.

b Work out the length of the prism.

⑥ This diagram shows a triangular prism.



a **i** Work out the areas of the three rectangular faces.

ii What is the total area of their faces?

b **i** Work out the perimeter of the triangular cross-section.

ii Multiply the perimeter by the length of the prism.

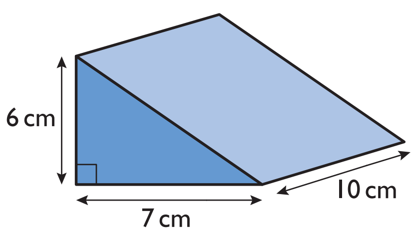
c Comment on your answers to **a ii** and **b ii**.

d **i** Work out the area of the triangular cross-section.

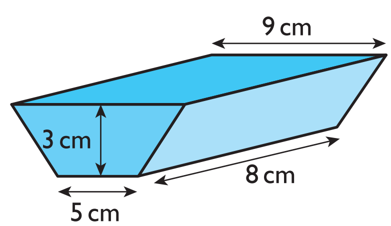
ii What do you find when you multiply the area by the length of the prism?

⑦ Work out the volume of each of these prisms in cubic centimetres.

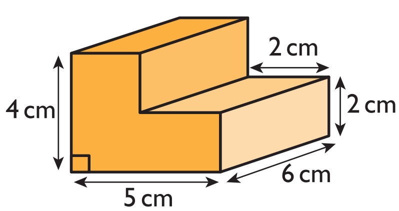
a



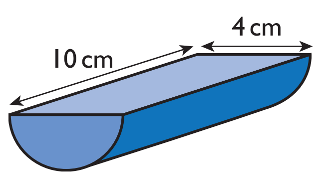
b



c

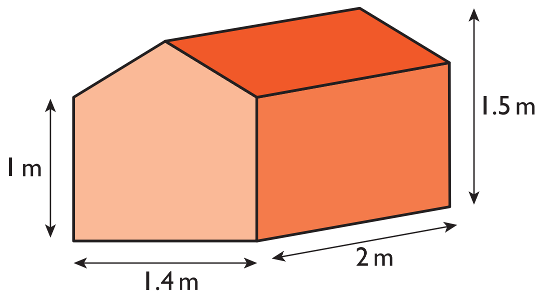


d

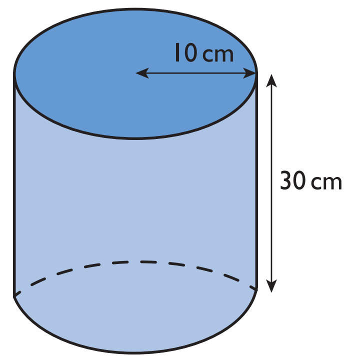


⑧ This is a sketch of a child's playhouse.

Work out the volume of the playhouse.



⑨ This cylinder has a radius 10 cm and height 30 cm.



a Work out the circumference of the cross-section.

b Calculate the surface area of the walls without the top and bottom.

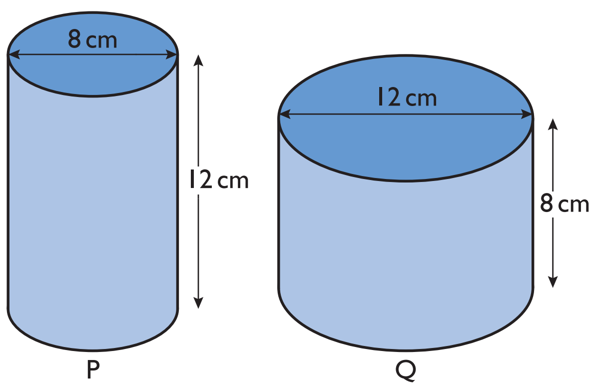
c Work out the area of the top.

d Work out the total surface area (the walls, top and bottom).

⑩ A tin of beans has diameter 6.8 cm and height 10.2 cm.

Work out its capacity (volume).

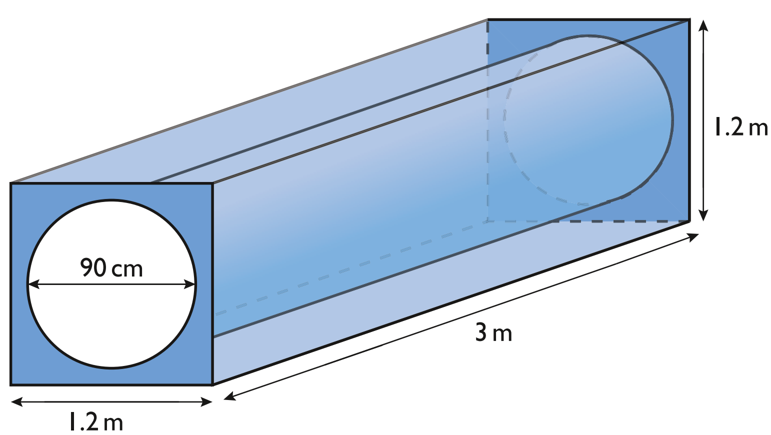
⑪ Susan has two cylindrical containers, P and Q.



a Which has the greater volume, P or Q?

b What is the difference in volume between the two containers?

⑫ At the park, a cylinder is cut from a wooden cuboid to make a tunnel for children to crawl through.



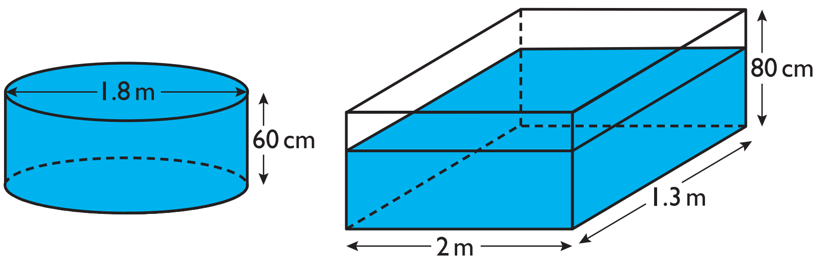
a Work out the volume of wood remaining.

b Work out the surface area of the inside of the tunnel.

⑬ The diagram shows two paddling pools.

Zara's pool is in the shape of a cylinder and Anna's is in the shape of a cuboid.

Zara fills her pool. Anna's pool is three-quarters full.



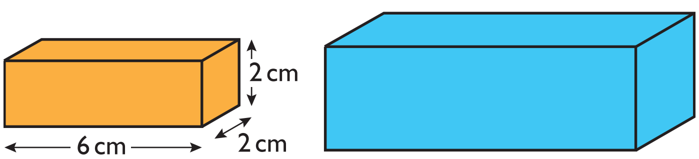
a Which pool contains more water?

b What is the difference in the volume of water in the two pools?

⑭ The diagram shows two cuboids.

The edges of the larger one are $2\frac{1}{2}$ times the lengths of those in the smaller one.

The smaller cuboid has dimensions of 2 cm × 2 cm × 6 cm.



a Write down the dimensions of the larger cuboid.

Calculate

b the surface area of the smaller cuboid

c the surface area of the larger cuboid

d the volume of the smaller cuboid

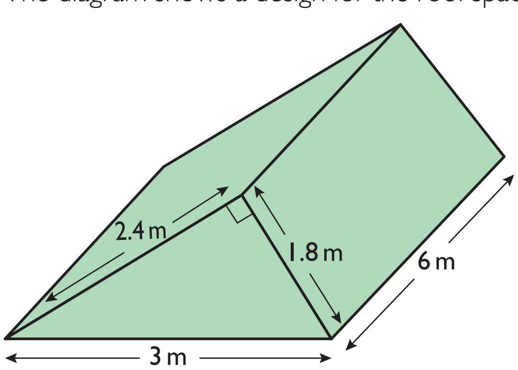
e the volume of the larger cuboid

f the ratio of the surface areas of the two cuboids in its simplest form

g the ratio of the volumes of the two cuboids in its simplest form.

Problem solving exercise

① The diagram shows a design for the roof space of a greenhouse.



The roof space is in the shape of a triangular prism of length 6 m.

The cross-section of the roof space is a right-angled triangle.

The sides of the triangle are 2.4 m, 1.8 m and 3 m.

Calculate

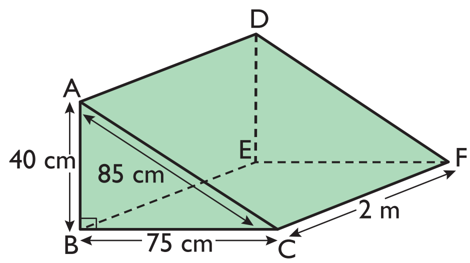
a the area of the glass used

b the volume of the roof space.

② The diagram shows a prism-shaped cold frame.

Rectangle BCFE is open. All of the other surfaces are made of glass.

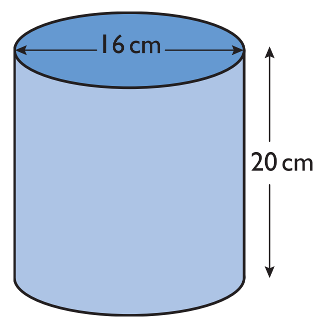
The dimensions of the cold frame are shown on the diagram.



a Work out the total area of glass.

b Calculate the volume of the cold frame.

③ Here is a container used to mix chemicals. It is a closed cylinder with the dimensions shown.



The inside surface of the cylinder has to be covered with a special chemical.

a Work out the area of each end of the cylinder.

b What is the area of the inside wall of the cylinder?

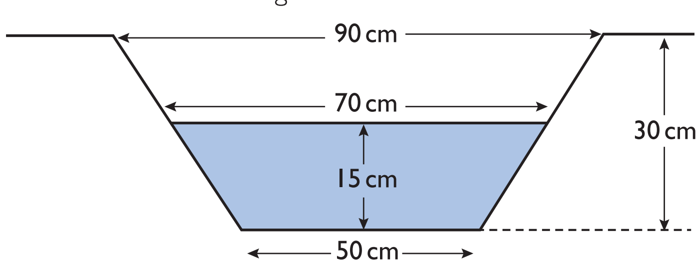
It costs £1.20 to cover each square centimetre with the special chemical.

c Work out the total cost of covering the inside surface of the cylinder.

④ The diagram shows the cross-section of a water channel.

The cross-section is a trapezium.

The channel is 2 km long.



a One day the water is 15 cm deep, as shown in the diagram.

i Work out the cross-sectional area of the water.

ii Calculate the volume of water in the channel.

b On another day, the channel is full. How much water does it contain?

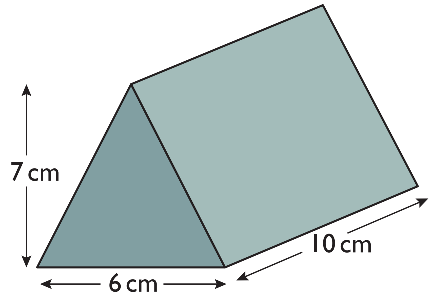
Do I know it now?

① Here are some prisms. For each prism, work out

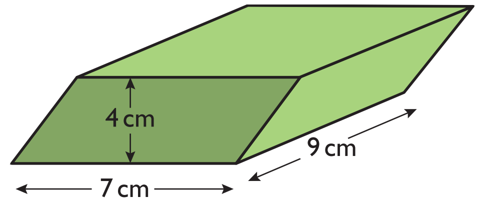
i the area of the cross-section

ii the volume.

a



b



② A prism has volume 5520 cm^3 and length 1.2 m.

Work out its cross-sectional area.

③ A cylinder is a prism. The cross-section is a circle.

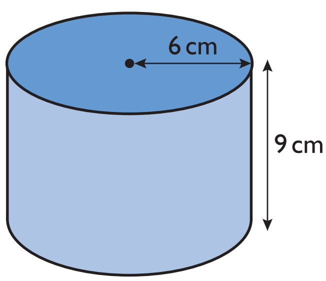
For each cylinder, work out

i the area of the circular face (Area $= \pi r^2$)

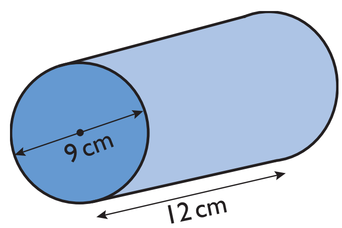
ii its volume

iii its total surface area.

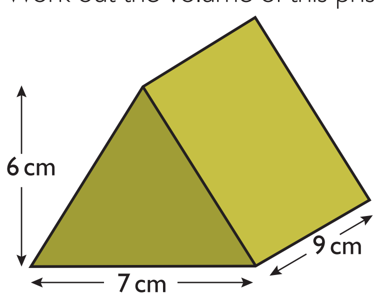
a



b



④ Work out the volume of this prism.



Can I apply it now?

① A company digs a tunnel through a mountain.

The tunnel is a cylinder with diameter of 8 metres.

The length of the tunnel is 1 kilometre.

a The material removed is taken away by lorries. Each lorry can carry 30 m^3 of material.
How many lorry loads are needed to take away all the material?

b The walls inside the tunnel are covered in concrete.

i What area of concrete is needed?

ii The concrete is 10 cm thick. What volume of concrete is needed?

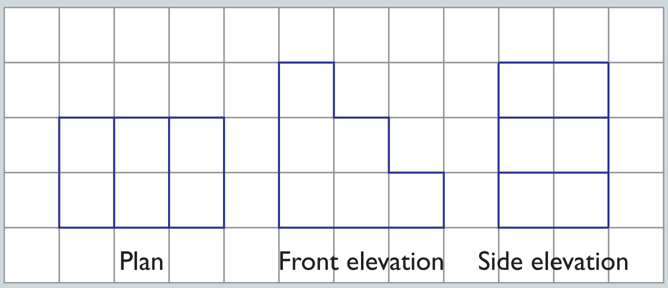
18.2 Constructing plans and elevations

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with this question.

1 Here are the plan and elevations of a 3-D shape.



a Make an isometric drawing of this 3-D shape.

The solid is made from centimetre cubes.

b Work out

i the volume of the shape

ii the surface area of the shape.

What you need to know

Did you know?

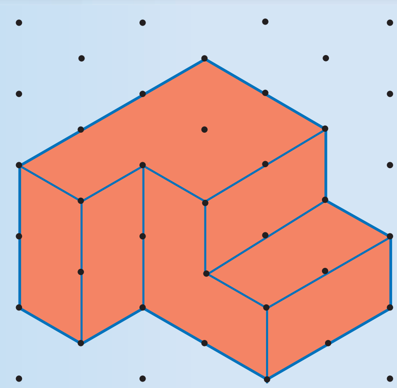
Architects draw plans and elevations to communicate to builders and prospective buyers how a building should look.

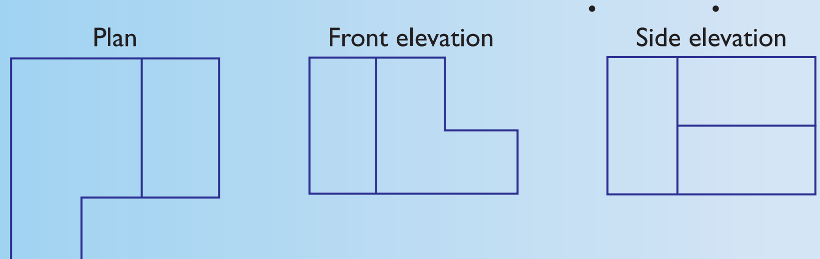
The most common types of three-dimensional drawings are **isometric drawings** and **plans** and **elevations**.

An isometric drawing is often made on a triangular grid.

Vertical edges are drawn vertically, but horizontal edges are not drawn horizontally.

Plans and elevations show the view from above, the front and the side.

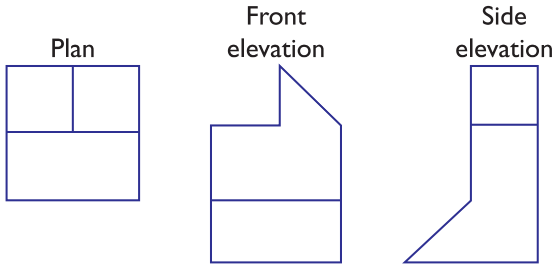




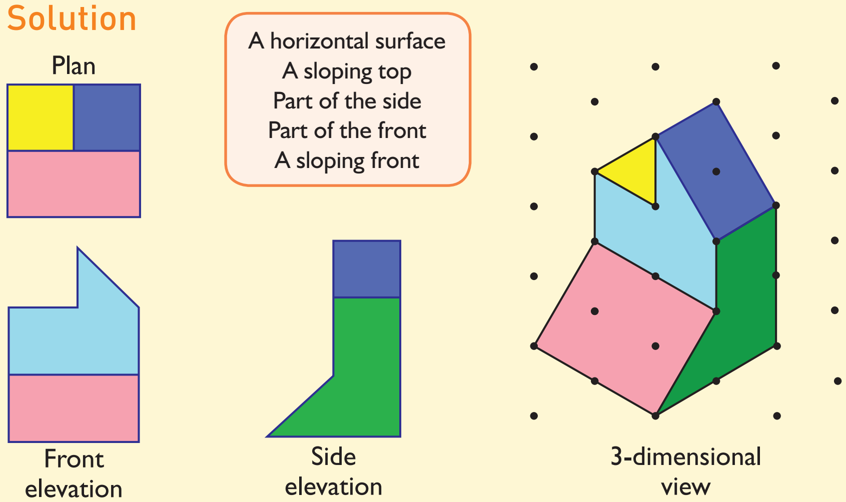
How to do it

➤ Isometric drawings

Here are the plan, front and side elevations of a shape.
Make an isometric drawing of the shape.

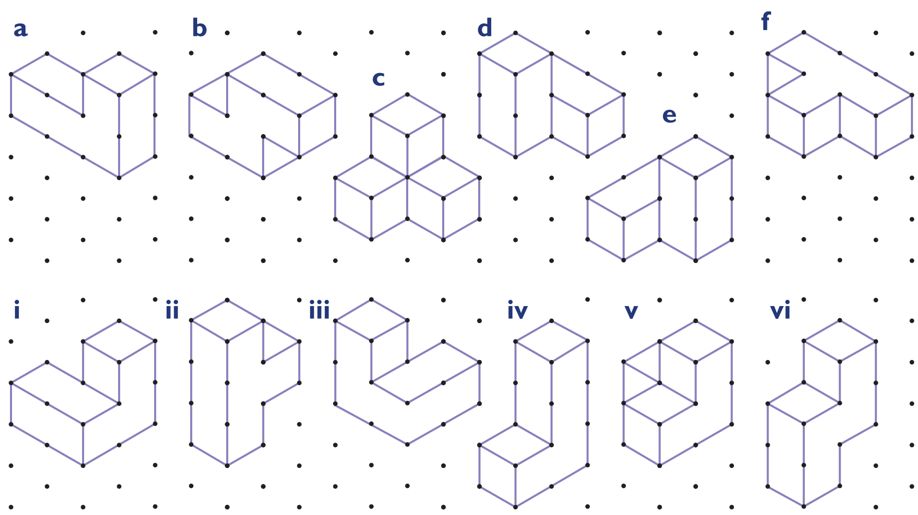


Solution



Learning exercise

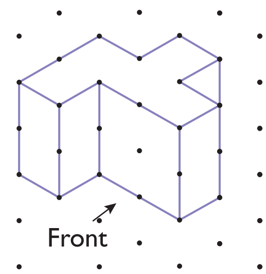
① Match each shape at the top with the same shape at the bottom.



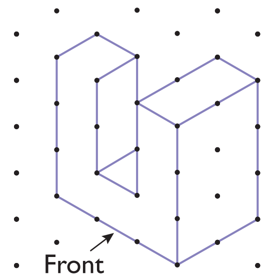
② These shapes are made of cubes.

Use isometric paper to draw the back view of each shape.

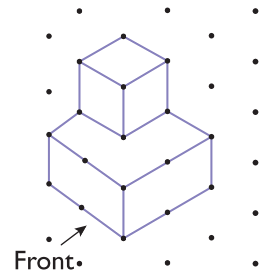
a



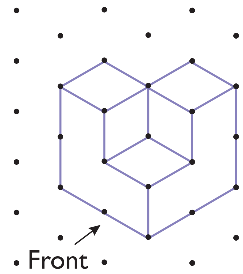
b



c

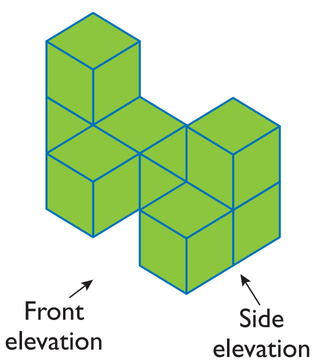


d

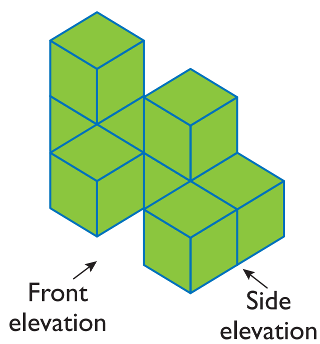


③ Match these isometric drawings to the views. Record your answers in a copy of the table.

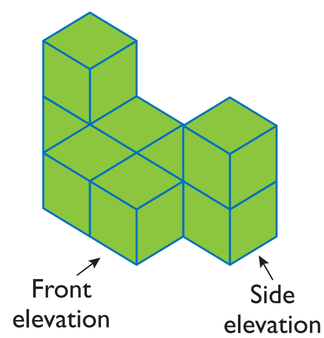
Shape A

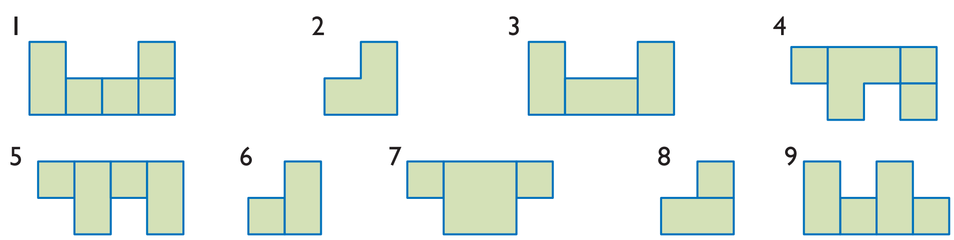


Shape B



Shape C

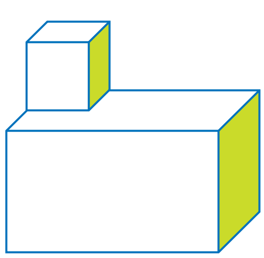




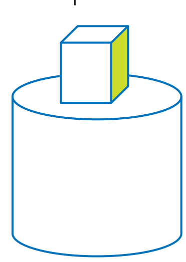
Shape	Front elevation	Side elevation	Plan
A			
B			
C			

④ Sketch the plan, front elevation and side elevation of each shape.

a

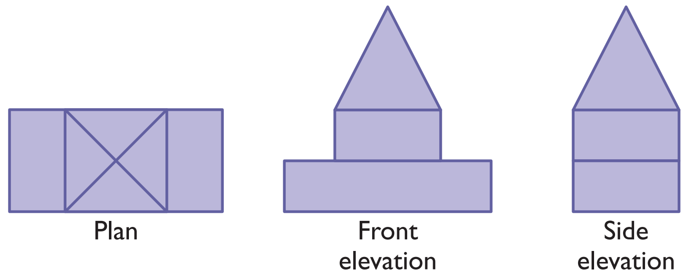


b

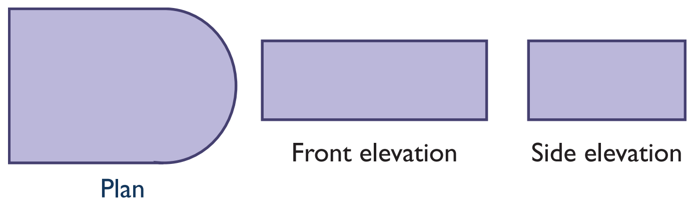


⑤ Make isometric drawings of the shapes shown in these views.

a



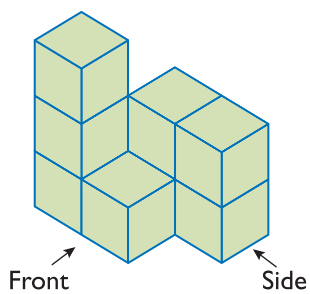
b



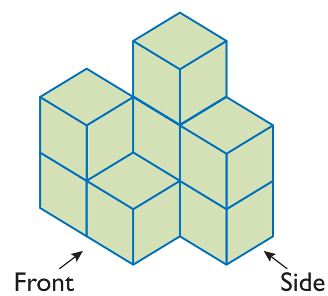
⑥ Each of these shapes is made of eight cubes.

Draw the plan, front and side elevations of each shape.

a

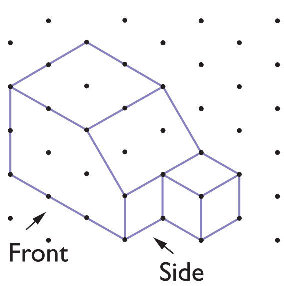


b

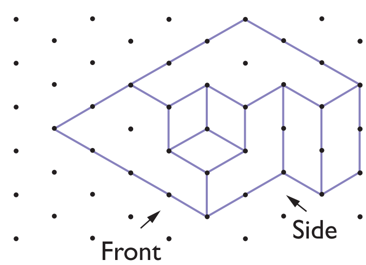


⑦ Draw the plan, front and side elevations of each shape.

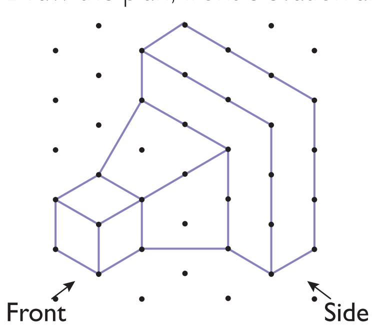
a



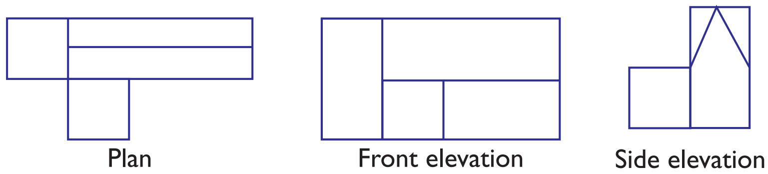
b



⑧ Draw the plan, front elevation and side elevation of this shape.

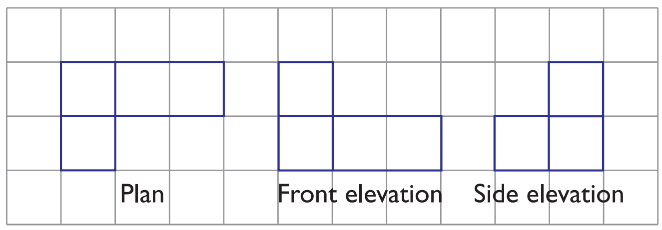


⑨ Make an isometric drawing of this shape.



⑩ Here are the plan and elevations of a 3-D shape.

The solid is made from centimetre cubes.

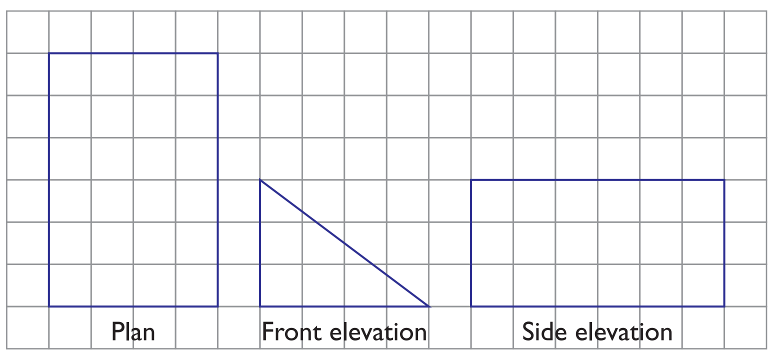


a Make an isometric drawing of this 3-D shape.

b Add one more cube to your drawing so that the drawing has plane symmetry.

Draw two different options.

⑪ Here are the plan and elevations of a 3-D shape. Each square represents 1 cm^3.



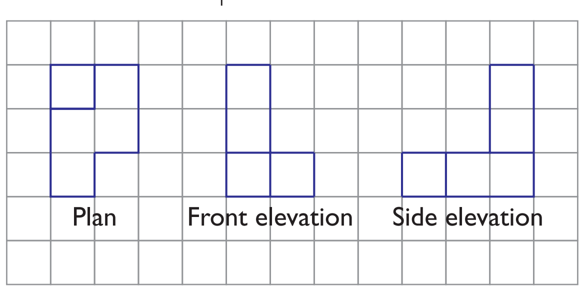
a Draw the 3-D shape on isometric paper. What is the mathematical name for this shape?

b Find the volume of the 3-D shape.

c Construct an accurate drawing of the net for the shape.

d Work out the surface area of the shape.

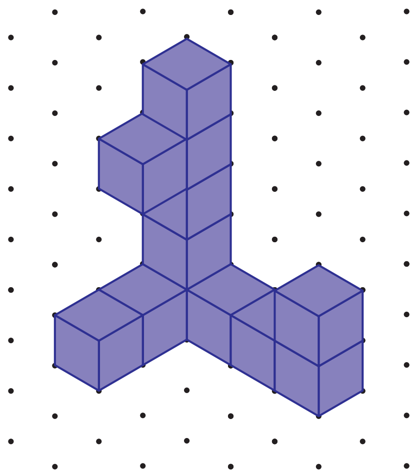
⑫ A child builds a shape out of centimetre cubes.



a Draw the 3-D shape on isometric paper.

b Add one cube to your diagram so that the resulting shape has a plane of symmetry.

⑬ **a** Draw the plan, front and side elevations of this 3-D shape.

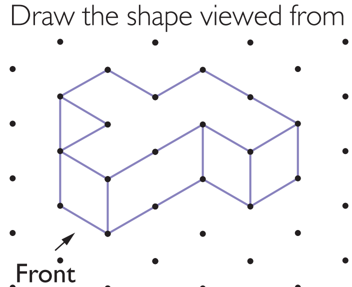


b Copy the 3-D shape on isometric paper.

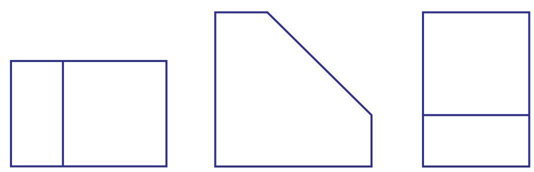
Draw three more cubes on your diagram so that the resulting shape has a plane of symmetry.

Do I know it now?

① This shape is made of cubes.
Draw the shape viewed from the back.

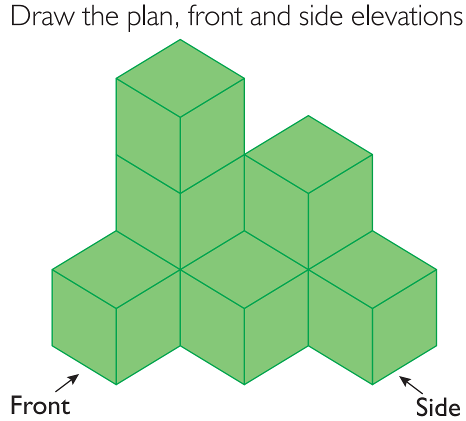


② Make an isometric drawing of the shape shown in these views.



Plan | Front elevation | Side elevation

③ This shape is made of eight cubes.
Draw the plan, front and side elevations of this shape.



Chapter 19 covers working with vectors. It can be found in the Next Steps section which contains material for students aiming to achieve a higher grade.

CHAPTER 20

ESSENTIAL TOPICS – STATISTICS AND PROBABILITY

Statistical measures

JUST IN CASE

Mode, median, mean and range

The **mode** of a set of data is the value which appears most often.

For a list of numerical data written in order, the **median** is the value which has the same number of items before and after it.

- For an odd number of items, the median is the middle value.
- For an even number of items, the median is halfway between the middle two values. You can find this by adding the two items and dividing by two.

For numerical data, the **range** is the difference between the largest and the smallest value.

To find an **average**, use the mode, the median or the mean.

Find the **mean** by adding up all the items of data and dividing by the total number of items.

To find out how the data are **spread**, find the **range**.

Shoe sizes (boys)	Shoe sizes (girls)
5, 7, 3, 6, 7, 9, 7, 5, 7, 4, 10	7, 5, 6, 6, 7, 4, 8, 6, 6

a Draw and complete a table to show all the averages.

b Who has the bigger feet, the boys or the girls?

Which type of average did you use to decide?

Solution

a

	Boys	Girls
Mean	6.4 (70 ÷ 11 = 6.36…)	6.1 (55 ÷ 9 = 6.11...)
Mode	7	6
Median	7 (3, 4, 5, 5, 6, $\boxed{7}$, 7, 7, 7, 9, 10)	6 (4, 5, 6, 6, $\boxed{6}$, 6, 7, 7, 8)

b You can use any of the averages but you must state which one it is. They all show that the boys have bigger shoe sizes.

Using frequency tables

A **frequency table** has two rows (or columns).

The first shows all the different values that the data can take.

The second shows the frequency of each value.

It is useful to add an extra column when calculating the mean.

Number of packets	Frequency	Total number of packets
0	9	0
1	11	11
2	9	18
3	4	12
4	11	44
5	10	50
Total	54	135
Mean	$\frac{135}{54} = 2.5$	

$0 \times 9 = 0$
$1 \times 11 = 11$
$2 \times 9 = 18$

When deciding which average to use, remember that the mean is the only one which uses all of the data. The median shows where the data split into two halves and the mode only uses the most common value.

The median can also be found from the frequency table.

There are 54 items of data so the median will be between the 27th and 28th which are both 2, so the median is 2 packets.

SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Mode, median and range

Six friends each buy a new dress for a party. Here are the costs of the dresses.

£16	£136	£35	£16	£55	£100

a Work out the mode and the median values.

b What is the difference in cost between the most expensive dress and the least expensive dress?

c What is the mathematical name for the amount you worked out in part **b**?

Using mean, median, mode and range

Nadir tries to find the averages of these items of data.

27	23	37	23	30

He gets them all wrong.
Here is his working.

mean = 27 + 23 + 37 + 23 + 30 ÷ 5
= 116 ✗
mode = 2 ✗
median = 37 ✗

What should the answers be?
Explain what Nadir has done wrong in each case.

Using frequency tables

Karl has some tomato plants. One set are treated with Grow-well fertiliser, the other set is not. Karl counts the number of ripe tomatoes he picks from each plant per day.

With Grow-well	5, 4, 4, 2, 7, 8, 10, 2, 3, 5, 4, 6, 6, 10, 7, 9, 8, 8, 8, 2
Without Grow-well	3, 5, 3, 5, 6, 9, 3, 3, 5, 4, 6, 5, 6, 5, 3, 6, 5, 6, 6, 3

a Construct a frequency table for each set of data.

b For each set of data find

i the mean **ii** the median

number of tomatoes picked per day for each plant.

c Do the data support the claim that Grow-well increases yield? Explain your answer.

Applying the knowledge

① A car service at Molly's Garage costs £200 plus parts. One day, the final bills were as shown in the sheet below.

	A	B	C	D	E	F	G	H
1	£227	£265.50	£285.30	£212.70	£202.60	£212.70	£217	£215.30
2								

a Calculate the mean cost of a service.

b Work out the median cost of a service.

c Which is more useful for a customer to know, the mean or the median?
Give a reason for your answer.

d What other statistic about these data would be useful for a customer? Explain why.

② The Human Resources department of a small company records the number of absences for each employee.

The results shown in the frequency table.

Number of absences	Frequency
0	20
1	6
2	8
3	12
4	9
5	5
6	0
7	2
8	1

a How many employees are there?

b Work out the mode, mean and median of the data.

c Which average do you think represents the data best?
Give a reason for your answer.

20.1 Using grouped frequency tables

SKILLS CHECK

→ Do I need to do this section?

Complete this section if you need help with the question below.

① 190 people take a general knowledge test with 100 questions.
They are hoping to go on a television show.
The table below shows the number of correct answers.

Number of correct answers, q	Frequency
$30 \leqslant q < 40$	80
$40 \leqslant q < 50$	40
$50 \leqslant q < 60$	30
$60 \leqslant q < 70$	20
$70 \leqslant q < 80$	10
$80 \leqslant q < 90$	8
$90 \leqslant q < 100$	2
Total	

a Calculate an estimate for the mean number of correct answers.
b What is the modal group?
c In which group does the median fall?
d Estimate the range.
e People with 75 or more correct answers will appear on the television show. Estimate how many people this is.

What you need to know

Did you know?

The information collected when people do market research is often put into a grouped frequency table to aid interpretation.

Use a **grouped frequency table** for large amounts of data.

Between 5 and 10 groups (or classes) is usually most suitable.

Show classes for continuous data using less than ($<$) or less than or equal to ($\leqslant$).

The **modal class** is the class with the highest frequency (if the class widths are all the same).

To **estimate the mean** from a grouped frequency table, multiply the mid-interval value for each group by the frequency for that group, add the results and then divide by the total frequency. Round your answer to a suitable degree of accuracy.

How to do it

Finding the mean of continuous data

The maximum temperature, in °C, is recorded in Buenos Aires for each day in June one year.

16.4	12.8	17.6	19.1	16.6	15.5
11.2	18.7	19.5	16.1	15.3	14.2
15.8	15.7	14.9	14.4	13.4	12.1
13.9	11.9	13.1	12.6	10.9	13.5
14.2	15.4	16.6	15.9	15.6	14.3

a Present these data in a copy of the grouped frequency table.

Temperature, T °C	Tally	Frequency
$10 \leqslant T < 12$		
$12 \leqslant T < 14$		
$14 \leqslant T < 16$		
$16 \leqslant T < 18$		
$18 \leqslant T < 20$		

b Calculate an estimate for the mean temperature.

Solution

a

Temperature, T °C	Tally	Frequency
$10 \leqslant T < 12$	$\vert\vert\vert$	3
$12 \leqslant T < 14$	𝍸 $\vert\vert$	7
$14 \leqslant T < 16$	𝍸 𝍸 $\vert\vert$	12
$16 \leqslant T < 18$	𝍸	5
$18 \leqslant T < 20$	$\vert\vert\vert$	3

b

Temperature, T °C	Midpoint, m	Frequency, f	$m \times f$
$10 \leqslant T < 12$	11	3	33
$12 \leqslant T < 14$	13	7	91
$14 \leqslant T < 16$	15	12	180
$16 \leqslant T < 18$	17	5	85
$18 \leqslant T < 20$	19	3	57
	Totals	30	446

Estimated mean $= 446 \div 30 = 14.866 = 14.9$ °C (1 d.p.)

Learning exercise

① The table shows the heights of 22 footballers about to play a match.

Copy and complete the table and use it to estimate the mean height of these footballers.

Height, h (cm)	Frequency, f	Midpoint, m	$m \times f$
$150 \leqslant h < 156$	3		
$156 \leqslant h < 162$	6		
$162 \leqslant h < 168$	8		
$168 \leqslant h < 174$	3		
$174 \leqslant h < 180$	2		
Totals			

② Ruth's telephone bill shows the lengths, in minutes, of her last 20 calls.

47 min	30 min	19 min	44 min	32 min
18 min	41 min	12 min	57 min	24 min
28 min	36 min	9 min	42 min	17 min
16 min	46 min	32 min	33 min	29 min

a Copy and complete the frequency table.

Length of call , l (minutes)	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant l < 10$			
$10 \leqslant l < 20$			
$20 \leqslant l < 30$			
$30 \leqslant l < 40$			
$40 \leqslant l < 50$			
$50 \leqslant l < 60$			
Totals			

b In which group does the median lie?

c Estimate the value of the mean.

③ A group of people do a puzzle as part of an aptitude test.

Here are the times (in seconds) that it takes them to solve the puzzle.

24	83	114	84	90	103	74	176	61	40	162	49
77	92	108	124	185	89	63	79	37	91	65	19

a **i** Write down the times of the first and the last people to solve the puzzle.

ii What is the range?

b Write down the median for this data set.

c Copy and complete the frequency table.

Time, t (seconds)	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant t < 40$			
$40 \leqslant t < 80$			
$80 \leqslant t < 120$			
$120 \leqslant t < 160$			
$160 \leqslant t < 200$			
Totals			

d In which group does the median lie?

e Estimate the value of the mean.

f The fastest 25% of the group are accepted and the slowest 25% are failed. The others have to do more tests.

What can you say about the marks of those who do more tests?

④ A speed camera recorded the speed of cars, v mph, on a road through a housing estate. Here are the results.

Speed, v (mph)	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant v < 10$	1		
$10 \leqslant v < 20$	12		
$20 \leqslant v < 30$	22		
$30 \leqslant v < 40$	4		
$40 \leqslant v < 50$	1		
Totals			

a Complete a copy of the table and estimate the mean speed.

b What do you think the speed limit is?

c Local residents say that any speed above 25 mph is unsafe.

Estimate the percentage of cars that are travelling at 25 mph or faster.

⑤ A group of students record how many hours each week they spend on homework.

Here are their results.

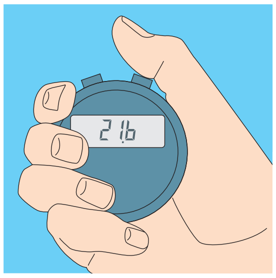
Hours, h	$0 \leqslant h < 4$	$4 \leqslant h < 8$	$8 \leqslant h < 12$	$12 \leqslant h < 16$	$16 \leqslant h < 20$
Frequency	3	15	14	9	4

a Estimate the mean number of hours a student spends doing homework.

b Most students do homework on six days. What is the average amount of homework they do each day?

⑥ A running club records the time members take to complete a cross-country race.

Time, t (minutes)	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant t < 10$	0		
$10 \leqslant t < 20$	1		
$20 \leqslant t < 30$	8		
$30 \leqslant t < 40$	14		
$40 \leqslant t < 50$	7		
Totals			



a Copy and complete the table then estimate the mean time.

b What can you say about the range?

c Another club had a mean of 27.9 minutes and a range of 55 minutes.

Compare the performance of the two clubs.

⑦ Harry travels to work by train.

He records the time, in minutes, that he waits for the train.

Waiting time (minutes), t	Frequency, f
$0 \leqslant t < 5$	6
$5 \leqslant t < 10$	5
$10 \leqslant t < 15$	11
$15 \leqslant t < 20$	0
$20 \leqslant t < 25$	9
$25 \leqslant t < 30$	4
Totals	

a Complete a copy of the table, adding and completing extra columns as necessary, and then estimate the mean waiting time.

b How many times does Harry have to wait at least 20 minutes?

c One day the waiting time was 72 minutes.

Can you explain what might have happened? Why did Harry exclude this time from his calculation?

⑧ The table shows the distance jumped by junior long jumpers at a club training session.

Distance (metres), d	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant d < 1$	1		
$1 \leqslant d < 2$	5		
$2 \leqslant d < 3$	12		
$3 \leqslant d < 4$	9		
$4 \leqslant d < 5$	8		
Totals			

a Complete a copy of the table and use it to estimate the mean distance jumped.

b There were 14 jumpers and they had 3 jumps each. How many foul jumps were there?

c The qualification for the club junior team is a jump over 4.5 m.

How many jumpers qualified at this session?

9 Alan plants two types of early potato, A and B. He measures the mass of the potatoes, w grams, that each plant produces.

The results are summarised in this table.

Compare the yields of the two types of potatoes.

	$700 \leqslant w < 750$	$750 \leqslant w < 800$	$800 \leqslant w < 850$	$850 \leqslant w < 900$	$900 \leqslant w < 950$	$950 \leqslant w < 1000$
A	5	8	4	2	5	0
B	2	4	5	7	3	3

10 An IT company gives all applicants for a vacancy an aptitude test.

The applicants score s marks on the test.

The table shows the results.

Score, s	Frequency, f	Midpoint, m	$m \times f$
$0 \leqslant s < 10$	11		
$10 \leqslant s < 20$	6		
$20 \leqslant s < 30$	12		
$30 \leqslant s < 40$	10		
$40 \leqslant s < 50$	9		
Totals			

a Copy and complete the table. Calculate an estimate of the mean mark.

b The IT company interviews all applicants who achieved 5 marks more than the mean. Estimate the number of applicants interviewed.

c Explain why your answers are estimates.

11 The grouped frequency table gives information about the distance that 75 commuters travel to work.

Distance travelled, d km	Frequency, f	Midpoint, m	$m \times f$
$20 < d \leqslant 30$	6	25	
$30 < d \leqslant 40$			420
$40 < d \leqslant 50$	20	45	900
$50 < d \leqslant 60$			
$60 < d \leqslant 70$	11		715
Total	75	Total	

a Copy and complete the table.

b In which group does the median lie?

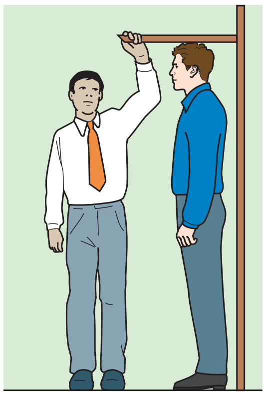
c Estimate the mean distance travelled by the commuters.

d A commuter expects that their journey will take $1\frac{1}{2}$ minutes for every kilometre commuted.

Estimate the mean time spent commuting.

⑫ A doctors' surgery surveys a group of adult male patients to find out their heights.

Here is a frequency table of the results.



Height in cm	Frequency
$150 < h \leqslant 160$	4
$160 < h \leqslant 170$	20
$170 < h \leqslant 180$	52
$180 < h \leqslant 190$	17
$190 < h \leqslant 200$	5
$200 < h \leqslant 210$	2

a How many people took part in the survey?

b In which class does the median lie?

c Dr Smith says the range of heights is 60 cm.

Is Dr Smith correct? Give a reason for your answer.

d Estimate the mean height of the patients.

e Dr Smith decides a patient is classified as tall if he is 9.5 cm taller than the mean.

Estimate the percentage of tall patients at the surgery.

⑬ The production team for a TV quiz show run a general knowledge test to identify suitable contestants.

The table below shows the number of correct answers from a group of hopeful contestants.

Number of correct answers, c	Frequency
$30 < c \leqslant 40$	41
$40 < c \leqslant 50$	16
$50 < c \leqslant 60$	10
$60 < c \leqslant 70$	7
$70 < c \leqslant 80$	8
$80 < c \leqslant 90$	10
$90 < c \leqslant 100$	8
Total	

a What is the modal group?

b In which group does the median fall?

c Estimate the mean number of correct answers.

The test has 100 questions.

A correct answer scores 2 points, an incorrect or missing answer scores −1 point.

d Estimate the mean score.

Contestants need a score of 125 or more to appear on the television show.

e Estimate the probability that a person chosen from the group at random will be selected for the show.

Do I know it now?

① A group of people take an aptitude test for a flying school. Here are the results.

Score, s	**Frequency, f**	**Midpoint, m**	$m \times f$
$0 \leqslant s < 20$	6		
$20 \leqslant s < 40$	8		
$40 \leqslant s < 60$	15		
$60 \leqslant s < 80$	14		
$80 \leqslant s < 100$	5		
Totals			

a Complete a copy of the table and then estimate the mean score.

b The pass mark for the flying school is 75. Estimate the number of candidates that passed.

c The lowest $\frac{1}{3}$ of the group of people are told they cannot try again.
What can you say about the marks of this group?

CHAPTER 21

ESSENTIAL TOPICS – STATISTICS AND PROBABILITY

Statistical diagrams

JUST IN CASE

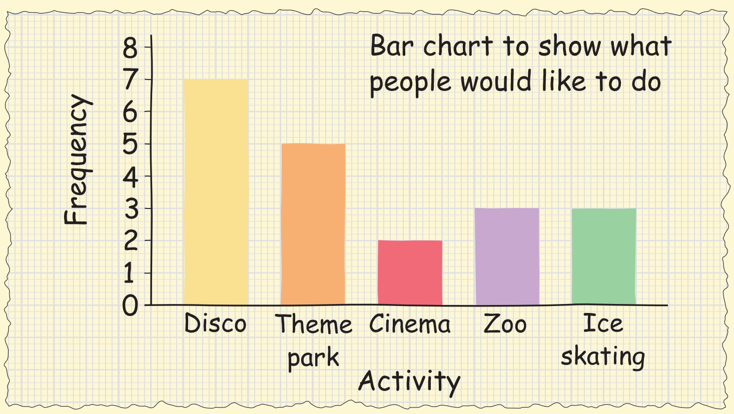
Drawing a bar chart

Hannah is organising an end-of-year event for her year group.
She asks everyone in her class what they would like to do and records their suggestions.

Activity	Disco	Theme park	Cinema	Zoo	Ice skating
Frequency	7	5	2	3	3

Draw a bar chart of Hannah's data.

Solution

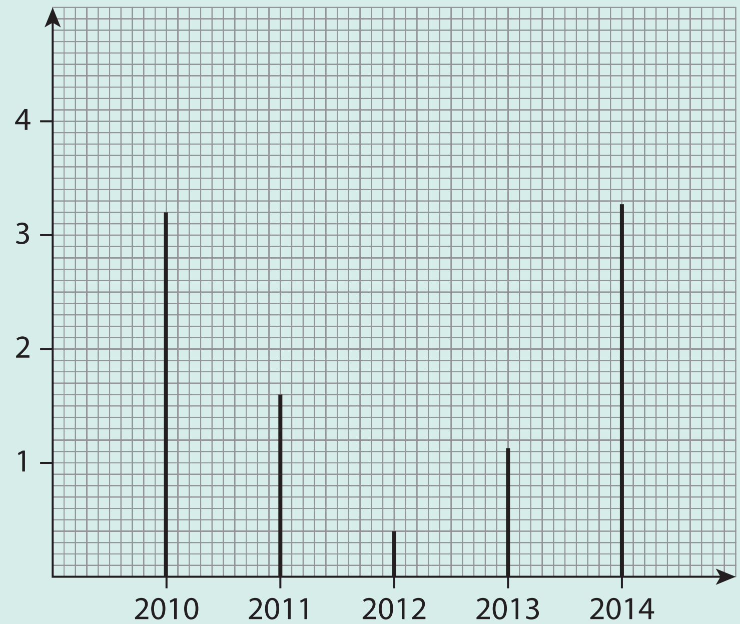


Bars are the same width.
Frequency goes on the vertical axis.
It is usual to have spaces between the bars.

Vertical line charts

A vertical line chart is used to show data that are obtained at intervals of time or place.
The scales on the axes do not have to be the same as each other. They will usually represent quite different things.
The scale must be the same all along each axis, so the numbers must be evenly spaced.
If time is involved it goes along the horizontal axis.

A company declares its profits on 1st June each year. Profits for 2010 to 2014 are shown here.



SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Using tables and charts

Martin is a catering student. He asks 50 friends what their favourite takeaway meal is. Here are his results.

Type of takeaway	Chinese	Indian	Pizza	Fish and chips
Frequency	13	11	17	9

a Draw a bar chart to show these results.

b Are there any other takeaway meals you think Martin could have included in his survey?

c What difference might age have on people's takeaway choices?

d Suggest some improvements to the way Martin carried out his survey.

➜ Vertical line charts

This vertical line chart shows the noon temperature in Sheffield in the first week of August one year.

a What is the lowest midday temperature?

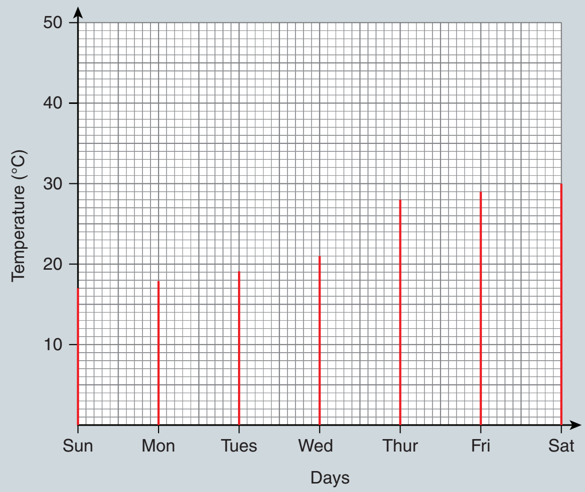
b Which is the highest midday temperature?

c Tuesday is 1 °C hotter than Monday. How much hotter is Wednesday than Tuesday?

d When is the greatest temperature change?

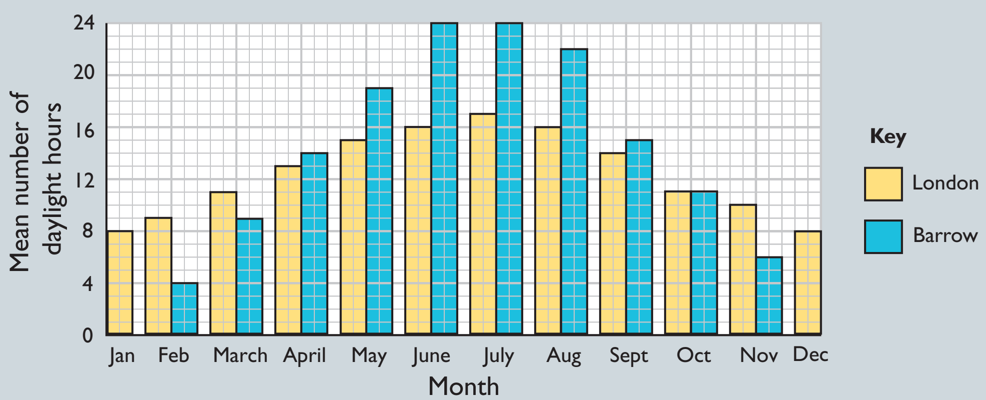
e **i** Describe what happens to the temperature during the week.

ii How is this seen on the graph?



➜ Applying the knowledge

① The bar chart shows the monthly mean number of daylight hours per day in London, UK, and the city of Barrow, Alaska.



a In which month is the mean number of daylight hours the same in London and Barrow?

b Work out the range in the mean number of daylight hours in London.

c How long is the average night in

i July in Barrow

ii December in Barrow?

d Work out the total number of daylight hours in London in June.

e Why has a multiple bar chart been used to display the data?

② The midday temperature in Brighton was recorded each day for a week.

Day	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Temperature (°C)	21	23	25	21	19	18	22

a Draw a vertical line chart for this information.

b Sam says, 'The hottest day was Wednesday.'

Present an argument to show this may not be true.

21.1 Pie charts

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Melanie is doing a survey about sport.

Boy ☐ Girl ☐

Which is your favourite sport from the following list?

athletics ☐ football ☐ hockey ☐

swimming ☐ tennis ☐

Here are her results.

	Athletics	Football	Hockey	Swimming	Tennis
Boys	4	19	2	10	5
Girls	5	3	11	9	8

a **i** How many boys did Melanie ask?

ii If she draws a pie chart for the boys' results, how many degrees should she use to represent each boy?

b **i** How many girls did she ask?

ii If she draws a pie chart for the girls' results, how many degrees should she use to represent each girl?

c Draw two pie charts to represent the data.

d Write a few sentences to explain what your pie charts show about girls' and boys' sport preferences.

What you need to know

Did you know?

Pie charts are often used in the media and workplace to illustrate how money is spent within an organisation.

A **pie chart** shows the parts of a whole.

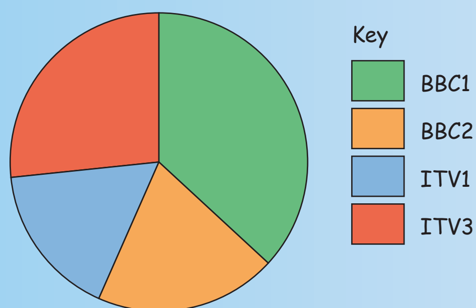
This pie chart shows that $\frac{1}{2}$ of the crisps sold were cheese and onion and $\frac{1}{4}$ were ready salted. The other two flavours are both the same size and take up $\frac{1}{4}$ of the pie chart in total. This means that they are each half of $\frac{1}{4}$, that is $\frac{1}{8}$.

To calculate the size of each sector

- work out the angle that represents one individual by dividing 360° by the total
- multiply by the frequency for each category.

The data show which of four television channels thirty people were watching on Monday evening.

	BBC1	BBC2	ITV1	ITV3	Totals
Frequency	11	6	5	8	30
Pie chart angle	132°	72°	60°	96°	360°

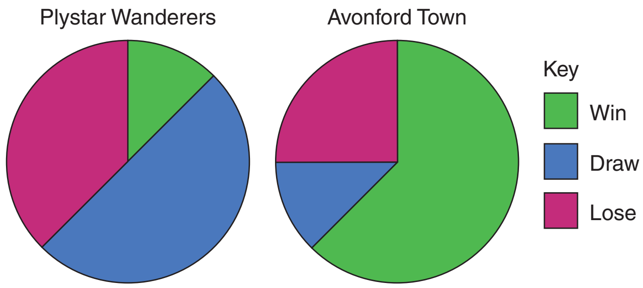


$360 \div 30 = 12°$ for one person, $12 \times 11 = 132°$

How to do it

Reading a pie chart

These pie charts show information about two football teams in the same season.



a Which team played better?

b What fraction of their matches did Plystar Wanderers draw?

c **i** What fraction of their matches did Avonford Town lose?

ii Which angle is used to show this?

d Each team has played 32 matches. How many matches did Avonford Town lose?

Solution

a Avonford Town, as their pie chart shows a greater proportion of wins.

b $\frac{1}{2}$

c **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ of $360° = 90°$

d $\frac{1}{4}$ of $32 = 8$ matches

Drawing a pie chart

Mark has made a list of the favourite types of music of his friends.

Dance	Pop	RnB	Dance	RnB	RnB	Dance	RnB
RnB	RnB	RnB	Garage	RnB	Pop	Dance	
Garage	RnB	Garage	Dance	RnB	Dance	RnB	
RnB	RnB	RnB	Garage	Pop	RnB	Pop	
RnB	Dance	RnB	Dance	Pop	RnB	Dance	

a Make a frequency table with a tally column for Mark's data.

Mark draws this pie chart to show the data.

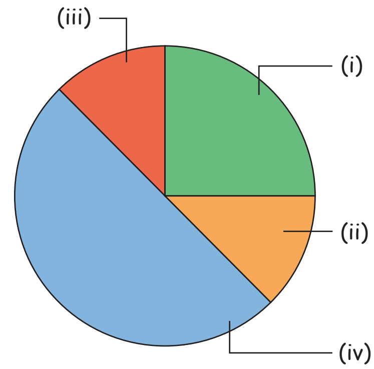
b What angle represents one person?

c Label each section (**i**) – (**iv**) with the correct type of music.

d What fraction chose Dance?

e What is the angle for Pop?

f What is the angle for RnB?



Solution

a

Favourite music	Tally	Frequency
Dance	卌 ||||	9
Pop	卌	5
RnB	卌 卌 卌 |||	18
Garage	||||	4
Total		36

b There are 36 people. The angle for each person is $360 \div 36 = 10°$.

c **i** Dance
ii Pop
iii Garage
iv RnB

d $\frac{9}{36} = \frac{1}{4}$

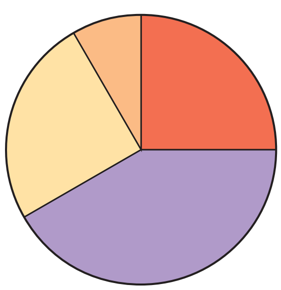
e $10° \times 5 = 50°$

f $10° \times 18 = 180°$

Learning exercise

① A group of school leavers were asked about their plans for next year.

The pie chart shows their responses.



a Which was the most popular response?

b Which angle represents those who chose gap year?

c What fraction of the group chose gap year?

d 168 school leavers were questioned.

Estimate how many chose each of the four options.

② A rail user group do a survey of how people get to their local station.

Here are their results.

Method	Walk	Bus	Taxi	Own car	Lift
Frequency	10	5	6	14	5

They want to show the results on a pie chart.

a How many degrees should they use to represent one person?

b Draw the pie chart.

c The rail user group are campaigning for more car parking space.

1200 people use the station each day. How many of them do you expect to need a parking space?

③ Jamini is a cat. Her owners watch how she spends her day.

Activity	Sleeping	Prowling	Eating	Grooming
Time (hours)	16	5	1	2

They show this on a pie chart.

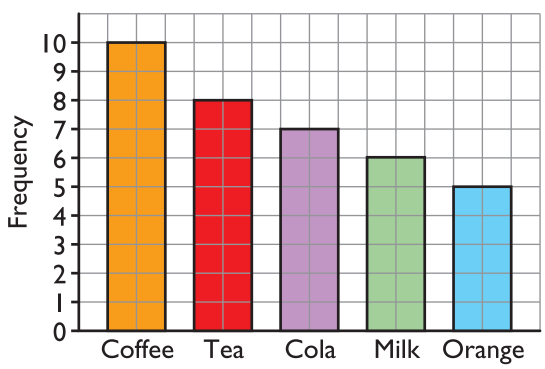
a How many degrees do they use for one hour?

b Draw the pie chart to show how Jamini spends her day.

c Jamini's owners watch her some more. They decide that she spends $\frac{1}{2}$ an hour playing, $6\frac{1}{2}$ hours prowling and less time sleeping.

Describe the changes they must make to the pie chart.

④ The bar chart shows the types of drink that Sally sold in her café one day.



a Draw a pie chart to show this information.

b Which diagram do you find more helpful – the bar chart or the pie chart?

⑤ A travel operator carried out a survey of British people in a bar in a Spanish holiday resort. She asked them how they had travelled to the resort.

Here are her results.

Method	Air	Drive	Coach	Live here	Other
Frequency	32	28	20	9	1

a How many people responded to the survey?

b The travel operator draws a pie chart to show the information.

How many degrees does she use to represent one person?

c Draw the pie chart.

d Draw a bar chart showing the same information.

e The travel operator noticed that about $\frac{1}{3}$ of the people arrived by air.

Which of the two charts do you think she was looking at? Explain why.

⑥ The table shows the results of a fruit seller's survey about people's favourite fruits.

Fruit	Apple	Orange	Peach	Grapefruit	Pineapple	Banana	Other
Frequency	11	8	9	6	4	12	40

a How many people did she survey?

b The information is to be shown as a pie chart. How many degrees should be used to represent one person?

c Draw the pie chart.

d Comment critically on the data collection and the display of the results as a pie chart.

⑦ Emma works for an animal charity that re-homes pets.

She keeps a record of the types of animals the charity re-homes.

$\frac{3}{8}$ are dogs, $\frac{2}{9}$ are cats, $\frac{1}{4}$ are rabbits.

The rest (for example, snakes) are recorded in the category 'others'.

a Work out the angles for a pie chart to show this information.

b Draw the pie chart.

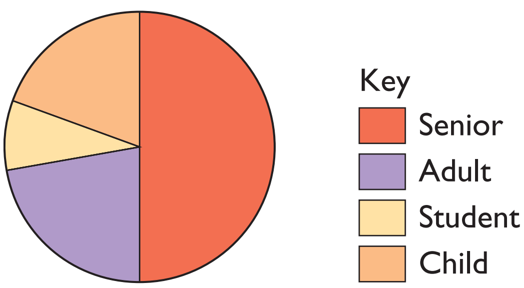
c Emma's friend looks at the pie chart. She thinks the charity re-homed 90 rabbits. Is she right? Explain your answer.

⑧ A cinema manager records the number of different types of tickets sold on Monday.

Tickets	Senior	Adult	Student	Child
Number sold	42	10	6	14

The manager draws a pie chart to represent these figures.

What mistakes did the manager make?



⑨ A careers adviser surveys a group of students to find out their career aims.

$\frac{7}{12}$ of the students plan to go to university.

$\frac{1}{4}$ of the students plan to start an apprenticeship.

The remainder plan to take a gap year.

a What fraction of students plan to take a gap year?

b Draw a pie chart to illustrate this data.

c 140 students plan to go to university.

How many students were surveyed altogether?

⑩ A clothes company collected data to find out why customers returned products to one of their stores last month. These are the results.

Reason	Frequency
Wrong size	27
Faulty	4
Wrong colour	16
Unwanted gift	5
Changed mind	8

a Draw and label a pie chart to represent this data.

b The clothes company has 15 000 returns annually.

Estimate how many items are returned because they are faulty.

Do I know it now?

① There are 224 children at a primary school.

$\frac{1}{8}$ of the children go to art club.

$\frac{1}{4}$ of the children go to football club.

a **i** What fraction of the children go to neither art club nor football club?

ii What angle would represent this on a pie chart?

b Draw the pie chart.

c How many children do not go to either art club or football club?

21.2 Displaying grouped data

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① Shamicka asks her pupils to record the number of minutes they spend playing computer games during May. Here are her results.

25	150	262	30	143	0	55	320	260	60
65	140	40	170	74	130	45	125	300	220
96	132	90	185	89	167	68	50	160	82

a Copy and complete the table.

Time spent playing computer games, t minutes	Tally	Frequency
$0 \leqslant t < 50$		

b Draw a frequency diagram to show this information.

c Shamicka plans to ask the same question in August. Predict the shape of the new diagram.

What you need to know

Ava

How many pets have you got?

0	2	3	0	1
4	2	0	1	6
5	0	3	2	2
1	0	0	3	4
2	4	3	1	1

14 s	13.5 s	20 s	16.7 s	14.96 s
15 s	19 s	16.75 s	14.8 s	17.63 s
13.9 s	17.2 s	18 s	21 s	15.87 s
18.2 s	17.3 s	20 s	14.24 s	13.1 s
16 s	18.12 s	14 s	16.4 s	17 s

Coley

Coley times his classmates running 100 m.

Ava's data are **discrete**. Each value must be a whole number. You cannot own 4.2 pets!

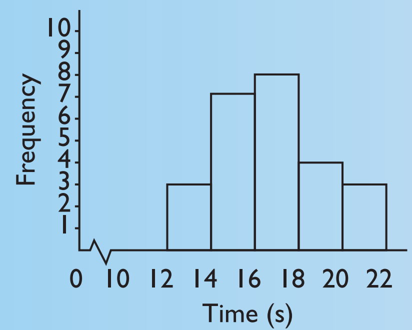
Coley's data values are **continuous**. Any sensible value is possible. The time taken to complete 100 metres could be 14 seconds, 15.2 seconds or 13.98 seconds.

You can group both types of data in a frequency table.

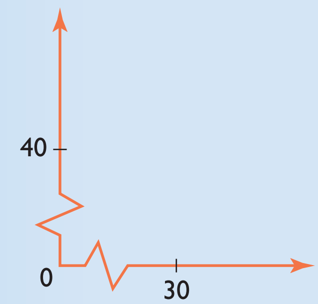
Time, t seconds	Frequency
$12 \leqslant t < 14$	3
$14 \leqslant t < 16$	7
$16 \leqslant t < 18$	8
$18 \leqslant t < 20$	4
$20 \leqslant t < 22$	3

A time of 14 seconds is included in the group $14 \leqslant t < 16$.

This **frequency diagram** shows the data in the table.



A jagged line is used to show the scale on an axis that does not start at zero. Sometimes just one axis has a **broken scale**, sometimes it is both. A broken scale can make it easier to plot a graph but, occasionally, it can also mislead you.



How to do it

Plotting a frequency diagram

Alex and Emily measured heights, h m, of girls in their athletics club.

1.45	1.57	1.60	1.48	1.60	1.77	1.56	1.55	1.66	1.66
1.70	1.62	1.60	1.42	1.52	1.55	1.59	1.72	1.52	1.62
1.80	1.52	1.75	1.55	1.70	1.63	1.44	1.73	1.50	1.54
1.36	1.62	1.54	1.64	1.55	1.82	1.47	1.68	1.55	1.70
1.60	1.75	1.63	1.75	1.44	1.60	1.60	1.42	1.58	1.80

a Display this information in a grouped frequency table.

b Use your grouped frequency table to plot a frequency diagram.

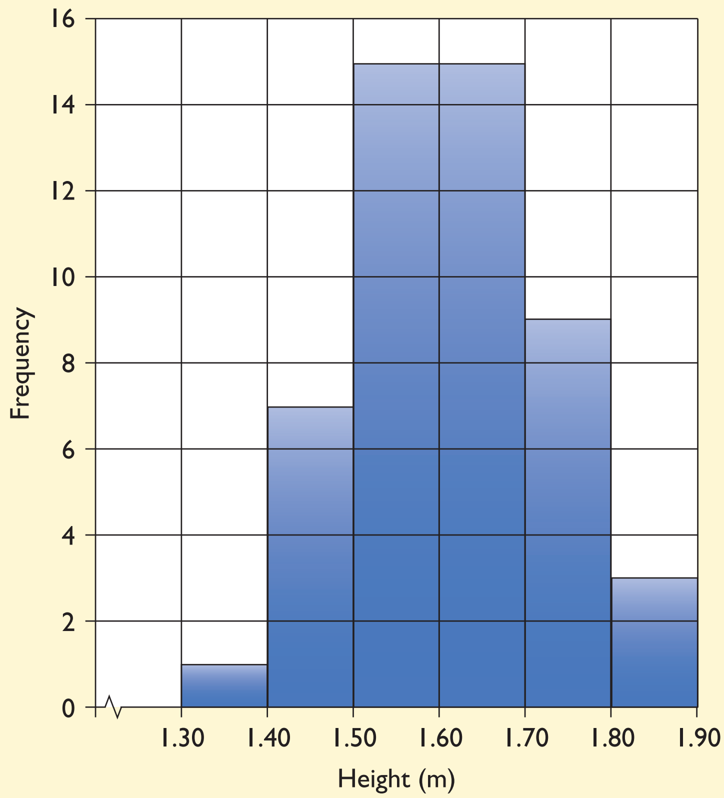
c Describe the distribution of the data.

Solution

a

Height, h m	Frequency
$1.30 \leqslant h < 1.40$	1
$1.40 \leqslant h < 1.50$	7
$1.50 \leqslant h < 1.60$	15
$1.60 \leqslant h < 1.70$	15
$1.70 \leqslant h < 1.80$	9
$1.80 \leqslant h < 1.90$	3

b Frequency diagram showing the heights of girls in an athletics club.



c Most of the girls are between 1.5 and 1.7 m tall. A few are shorter than this and some are taller.

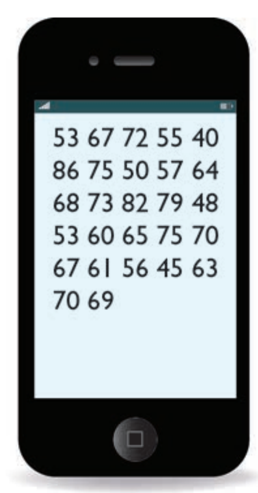
Learning exercise

① State whether each type of data is discrete or continuous.

a **i** The number of cars passing a camera on a motorway
ii The speed of cars passing the camera on a motorway
iii The numbers of people in the cars

b **i** The number of passengers on an aeroplane
ii The masses of the passengers on an aeroplane
iii The average age of the passengers on an aeroplane
iv The length of the aeroplane flight

② Trevor has recorded the masses of some people.
Here are the results, in kilograms.



a Copy and complete the tally chart.

Mass, w kg	Tally	Frequency
$40 \leqslant w < 50$		
$50 \leqslant w < 60$		
$60 \leqslant w < 70$		
$70 \leqslant w < 80$		
$80 \leqslant w < 90$		

b Which class has the highest frequency?

c Draw a frequency diagram to show Trevor's data.

d Which shows the data most clearly: the tally chart, the frequency diagram or the original information?

③ These are the numbers of people visiting a gym on each of 21 days.

23, 45, 31, 37, 63, 54, 36, 64, 60, 49, 50, 32, 45, 40, 38, 37,
41, 53, 71, 57, 62

a Copy and complete the tally chart.

Number attending, n	Tally	Frequency
$20 \leqslant n < 30$		
$30 \leqslant n < 40$		
$40 \leqslant n < 50$		
$50 \leqslant n < 60$		
$60 \leqslant n < 70$		
$70 \leqslant n < 80$		

b Which is the modal class?

c On how many days did fewer than 40 people attend?

d What can you say about the numbers of people who went to the gym over this period?

④ Sally records the reaction times for a group of people.

Here are her results, in seconds.

0.34	0.56	0.72	0.20	0.65	0.57	0.36	0.43
0.81	0.73	0.27	0.30	0.52	0.48	0.61	0.59
0.28	0.28	0.44	0.62	0.64	0.50	0.28	0.33
0.58	0.46	0.44	0.51	0.26	0.38		

a Make a grouped frequency table using class intervals $0.20 \leqslant n < 0.30$, $0.30 \leqslant n < 0.40$, and so on.

b Draw a frequency diagram to show the data.

c What percentage of the group have a reaction time of less than 0.30 seconds?

d Describe the shape of the distribution.

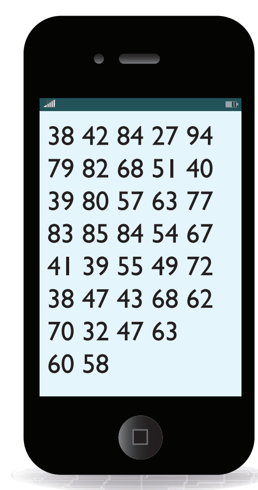
⑤ Henry plays a computer game and keeps a record of his scores.

a Make a grouped frequency table using class intervals $20 \leqslant n < 30$, $30 \leqslant n < 40$, and so on.

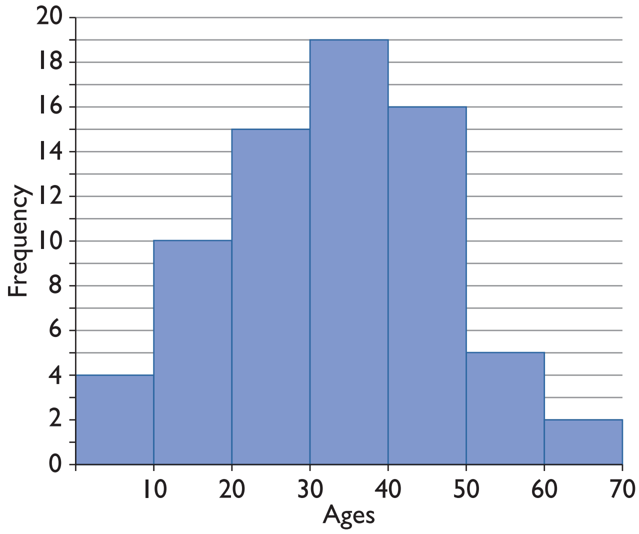
b Draw a frequency diagram to show the data.

c Which is the median value? Explain, in writing, the easiest way to find this value.

d Describe the shape of the distribution. Is it easier to use the original data or the grouped data?



⑥ Sam recorded the ages of people using the local swimming pool. His results are shown in this frequency diagram.



a How many people in the survey were aged 40 to 60?

b Draw and complete a frequency table for the data.

c Explain why it is likely that this survey was taken in the evening.

⑦ The junior members of a cricket club record how long they can stay underwater, t seconds.

Boys	24	15	21	18	9	0	45	33	54	31	7	25	27	31	30	24	42
Girls	48	36	24	16	42	50	44	28	30	20	16	35	42	52	34		

a Make grouped tally charts to show

i the boys' data

ii the girls' data.

Use the groups $0 \leqslant t < 10$, $10 \leqslant t < 20$, and so on.

b Write down the median times for boys and for girls.

c Draw separate frequency diagrams for the boys and the girls.

d Who can stay underwater longer, boys or girls? Explain your answer.

⑧ Pat is a biologist. She is investigating the distribution of poppies in a meadow. She divides the meadow into equal-sized patches and counts the number of poppies in each patch.

Pat records her results as follows.

72	67	47	65	55	52
58	15	74	69	66	31
22	91	48	84	82	73
150	71	33	44	65	15
52	50	64	49	88	76

a One of the values is an outlier.

i Which one is it?

ii Give one reason why she might exclude it and one reason why she might accept it.

Pat thinks she made a mistake and it should have been 15. So she changes it to 15.

b Make a grouped frequency table, using intervals of class width 10, starting at 0.

c Draw a frequency diagram to represent the data.

d **i** Write down the mode and the median of the data.

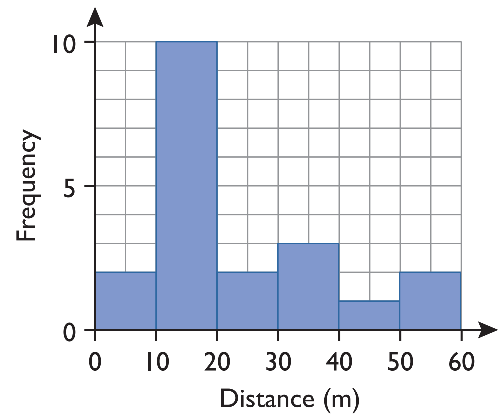
ii Is the mode or median more representative of the data? What about the modal class?

e The meadow is said to be 'in good heart' where there are 60 or more poppies per patch. Estimate the percentage of the meadow that is 'in good heart'.

(9) Zubert is a diver. He holds a competition to find out how far his friends can swim underwater without taking a breath.

He records the results in a frequency table.

Distance, metres		Frequency
At least	**Less than**	
0	10	2
10	20	11
20	30	4
30	40	2
40	50	1



a Display this information as a frequency diagram.

b Describe the distribution.

c Calculate the percentage of people in the modal group.

Zubert shows his friends some techniques and then tests them again.

The frequency diagram for the re-test is above.

d Describe the effect of Zubert's teaching.

(10) It can be very difficult to tell whether a newly-hatched chicken is male or female.

Owen keeps a particular rare breed of chicken and he does an experiment to see if newly-hatched cockerels are heavier than newly-hatched hens. He weighs and marks newly-hatched chickens and then records whether each one turns out to be a cockerel or a hen.

Owen's measurements are summarised in this table.

Mass, m g	Female	Male
$25 \leqslant m < 30$	1	0
$30 \leqslant m < 35$	6	0
$35 \leqslant m < 40$	18	2
$40 \leqslant m < 45$	23	10
$45 \leqslant m < 50$	7	27
$50 \leqslant m < 55$	3	26
$55 \leqslant m < 60$	2	13
$60 \leqslant m < 65$	0	2
Total	60	80

a Draw frequency diagrams for females and males.

b Describe what your diagrams show.

Owen says, 'As a rule of thumb, any chick under 45 g will turn into a hen and any chick over 45 g will turn into a cockerel.'

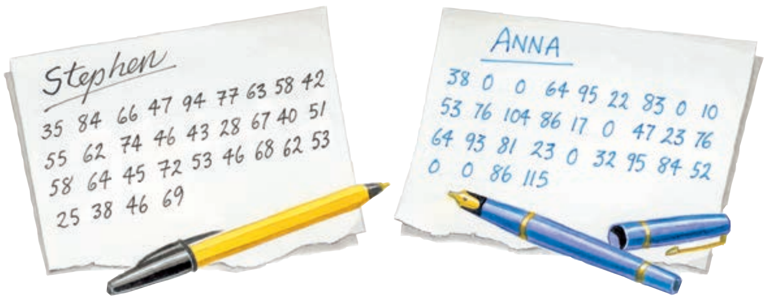
c Estimate the percentage of

i female chicks that Owen would judge to be male

ii chicks whose gender Owen would predict correctly.

⑪ Stephen and his sister Anna share a computer.

Stephen and Anna each keep a record of the length of time, in minutes, that they use the computer each day for one month.

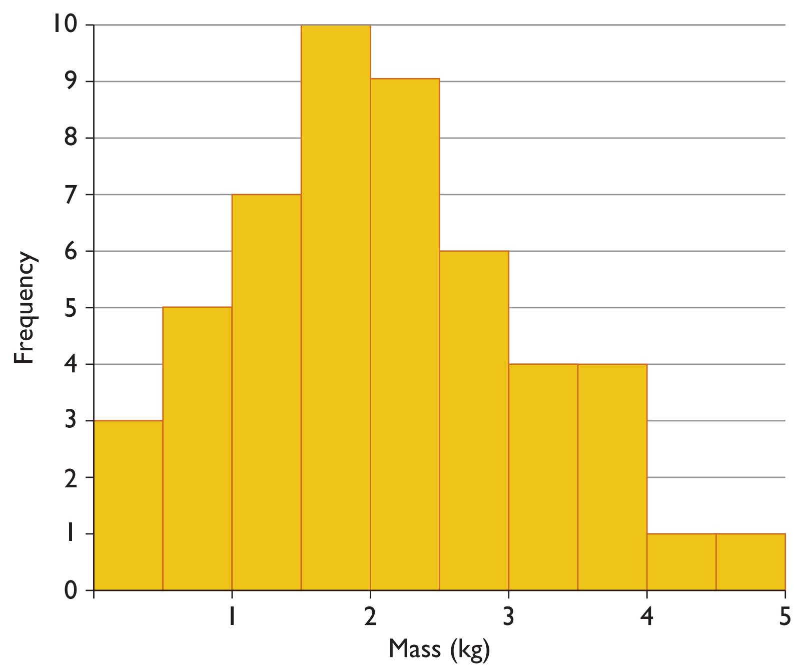


a Compare the distributions of the lengths of time Stephen and Anna use the computer this month.

b Stephen and Anna want their parents to buy them a second computer. They say, 'We often have to stay up late to do our work on the computer.' Comment on what they say.

Do I know it now?

① The frequency diagram shows the masses of cats taken into a vet's surgery one week.



a Tabby weighs 832 g. Which group is she in?

b Which is the modal group?

c Draw a grouped frequency table for the data.

d How many cats were taken in to see the vet that week?

21.3 Scatter diagrams

SKILLS CHECK

➜ Do I need to do this section?

Complete this section if you need help with the question below.

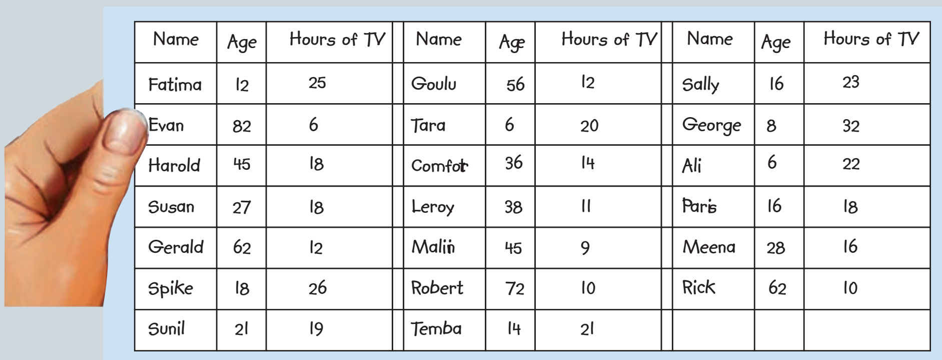
① Vicki thinks that older people watch more television than younger people.

She carries out a survey.

She chooses 20 people of varying ages.

They each keep a record of how much TV they watch during one week.

Here are her results.



Name	Age	Hours of TV	Name	Age	Hours of TV	Name	Age	Hours of TV
Fatima	12	25	Goulu	56	12	Sally	16	23
Evan	82	6	Tara	6	20	George	8	32
Harold	45	18	Comfort	36	14	Ali	6	22
Susan	27	18	Leroy	38	11	Paris	16	18
Gerald	62	12	Malin	45	9	Meena	28	16
Spike	18	26	Robert	72	10	Rick	62	10
Sunil	21	19	Temba	14	21			

a Draw a scatter diagram to show the data.

b **i** Is there any correlation between age and the amount of time spent watching TV?

ii What does this mean?

c Draw a line of best fit on your scatter diagram.

d Jane Smith is 58 years old. Estimate how much television she watches.

e Scott McKenzie watches 8 hours of television per week. Estimate his age.

What you need to know

Did you know?

Many people think that correlation implies causation.

For example, there is correlation between the amount of rain and the number of umbrellas: when there is more rain, there are more umbrellas. Does putting up umbrellas cause it to rain?

Scatter diagrams are used to investigate possible relationships between **two variables** affecting the same data (called **bivariate data**).

You do not join up the points on a scatter diagram.

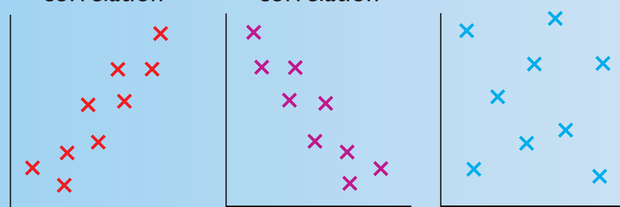
The word **correlation** describes the relationship between the values of the two variables.

If the variables increase together there is **positive correlation**.

If one variable decreases when the other increases, there is **negative correlation**.

If there is correlation between the variables, you can draw a **line of best fit** through the points. This is a straight line that best represents the trend of the data.

Positive correlation | Negative correlation | No correlation



How to do it

Plotting a scatter diagram

Matthew and Aneesa are having an argument.

I think that people with long legs can jump further than people with short legs.

Rubbish! The length of a person's legs does not affect how far they can jump.

They decide to collect data from their friends to find out who is right.

	Alan	Barry	Claire	Dipak	Ernie	Flora	Gurance	Habib	Ivan
Inside leg measurement (cm)	60	70	50	65	65	70	55	75	60
Standing jump distance (cm)	85	90	65	90	80	100	80	95	70

a Draw a scatter diagram for the data.

b **i** Describe the correlation in the scatter diagram.

ii Does the data support what Matthew says or what Aneesa says?

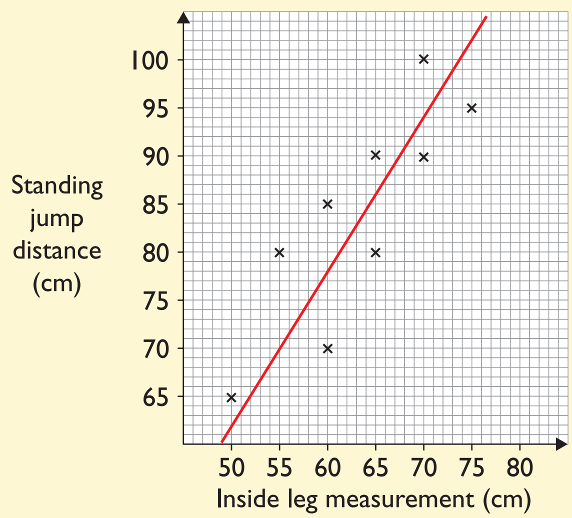
c **i** Draw a line of best fit.

ii Jemima has an inside leg measurement of 70 cm.
Use your graph to estimate what distance she is likely to jump.

d Do Matthew and Aneesa have enough data to be certain about their findings?

Solution

a



b **i** The graph shows positive correlation: as the inside leg measurement increases, so does the distance jumped.

ii This supports Matthew's claim.

c **i** A line of best fit is drawn on the graph. The line of best fit leaves an even distribution of points on either side of the line. The line of best fit may go through some points or none at all.

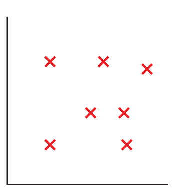
ii From the graph we can predict that Jemima should jump about 94 cm.

d They do not have enough data to be certain. More points on the graph would help.

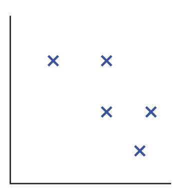
Learning exercise

① Describe the correlation shown in each scatter diagram.

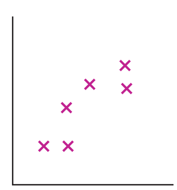
a



b

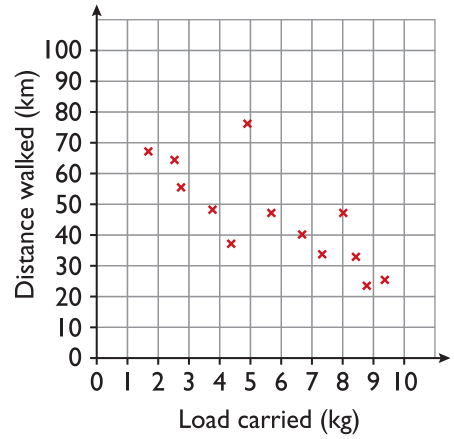


c



d

② The scatter graph shows the distances walked and the loads carried by a number of people on a hiking trip.



a Describe the correlation shown in the diagram.

b Copy the diagram and draw the line of best fit.

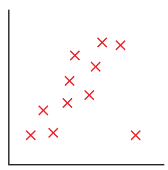
c Use the line of best fit to estimate the distance walked by someone with a 6 kg load.

d Use the line of best fit to estimate the load carried by someone who walked 70 km.

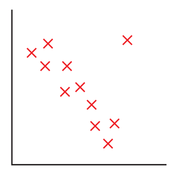
③ A car salesman draws three scatter graphs to show information about the cars on his forecourt.

The axes on the scatter graphs have no labels.

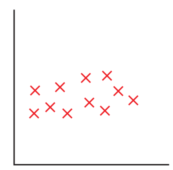
i



ii



iii



a How many cars does the salesman have on his forecourt?

b Choose an appropriate label from the box below, for each axis.

Age of car (years)	Length of car (m)	Petrol tank capacity (litres)
Mileage (1000s of miles)	Mass of car (kg)	Value of car (£)

c Describe the correlation shown in each graph.

d Two of the scatter graphs have an outlier. Give a possible explanation for the outlier in each case.

④ The table shows the marks (out of 10) given by two judges at a local flower show.

Entrant	A	B	C	D	E	F	G	H	I	J	K	L
Judge 1	10	7	2	4	8	4	6	7	0	2	9	3
Judge 2	9	8	3	5	6	4	7	7	1	4	8	3

a Draw a scatter diagram to show the marks of Judge 1 and the marks of Judge 2.

b Describe the correlation between the two judges' marks.

c Draw a line of best fit.

d Use the scatter diagram to estimate Judge 2's score for an entrant awarded 5 marks by Judge 1.

⑤ These data show the scores from two assessments taken by some school leavers who want a career in the media.

Assessment 1	**Assessment 2**
68	52
69	58
43	45
57	60
38	27
41	38
83	76
27	27

a Plot the data and draw a line of best fit for the graph.

b Fauzia scored 50% in the first assessment, but she missed the second assessment. Use your graph to predict what score she might have achieved.

6 A group of language students took oral and written tests.

Oral (%)	16	30	65	32	62	55	45	74	63	33	67
Written (%)	27	32	62	47	73	57	43	82	76	32	51

a Draw a scatter diagram to show the data.

b Describe the correlation between the two tests.

c **i** How many students scored a higher mark in the oral test than the written test?

ii How can you tell this easily from your graph?

d Draw a line of best fit.

e Use your line of best fit to estimate

i the oral mark for a student who scored 60 in the written test

ii the written mark for a student who scored 40 in the oral test.

7 These data are from a football league. They show the numbers of goals some of the teams scored and the numbers of points they received.

Goals	79	36	63	50	54	81	31	58	46	68
Points	31	28	42	24	37	72	16	51	42	61

a Draw a scatter diagram to show the data.

b Describe the correlation between the goals scored and points received.

c Draw a line of best fit.

d Use your line of best fit to estimate the number of

i points for a team scoring 60 goals

ii goals for a team gaining 45 points.

8 Paula is looking at some crime statistics.

She records the numbers of police officers in a small sample of police forces and the numbers of reported crimes in their areas in one month.

Number of police officers	58	38	16	72	34	57	78	12	33	42
Number of reported crimes	68	110	125	177	107	93	48	146	134	83

a Draw a scatter diagram to show the data.

b Which point represents a large police force operating in a high crime area?

c Describe the correlation between the number of police officers and the number of reported crimes.

d Draw a line of best fit.

e Estimate the likely number of reported crimes if a police force employs 50 police officers.

9 A health visitor is investigating the relationship between the heights of mothers and their daughters.

Height of mother (cm)	165	152	164	158	169	164	174	170	155	158	163	177	161	159	162
Height of daughter (cm)	167	154	161	159	173	160	177	169	161	160	164	175	162	162	164

a Draw a scatter graph to illustrate the data.

b Describe the correlation shown.

c **i** Find the median height of the group of mothers from your scatter graph. Explain how you did this.

ii Does the mother who is of median height have a daughter of median height?

⑩ An athletics coach thinks that athletes who are good at running are also good at high jump.

The coach collects some data from his club.

Time to run 200 m (s)	25.3	27.4	26.4	27.5	29.0	30.2	27.3	26.6	30.1	32.0	28	28.3	29.4	27.0	31.3	31.7
High jump (m)	2.24	2.15	2.18	2.01	1.89	1.76	1.90	2.10	2.06	2.26	2	1.96	1.92	2.00	1.80	1.75

a Draw a scatter diagram to show the coach's data.

b Describe the correlation. Is the coach right?

c Find the median time taken to run 200 m.

d **i** What is the median height jumped?

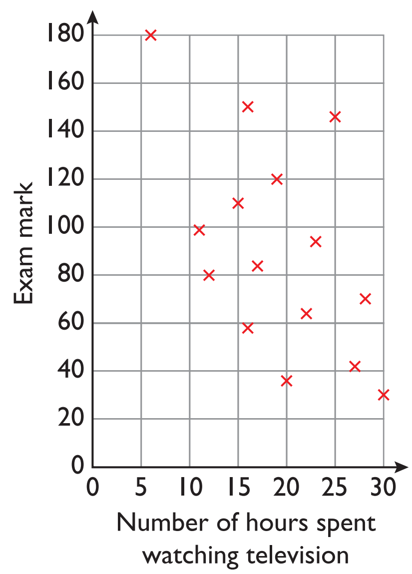
ii Is this the same person as in part **c**?

e The coach decides the top 25% of athletes in each sport should form an elite squad.

How many athletes make it on to the squad?

⑪ Mrs Jones collects information about how many hours students spent watching television in the week before a biology exam and their exam mark.

The scatter diagram shows the information.



Mrs Jones thinks that there is a connection between the number of hours spent watching television and the biology exam mark.

a Does the scatter diagram support Mrs Jones' thinking?

Describe the correlation shown.

Mrs Jones collects some more data and finds that there is strong positive correlation between the number of hours spent revising and the mark achieved in the exam.

b **i** Sketch a scatter graph to illustrate this.

ii Mrs Jones says, 'This shows you either watch television or you revise for biology, and you know which will definitely get you the better mark.'

Is Mrs Jones right? Explain your reasoning.

Do I know it now?

① Avonford Swimming Club keeps a record of the age of each swimmer and how many lengths they can swim without taking a break. This table shows the data for a few of the swimmers.

Age	16	38	53	36	63	46	22	55	58
Number of lengths	58	45	68	30	12	33	46	34	21

a Draw a scatter diagram to show the data.

b Describe the correlation between the age of the swimmer and the number of lengths they can swim.

c Draw a line of best fit.

d Use the scatter diagram to estimate

i the number of lengths that a person aged 50 may swim

ii the likely age of a person who can swim 40 lengths.

21.4 Use and limits of lines of best fit

SKILLS CHECK

Do I need to do this section?

Complete this section if you need help with the question below.

① The table gives data for a number of diesel cars. It shows the engine size for each car and the distance it travels on 1 litre of diesel.

Engine size (litres)	3.0	.6	1.2	1.8	2.8	1.5	1.0	2.5
Distance (km)	6	15	13	10	7	11	12	9

a Draw a scatter diagram for the data.

b Comment on the correlation.

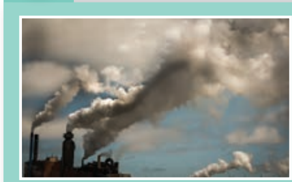
c Draw a line of best fit.

d Estimate the distance travelled for a car with an engine size of 1.4 litres.

See below (What you need to know 21.4 Use and limits of lines of best fit) for more explanation and practice on this.

What you need to know

Did you know?

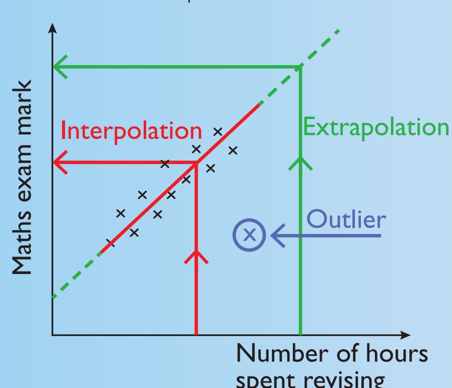


Collecting data and organising them using statistical techniques enables us to monitor pollution and global warming. We can use the information to predict when it will pass key milestones and become a significant problem.

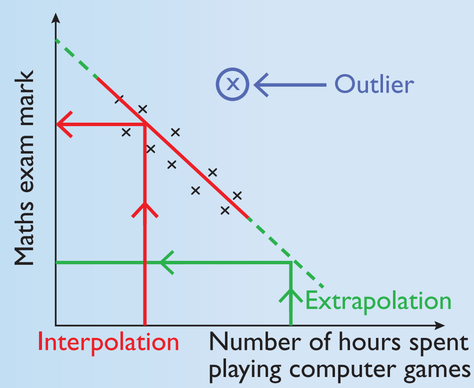
Scatter graphs are used to show the relationship between **bivariate data** – this means the data contain pairs of values.

A **line of best fit** shows the overall trend of the data. It does not have to go through the origin.

An **outlier** is a pair of values that does not fit the overall trend.



Positive correlation



Negative correlation

Interpolation is where you use your line of best fit to estimate a value within the range of the data. Interpolation is reliable if there is a strong correlation.

Extrapolation is where you extend your line of best fit to estimate a value beyond the range of the data. Extrapolation is not very reliable because you cannot tell whether the trend will continue.

A strong correlation does not prove **causation**.

Spending more time revising will cause your maths mark to go up.

Computer games do not cause people to do badly in a maths test, but they might cause them to revise less!

How to do it

Extrapolation and causation

Country	Life expectancy at birth	Birth rate per 1000
Australia	82.0	12.2
Barbados	75.0	12.0
Cambodia	63.8	24.4
Canada	81.7	10.3
Czech Republic	78.3	9.8
Guatemala	71.7	25.4
India	67.8	19.9
Italy	82.0	8.8
Libya	76.0	18.4
Samoa	73.2	21.3
Syria	68.4	22.8
Tajikistan	67.1	25.0
Uganda	54.5	44.2
Uruguay	76.8	13.2

a Draw a scatter diagram to show the figures for life expectancy and birth rate.

b Describe the correlation.

c Draw a line of best fit.

d Are either of these two statements true?

A

The scatter diagram shows that a high life expectancy causes a low birth rate.

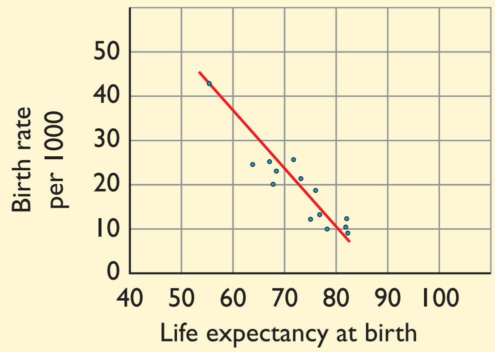
B

You can see that a high birth rate causes low life expectancy.

e Someone looking at the line of best fit says, 'It shows that when the life expectancy in a country reaches 88, people will stop having children.' Comment on this statement.

Solution

a and **c**



b The graph shows there is negative correlation between life expectancy and birth rate.

d A high life expectancy does not cause a low birth rate. Neither does a low birth rate cause a high life expectancy, although it could be true that more care can be given to older people if there are fewer babies to look after! It is more the case that both high life expectancy and low birth rate are the result of better access to medication and education.

Notice that correlation does not imply causation. In this case a common underlying factor, economic development, seems to lie behind both the observed effects of low birth rate and high life expectancy.

e Common sense tells us this is definitely not true. We should not therefore extrapolate the line of best fit beyond the values given in the data. We can be fairly certain the relationship exists within the region of evidence but we cannot assume the relationship will exist outside of that region.

Learning exercise

1. This table gives the power (in horsepower) and the fuel consumption (in miles per gallon) of a number of cars.

Power	Fuel consumption	Power	Fuel consumption
195	24.5	305	21.4
185	27.7	278	21.6
145	31.5	173	27.9
182	31.2	270	25.3
178	28.7	268	24.8
182	23.5	301	18.9
290	19.5	360	15.9
360	15.9	470	14.9
285	19.8	355	18.6
355	18.6	420	17
302	17.5	360	15.9
360	15.9	411	13.4
305	19.7	395	17.3
159	19.2	236	17

a Find the smallest and greatest values of the power and of the fuel consumption.

Use them to decide on suitable scales for a scatter diagram.

b Draw a scatter diagram for the data.

c Draw a line of best fit.

d **i** Describe the relationship between power and fuel economy.

ii Predict the power of a car that travels 15 miles on one gallon of petrol.

iii Predict the fuel economy of a car that produces 450 horsepower.

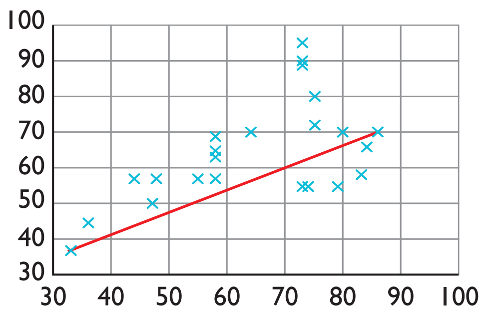
e Formula 1 racing cars produce around 800 horsepower. Can you use your graph to find their typical fuel consumption? Explain your answer.

2 Look at these four lines of best fit. They are all drawn for the same data.

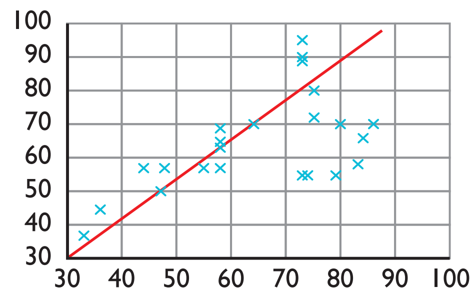
Rank them in order from what you consider to be the best line of best fit to the worst.

Explain why you've ordered them in this way.

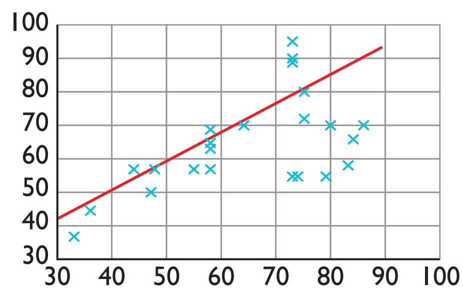
A



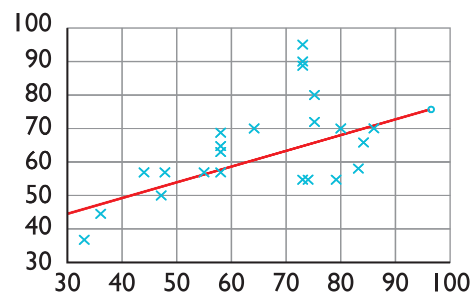
B



C



D



3 Here are the personal best performances of some athletes in a club.

Time to run 100 m (s)	11.57	11.12	10.88	11.54	11.25	10.73	11.92	11.24
Time to run 1500 m (s)	275	328	326	296	316	345	266	299
Distance jumped (m)	4.49	5.56	5.52	4.95	5.33	5.92	4.31	5.02

The data can be written as these pairs: 100 m and 1500 m; 100 m and long jump; 1500 m and long jump.

a Investigate the correlations shown by the data.

b What do these correlations tell you about who is good at these events?

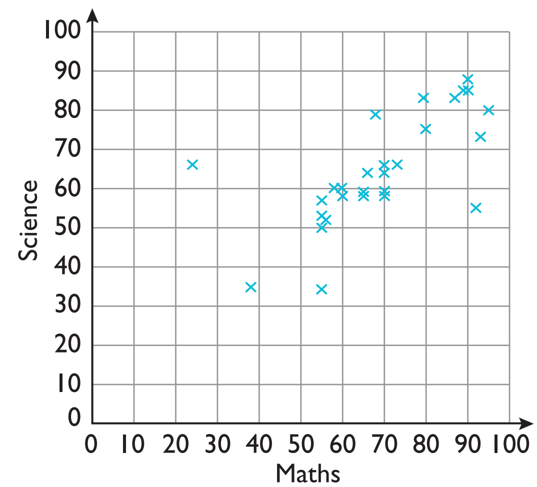
c How could you tell if the correlations hold for athletes in general?

d Comment on these statements from the team coach.

i 'One of these days we'll get a team member who can run the 100 m in 10 seconds. He'll be able to jump 7 m in the long jump.'

ii 'Being able to run the 100 m in a low time makes you able to run the 1500 m in a low time.'

4 This scatter diagram shows the science and maths marks of a class in two tests.



a Describe the correlation between the two sets of marks.

The science teacher says, 'This shows that doing well at science makes you good at maths.'

The maths teacher says, 'This shows that doing well at maths makes you good at science.'

b Which of them (if either) is right? Give your reasons.

5 A lifeguard kept a record of the highest daily temperature and the number of people swimming in the sea at 3 p.m.

The results are shown below.

Temperature (°C)	32	30	31	34	27	26	25	24	25	23	22	23	24	22
Number of people swimming	203	180	188	144	171	164	155	140	145	135	120	132	129	110

a Show the data on a scatter diagram.

b Describe any correlation between the two variables.

c There is an outlier in the data. State which it is and suggest a reason for it.

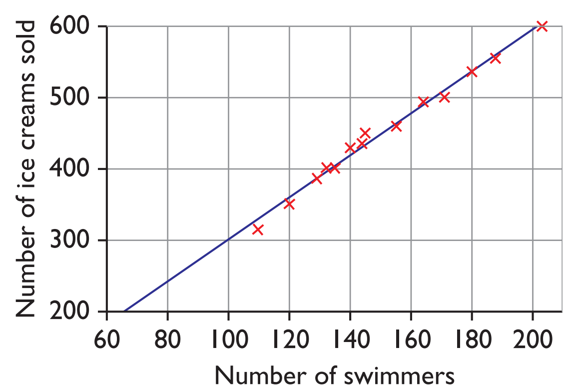
d Draw a line of best fit on your scatter graph.

e One day the temperature is forecast to be around 15 °C.

i Use your line of best fit to predict the number of swimmers.

ii Comment on the reliability of your estimate, giving a reason for your answer.

An ice-cream vendor records the number of people in the sea at 3 p.m. and the number of ice creams he sells each day. Here is the ice-cream vendor's graph.



f The ice-cream vendor says, 'My graph proves that swimming causes people to eat more ice creams.' Explain why the vendor is not correct.

6. The table gives some information about different types of aircraft.

Aircraft	A320	A318	B757	A350	B747	MD-11	A310	B737	AN225
Wingspan (m)	35.8	34.1	64.8	65	38	52	44	36	88
Maximum landing mass (tonnes)	66	57.5	251	205	88	185	124	66	640

a Draw a scatter diagram to show the data.

b Is there evidence from the scatter diagram to suggest there is a correlation between the wingspan of an aircraft and the maximum landing mass of the aircraft? Explain your answer.

c Draw a line of best fit.

d Use your line of best fit to estimate the maximum landing mass of an aircraft with wingspan 58 m.

e Peter has an aircraft with a wingspan of 12 m. Would it be sensible for him to use the line of best fit to estimate the maximum landing mass of his aircraft? Explain your answer.

7. Megan is carrying out a science experiment.

She hangs a weight from a spring and measures the resulting length of the spring.

Megan repeats the experiment for different masses.

Here are Megan's results.

Mass (g)	50	100	150	200	250	300	350	400	450	500
Length of spring (cm)	6.5	8.3	10.1	12.2	14.3	20.1	18.3	20.3	22.6	24.1

a Draw a scatter diagram to show the data.

b Megan made a mistake when she wrote down one of the results.

The correct result should be 16.2 cm.

Identify and correct the mistake.

c Draw a line of best fit.

d Use your line of best fit to estimate the length of the spring when a mass of 180 g is hung from it.

e Comment on the reliability of using your line of best fit to estimate the length of the spring when a 1 kg mass is hung from it.

f Estimate the original length of the spring.

Do I know it now?

1. This table shows the heights above sea level, in metres, and the temperatures, in Celsius, on one day in eight different places in Europe.

Height (m)	1300	275	800	360	580	540	1300	690
Temperature (°C)	10	20	14	20	27	17	12	17

a Plot a scatter diagram and describe the correlation.

b Identify any outliers and suggest a reason for them.

c Use your diagram to estimate the temperature at a height of 400 m.

d Use your diagram to estimate the height of a place with a temperature of 25 °C.

e Comment on the reliability of your answers in parts **c** and **d**.

CHAPTER 22

ESSENTIAL TOPICS – STATISTICS AND PROBABILITY

Probability

JUST IN CASE

Introduction to probability

Probabilities can be described in words or represented as percentages, decimals or fractions.

0%	25%	50%	75%	100%
Impossible	Unlikely	Evens	Likely	Certain
0	0.25	0.5	0.75	1
0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1

← less likely — more likely →

In the 0 to 1 probability scale, the 0 indicates 0% chance of an event occuring; a 1 indicates 100% chance of an event occuring.

Single event probability

An **event** is something which may or may not occur. The result of an experiment or a situation involving uncertainty is called an **outcome**, like the score on a die. The word **event** is also used to describe a combination of outcomes, like scores 5 or 6 on a die.

For any event with equally-likely outcomes, the probability of an event happening can be found using the formula:

$$P(\text{event happening}) = \frac{\text{total number of successful outcomes}}{\text{total number of possible outcomes}}$$

Mutually exclusive events are events that cannot happen together. For example, you cannot roll a 2 and a 5 at the same time on one die!

The probabilities of all mutually-exclusive outcomes of an event add up to 1.

P(event not happening) = 1 − P(event happening)

Kyle throws an ordinary die. He makes a list of all the possible outcomes.

1	2	3			

a Complete Kyle's list.

b Find the probability that Kyle gets

- **i** 6
- **ii** not a 6
- **iii** an even number
- **iv** 5 or more
- **v** less than 4
- **vi** a prime number.

Solution

a All possible outcomes: 1 2 3 4 5 6

b i $P(6) = \frac{1}{6}$ ← 1 possible throw out of 6 equally-likely possibilities.

ii $P(\text{not a } 6) = 1 - P(6) = 1 - \frac{1}{6} = \frac{5}{6}$

iii $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$ ← 3 possible throws: 2, 4 and 6

iv $P(5 \text{ or more}) = \frac{2}{6} = \frac{1}{3}$ ← 2 possible throws: 5 and 6

v $P(\text{less than } 4) = \frac{3}{6} = \frac{1}{2}$ ← 3 possible throws: 1, 2 and 3

vi $P(\text{prime}) = \frac{3}{6} = \frac{1}{2}$ ← 3 possible throws: 2, 3 and 5

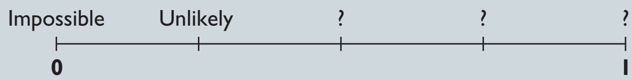
SKILLS CHECK

Warming up

Complete the skills checks below to make sure you are ready for this chapter. Further revision of this assumed knowledge is available in the online Dynamic Learning Resources.

Introduction to probability

a Copy the 0 to 1 probability scale and fill in the missing words.



b Add in the decimal, fraction and percentage probability values at the labelled points.

c Mark the probability of the outcome of each event on the probability scale.

- **i** You throw a die and it comes up 4.
- **ii** You will go abroad sometime in the next two years.
- **iii** You throw two dice and the total is 15.
- **iv** The shortest day next year will be 21 December.
- **v** You will fail your driving test first time.
- **vi** A baby will be born somewhere in England tomorrow.

➜ Single event probability

Steve has a bag containing only red, black and white counters.

The probability of picking a red counter, at random, is $\frac{3}{8}$.

a What is the smallest number of counters that could be in the bag?

The probability of picking a black counter is $\frac{2}{5}$.

b What is the probability of picking a white counter?

c What is the smallest number of counters that could be in the bag?

There are 45 red counters.

d How many black counters and how many white counters are there in the bag?

➜ Applying the knowledge

1. Kyle and Isabelle are playing a game with a set of 21 cards, numbered 1 to 21.

   Kyle selects the card with number 15 at random.

   Isabelle shuffles the remaining cards and picks one card at random.

   a What is the probability that the number on Isabelle's card is higher than 15?

   They put their cards back and play again. Kyle takes out a card at random.

   Isabelle shuffles the rest of the cards and picks one card at random.

   The probability that the number on Isabelle's card is higher than the number on Kyle's card is 0.6.

   b What is the number on Kyle's card?

   c What is the probability that the number on Isabelle's card is lower than 15?

2. A spinner has eight equal sections and four different colours.

   The probability of landing on red is greater than the probability of landing on green.

   There is an equal probability of landing on yellow or blue.

   Draw the three possible designs.

22.1 Combined events

SKILLS CHECK

Do I need to do this section?

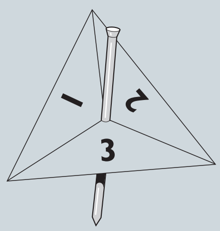
Complete this section if you need help with the question below.

① Jane has two spinners with sides numbered 1, 2 and 3.

She spins both spinners and adds the results.

Second spin

First spin +	1	2	3
1		3	
2			5
3			



a Copy and complete the table to show all the possible outcomes.

b What is

i the most likely total **ii** the least likely total?

c Work out the probability that the total is

i 5 **ii** less than 4 **iii** a square number **iv** a prime number.

If you can do the question above, try this one on problem solving.

② James and Kate decide to play a game with three coins. James wins if he gets three heads or three tails, otherwise Kate wins.

a Make a list of all the possible outcomes of throwing three coins.

b What is the probability that James wins?

c What is the probability that Kate wins?

d Write a set of rules to make the probability of each player winning equal.

If you need more help with problem solving use the Problem Solving chapters, starting on page 324, and practise using the exercise on page 313 (Problem solving exercise 22.1 Combined events).

What you need to know

The formula to calculate the probability of an event in which all outcomes are equally likely is

$$P(\text{event}) = \frac{\text{total number of successful outcomes}}{\text{total number of possible outcomes}}$$

To find how likely a combination of events with equally likely outcomes is, list all of the possible outcomes if you can.

Be systematic and change one item at a time.

If it is not possible to list all the outcomes, use a **possibility space diagram** or a **Venn diagram**.

How to do it

Listing all possible outcomes

Sam is choosing her breakfast. She can choose one cereal and one drink.

Cereals:	Wheatamix, Cornflakes or Sugarloops
Drinks:	tea or coffee

a Draw a diagram to show all the possibilities for Sam's breakfast.

b Sam selects her cereal and drink at random. What is the probability that she has Sugarloops and coffee?

c How would the table change if Sam had three options for drinks (for example, tea, coffee and orange juice)?

Solution

a

	Wheatamix	**Cornflakes**	**Sugarloops**
Tea	T, W	T, C	T, S
Coffee	C, W	C, C	C, S

b There are 6 possible outcomes and 1 successful outcome.

Probability $= \frac{1}{6}$

c There will be an extra row in the table in answer **a**.

Probability space diagrams

Hannie throws two dice, one red and one green. Her total score is the sum of the number shown on the top of each die.

a Draw and complete a table to show all the possibilities for her total score.

b What is the probability that her total score is

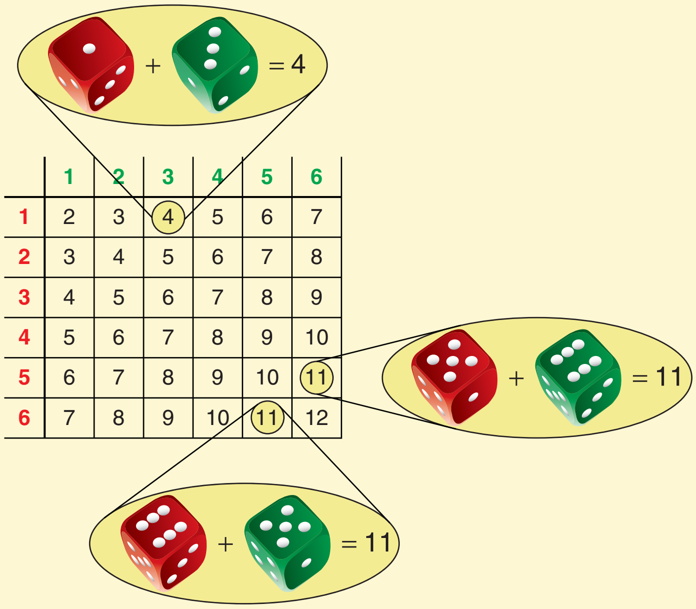
i exactly 3 **ii** 3 or less

iii greater than 12 **iv** a prime number?

c Would the answers be the same with two red dice?

Solution

a



	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

b There are 36 possible outcomes.

i $\frac{2}{36} = \frac{1}{18}$

There are 2 ways of scoring 3, 1 + 2 and 2 + 1, so 2 favourable outcomes.

ii $\frac{3}{36} = \frac{1}{12}$

There are 3 ways of scoring 3 or less.

iii 0

It is impossible to get more than 12.

iv $\frac{15}{36} = \frac{5}{12}$

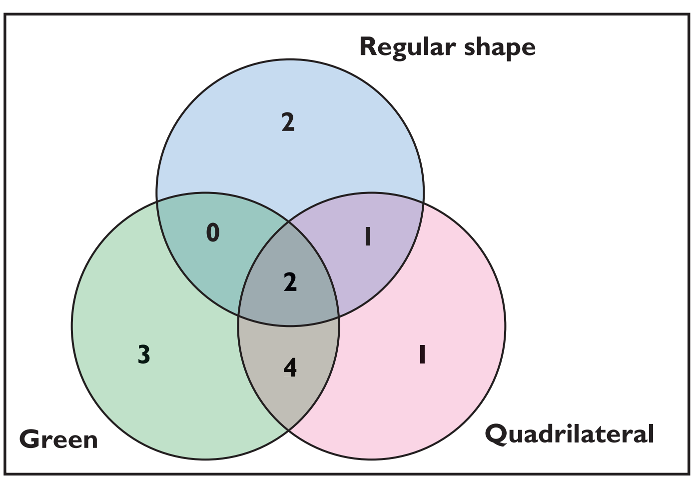
Prime numbers 2 (1 way), 3 (2 ways), 5 (4 ways), 7 (6 ways), 11 (2 ways).

c The answers would be the same. The colour of the dice does not affect the score.

Sometimes it is easier to see what is going on with different coloured dice.

Venn diagrams

Nick has some cards with different shapes printed on them.
This Venn diagram gives information about the shapes printed on the cards.



a How many cards are there in Nick's pack?

b Nick picks a card at random. What's the probability that his card is

i a quadrilateral **ii** a green quadrilateral **iii** not green?

c Draw the shape in the central intersection.

Write a sentence about the probability of picking it.

Solution

a There are 13 cards.

Add the number of cards in all the different regions.

b i P(quadrilateral) $= \frac{8}{13}$

$4 + 2 + 1 + 1$

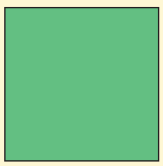
ii P(green quadrilateral) $= \frac{6}{13}$

$4 + 2$

iii P(not green) $= \frac{4}{13}$

$2 + 1 + 1$

c

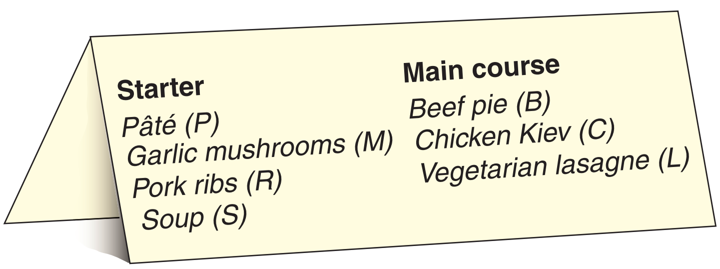


A regular quadrilateral is a square.
The shapes in the intersection are green.

The probability of picking a card with a green square at random is $\frac{2}{13}$.

Learning exercise

① A restaurant has four starters and three main course meals as shown on the menu card.



a List the possible combinations of starter and main course.

b How many meals are there in your list?

How could you work it out quickly without listing them all?

c Jamie says, 'Get me anything'.

If the starter and main course are selected at random, what is the probability he gets soup and beef pie?

② Two ordinary fair dice are rolled. One die is red and the other is blue. The total score is the product of the two numbers shown on the dice.

Red die

Blue die

×	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

a Copy and complete this table to show all the possible outcomes.

b What is the probability that the product is

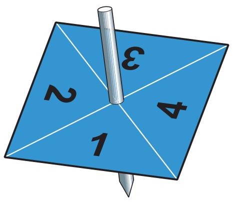
i 8 **ii** 25 **iii** 7 **iv** a number less than 37?

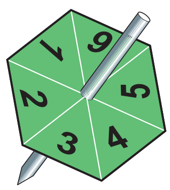
c Do the colours of the dice make any difference?

d What types of numbers between 1 and 36 have a probability of $\frac{1}{6}$?

③ Gloria has two spinners: the blue one has four equal edges numbered 1 to 4 and the green one has six equal edges numbered 1 to 6.

Gloria spins them and adds the two numbers together.





a Make a table to show all the possible outcomes.

b What is the probability that Gloria gets a score of

i 5 **ii** 12 **iii** 4?

c Which other numbers between 1 and 20 have the same probability as

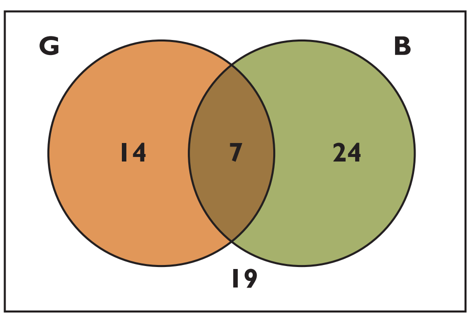
i 5 **ii** 12 **iii** 4?

④ Members of a sports club may play golf (G) or bowls (B).

Some play both and some play neither.

The numbers are shown on the Venn diagram.

The club holds a raffle. Every member has a ticket and the winner is chosen at random.



Find the probability that the winner

a plays golf

b plays bowls

c plays both golf and bowls

d does not play golf

e does not play bowls

f does not play either bowls or golf.

5 A cat has had a lot of kittens.

Her owner, Veronica, classifies them as follows.

	Tabby	Ginger	Black & White	Tortoiseshell
Female	3	0	4	3
Male	3	4	3	0

Veronica has a photo of each of the kittens.

She selects a photo at random. What is the probability that the photo is of

a a female kitten **b** a tabby kitten **c** a female tabby?

6 Geoff is on holiday.

In his case, he has four shirts and three pairs of shorts.

Shirts	Shorts
Green	Black
Red	Grey
Black	Cream
Cream	

a Make a list of all the possible combinations he can wear.

b Geoff grabs one shirt and one pair of shorts at random.

What is the probability that he has

i a red shirt and grey shorts

ii a shirt and shorts of the same colour?

7 Amanda has two bags each containing coloured balls.

Bag 1 contains 2 red and 3 blue balls.

Bag 2 contains 4 red and 2 blue balls.

She selects a ball from each bag at random.

a Copy and complete this table to show all possible outcomes.

Bag 1

Bag 2	R	R	B	B	B
R					
R					
R					
R					
B		RB			
B					

b What is the probability that the balls she selects are

i one of each colour

ii both red

iii both blue?

Problem solving exercise

① A factory manager records whether employees arrived late or on time one Friday.

She also records how employees got to work that day.

The table shows the results.

	Walk	Bus	Car	Cycle
Late	5	28	6	7
On time	95	85	142	32

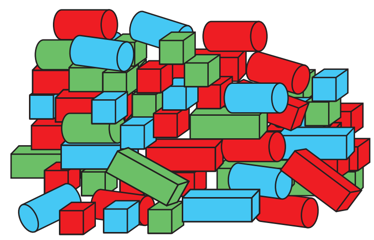
a An employee is selected at random. Work out the probability that they

i were on time **ii** walked and were late **iii** were late and came by bus.

b The manager says that the results show that employees are not making enough effort to arrive at work on time. Comment on this statement.

② Baby Ben has lots of wooden bricks, in different shapes and colours.

	Cube	Cuboid	Cylinder	Total
Red	14	40		71
Green	21		12	60
Blue		33		69
Total		100	42	



a Copy and complete the table.

b Find the probability that a randomly chosen brick is

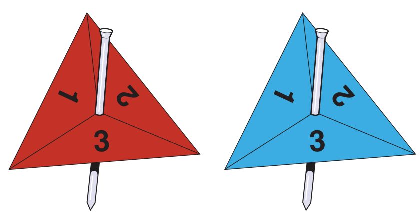
i a cuboid **ii** red **iii** a blue cube.

c Find the probability that a randomly chosen

i cuboid is green **ii** green brick is a cuboid.

③ Isobel and Peter are playing a game.

They each spin a spinner with sides numbered 1, 2 and 3.



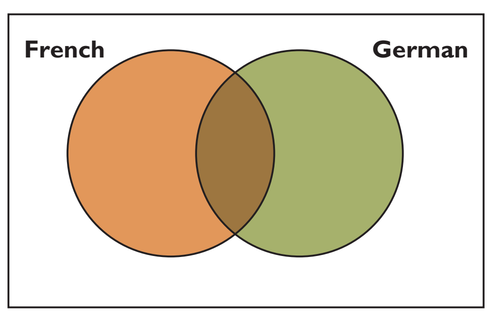
a List all the possible outcomes of the two spinners.

b Isobel wins the game when one spinner shows an odd number and the other spinner shows an even number.

Work out the probability that Isobel wins the game.

c How could they change the game so that they each have an even chance of winning?

④ A school has 200 Year 11 students. Of these, 136 study French, 96 study German and 8 students study neither French nor German.



a Copy and complete the Venn diagram.

b Find the probability that a student selected at random studies

i French

ii French and German

iii German but not French.

⑤ A café records what their customers order one morning.

The table shows the results.

	Coffee	No coffee
Cake	82	35
No cake	90	43

a A customer is chosen at random.

Find the probability that they order

i coffee and a cake **ii** neither coffee nor a cake **iii** coffee.

b The café is open 6 days a week and expects to have 400 customers a day.

i Estimate the number of customers that order coffee in a week.

A 200 g bag of coffee beans makes 24 cups of coffee.

The café owner wants to order enough coffee to last for the next four weeks.

ii How many kilograms of coffee should she order? Give your answer to the nearest kilogram.

Do I know it now?

① Steve and Bella are playing a game with a green die and a blue die.

The green die is numbered 1 3 3 4 6 6.

The blue die is numbered 2 3 4 4 5 5.

The dice are rolled and the scores on the two dice are added together.

a Make a table showing all the possible outcomes.

b Find the probability of getting

i a 3 on both dice **ii** a total of 7 **iii** a double

iv a total of at least 6

v a number on the blue die that is one more than that on the green die.

Can I apply it now?

① In a game show, contestants spin a wheel of fortune and pick a card to seal their fate.

Calculate

a P(winning £100)

b P(losing £400)

c P(no money is lost or won)

d P(wheel lands on £400)

e P(winning some money).

Essential Topics Test Paper 1

Total number of marks: 50

Suggested time allowed: 1 hour

① Hilary and three of her friends dine at the Red Fox Hotel. The waiter gives this bill to Hilary.

Red Fox Hotel

2 Prawn cocktails @ £5.65 ------- £11.30
1 Soup of the day @ £4.20 -------- £4.20
1 Liver pâté @ £6.85 ------------- £6.85
2 Fillet steaks @ £18.00 --------- £36.00
2 Roast duck @ £15.95 ----------- £31.90
Drinks from the bar ------------ £27.80

Total ----------- £150.05

She does a quick mental calculation and then tells the waiter that the total is incorrect.

Write down a calculation that Hilary may have done to be sure that the total of their bill was incorrect. [3]

② Mo records the hair colour of the students in his class.

Here are his results.

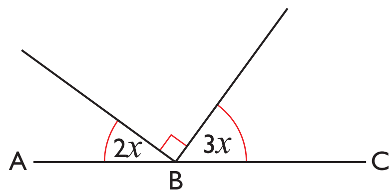
black	brown	red	blonde	brown
red	brown	blonde	brown	black
black	red	brown	blonde	brown
brown	black	brown	brown	black
black	brown	blonde	black	blonde

a Draw a suitable diagram to show this information. [4]

b **i** Which average do you think describes the data best?
Give a reason for your answer. [1]

ii Find this average. [1]

③ ABC is a straight line. Find the value of x.



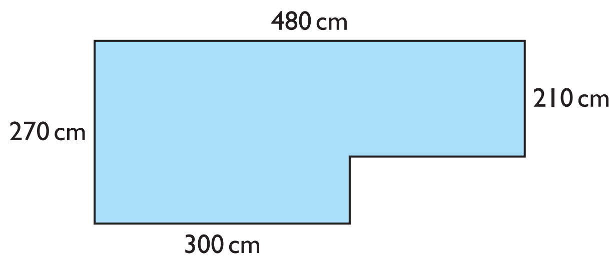
[3]

④ Janet bought a new car.

The car was advertised in a showroom at a price of £23 400. Janet bought the car on hire purchase over a period of five years. She had to pay a deposit of £3000 plus equal monthly payments. After five years, she will have paid £4800 more than the showroom price.

Work out how much Janet is paying each month. [4]

⑤ Lisa is going to tile her kitchen floor. The diagram shows a plan of the area of her kitchen floor that will be tiled.



The tiles are square and are sold in boxes. Each box contains 12 tiles and costs £40. Each tile is of side 30 cm.

Lisa has £450 to spend on tiles. Does she have enough money to buy the tiles required to tile her kitchen floor? [4]

⑥ A bag contains only black, white, yellow and red counters. Susie takes a counter from the bag at random. The table gives the probability of her taking, at random, a black, a white or a red counter.

Colour	black	white	yellow	red
Probability	0.2	0.15		0.4

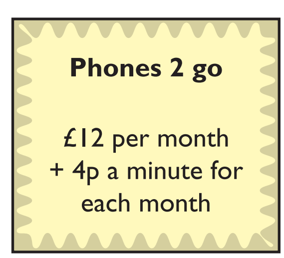
What is the probability that Susie takes a yellow counter? [2]

⑦ A train ticket for a child costs £n. A train ticket for an adult costs £5 more than a child's ticket. Mrs Jones buys two adult's train tickets and three child's train tickets. The total cost is £50.

a Write an equation in terms of n. [1]

b Solve the equation from **a** and work out the cost of an adult's train ticket. [2]

⑧ Bill is going to hire a phone from Phones 2 go.



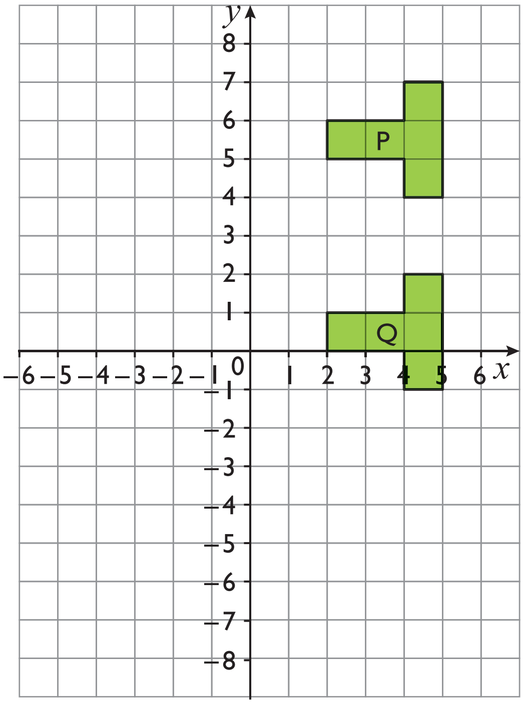
a Write an expression for the cost of using a phone for m minutes for one month. [2]

b Bill hires a phone for four months and pays a total of £64. How many minutes did he use the phone for? [2]

⑨ A company owns a fleet of eight cars. Each car emits 150 g/km of CO_2. Each car does on average 12 000 miles per year.

Work out the CO_2 emissions from the fleet of cars for a year. Use 5 miles = 8 km and 1000 kg = 1 tonne. [4]

⑩ Shape P is transformed to give shape Q.



a Describe fully a single transformation that does this. [2]

b Describe fully a different single transformation that does this. [2]

⑪ A circle and a square have the same perimeter.

The square has an area of $64\,cm^2$. What is the area of the circle? [6]

⑫ Three numbers appear on a number line.

One number is 3 units away from zero. Two of the numbers are 10 units apart. One number is equidistant from the other two numbers.

Write down all of the possible positions of the three numbers on the number line. [5]

⑬ Denzil is planning a holiday in Italy. He needs to hire a car.

If Denzil pays in cash, the cost will be €603 if he pays in Italy. If Denzil pays in cash in the UK, the cost to hire the same car will be £490. The exchange rate is £1 = €1.25.

In which country will Denzil pay less for the hire of the car? [2]

Essential Topics Test Paper 2

Total number of marks: 50

Suggested time allowed: 1 hour

1. Mary says the median of this set of data is 7   8, 1, 3, 2, 7, 4, 2, 4, 6

   Is Mary correct? Explain why. [2]

2. Elaine buys and sells antiques.

   The table shows information about some of Elaine's antiques.

Item	Cost price in £	Selling price in £	Profit or loss
Clock	25.80	46.00	Profit of £20.20
Tea set	32.70	65.40	
Silver watch	56.00		Loss of £12.20
Painting		124.00	Profit of £8.80
Coffee table	93.50	56.00	
Mason jug	8.60	23.90	

   a Copy and complete the table. [5]

   b How much profit or loss did Elaine make overall? [2]

3. These are the temperatures in some cities recorded on 1st December.

   Sydney: 22 °C, London: –2 °C, Reykjavik: –16 °C, Beijing: 4 °C

   a What was the difference between the temperature in Sydney and the temperature in London? [1]

   b On the same day, the temperature in Rome was exactly halfway between the temperature in Sydney and the temperature in Reykjavik. What was the temperature in Rome? [1]

   On 2nd December, the temperature in Reykjavik rose by 3 degrees and the temperature in London fell by 4 degrees.

   c What was the difference between the temperature in Reykjavik and the temperature in London on 2nd December? [1]

4. The table shows information about the selling price of televisions and their screen size.

Screen size (inches)	19	24	27	32	40	48	50	55	66
Price (£)	100	150	200	270	230	550	600	800	1000

   a Draw a scatter diagram for these data. [4]

   b Describe the correlation. [1]

   c Estimate the selling price of a television of screen size 44 inches. [1]

⑤ Nadia buys a retro scooter priced at £1590.

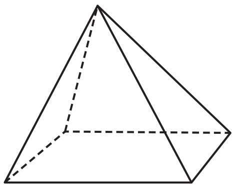
She pays a deposit of 40% and then the salesman tells Nadia that she will have to make 12 monthly payments of £80.

Nadia says that the monthly payments of £80 are too much. Is Nadia correct? [3]

⑥ Euler developed a relationship that connects the number of faces, vertices and edges of some solid shapes. He worked out that:

number of faces + number of vertices = number of edges + 2

a Show that this works for this pyramid.



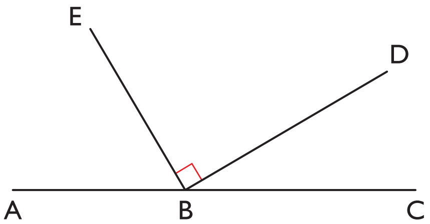
[1]

b What is the name of a shape that has 7 faces, 10 vertices and 15 edges? [1]

c What is the name of a shape that has 9 faces, 9 vertices and 16 edges? [1]

⑦ ABC is a straight line. The size of angle ABE is four times the size of angle DBC.

Work out the size of angle ABE.



[3]

⑧ The table shows the results of Peter's end-of-year tests.

Subject	Marks available	Mark obtained
English	20	14
Mathematics	40	32
Science	30	22
Humanities	30	18
Languages	20	15
Art & Craft	40	27
Design & Technology	10	8

a What fraction of the marks available in science did Peter obtain? [1]

b Write down Peter's subjects in ascending order, starting with the subject with the lowest performance. [4]

⑨ Anne and David go to a garden centre.

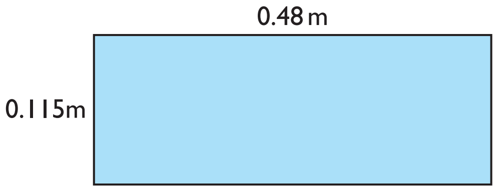
Anne buys 9 rose bushes. It costs her £54.

David buys 12 rose bushes and 4 pot plants. It costs him £88.

Work out the cost of one pot plant. [3]

⑩ This question was set in a test.

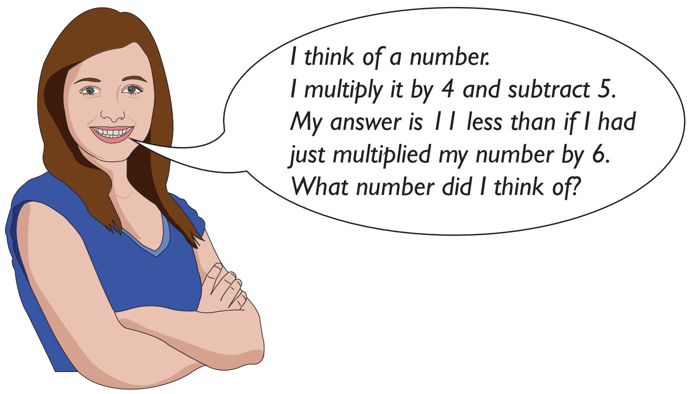
Work out the area of this rectangle.



Jane worked this out and gave an answer of $0.552\,\text{m}^2$.

Explain how Jane could have quickly checked her answer to show that it must be wrong. [2]

⑪



[3]

⑫ Explain why it is not possible for the angles of an equilateral triangle to have values $3x + 30°$, $2x + 40°$ and $8x - 22°$. [3]

⑬ David has two fair dice. On each of them are the numbers −3, −2, −1, 1, 2 and 3.
When he rolls the two dice, he adds the numbers together.

a Copy and complete the sample space diagram to show all the possible totals. [4]

+	−3	−2	−1	1	2	3
−3						
−2						
−1						
1						
2						
3						

b Work out the probability that David gets a total of:

i 0 [1]

ii 2 [1]

iii −3 [1]

Essential Topics Test Paper 3

Total number of marks: 50

Suggested time allowed: 1 hour

① Copy and complete these statements.

a $\sqrt{16} + \square = 22$ [1]

b $\square - 2^4 = 80$ [1]

c $\square^2 + \sqrt[3]{27} = 52$ [1]

② An apple costs x pence. A pear costs 10 pence more than an apple. An orange costs 5 pence less than an apple.

Write an expression in terms of x for the cost of each of these.

Write your expressions in their simplest terms.

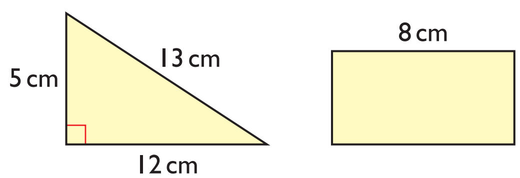
a 1 pear [1]

b 1 orange [1]

c 4 apples [1]

d 2 apples and 3 pears. [2]

③ The area of the triangle is the same as the area of the rectangle. Work out the perimeter of the rectangle.



[4]

④ Pedro asked 30 workers how many minutes it takes them to get to work. The table shows some information about his results.

Time, t (minutes)	Frequency
$0 < t \leqslant 10$	3
$10 < t \leqslant 20$	5
$20 < t \leqslant 30$	10
$30 < t \leqslant 40$	6
$40 < t \leqslant 50$	5
$50 < t \leqslant 60$	1

a Calculate an estimate for the mean number of minutes taken by the 30 workers. [3]

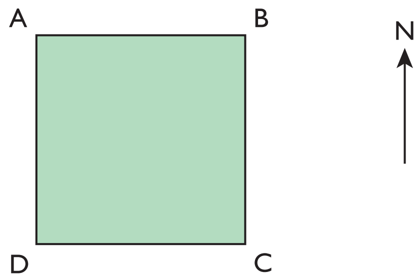
b Find the class interval that contains the median. [1]

c What is the modal class? [1]

⑤ In a sale, jackets priced at £40 are reduced by 20% and hats priced at £12 are reduced by 25%.

Mark buys a jacket and two hats. How much does he save in total? [4]

⑥ ABCD is a square.



Write down the bearing of

a B from A [1]

b C from A [1]

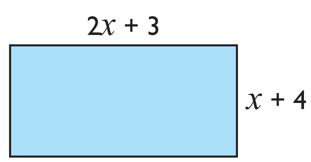
c D from B. [1]

⑦ A rectangle has length 5 inches and width 2 inches. It is enlarged by a scale factor of 3. Work out the area in cm^2 of the new rectangle. [3]

⑧ Dave is training for a marathon. In the month of March he trained on 15 days. In the month of April he trained on 18 days.

By what percentage has his number of days of training per month increased? [5]

⑨ A rectangle has length $(2x + 3)$ cm and width $(x + 4)$ cm.



a Write an expression, in its simplest form, for the rectangle's perimeter. [2]

The perimeter of the rectangle is 44 cm.

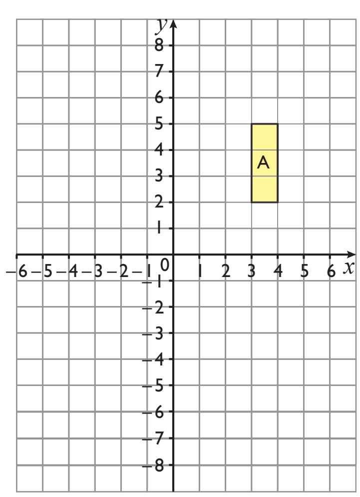
b Write an equation and solve it to find the area of the rectangle. [3]

⑩ The nth term of a sequence is $3n + 4$. The nth term of another sequence is $5n - 2$.

Which term is the same in both sequences? [2]

⑪ Rectangle A is reflected in the x axis and then translated $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$ to give rectangle B.

Describe fully a single transformation that maps B to A.



[3]

⑫ The ratio of the number of green counters to yellow counters in a box is 3 : 2.

Heather takes at random a counter from the box. What is the probability that the counter is yellow? [2]

⑬ Tom and Sam share £90 between them in the ratio 2 : 3. Sam splits his money equally between his three sons.

What fraction of the original amount of money does each son receive? [3]

⑭ Lewis Hamilton won the British Formula 1 Grand Prix of 2014.

The race was of 62 laps.

Lewis Hamilton's fastest lap time was 1 minute 37.176 seconds. The race started at 2.00 p.m. Estimate the time when Lewis Hamilton finished the race. [3]

CHAPTER 1

PROBLEM SOLVING

Getting started

What makes a problem a problem?

A problem is a problem when there is no obvious way to solve it. If you know what to do straight away, then it is not really a problem. This will be true for the problem solving questions in exams.

These problem solving chapters give you strategies to use when you don't know what to do.

In this chapter you will use three strategies to help you get started on a problem.

1 Make a guess

You have probably solved many problems in your everyday life by making a guess, so this approach should be familiar. Making a guess helps you get your head around the problem and can be the start of a trial and improvement process.

2 Draw (on) a diagram

If a diagram is given, then you can add lines or numbers to it. If not, make a sketch showing the important aspects of a context or draw a bar model, number line or shape to show what is happening.

3 Ask yourself 'What can I do?'

Look for something you **can** do with the information given in the problem. It is likely to be useful at some point and should help you understand the question more clearly.

Each of the four problem solving examples below is set in a different area of mathematics. Work through them, following the prompts that apply the three strategies outlined above. You will continue to work on these problems in Problem Solving Chapters 2 to 4, applying additional strategies each time until you have solved the problems.

Answer the questions within each example to see how the strategies work.

A profit problem

Ed buys 200 bottles of lemonade for £160.

He sells 120 of them for £1.20 each.

Ed wants to make a 30% profit overall.

How much should he charge for the remaining bottles?

1 Make a guess

Example: I guess the remaining bottles are sold for £1 each. ← There is no one correct guess.

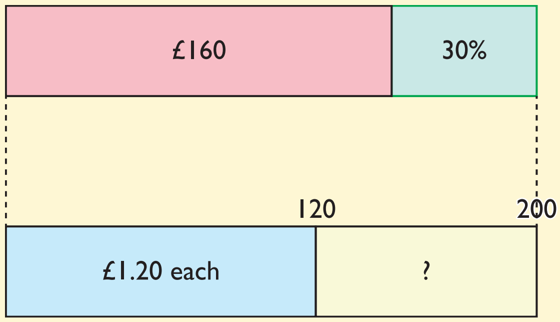
Based on this guess:

a How much would Ed receive in total for the remaining bottles?

b What could you work out next?

❷ Draw a diagram

Example: Draw a bar diagram to show the information.



There may be a choice of useful diagrams.

a Label the diagram to show what each number represents.

b What diagram would you have drawn to represent the information?

❸ Ask yourself 'What can I do?'

Example: I can

A *work out the total sum he received from the 120 bottles he has already sold*

B *work out how many bottles are left to sell*

C *work out how much each bottle cost*

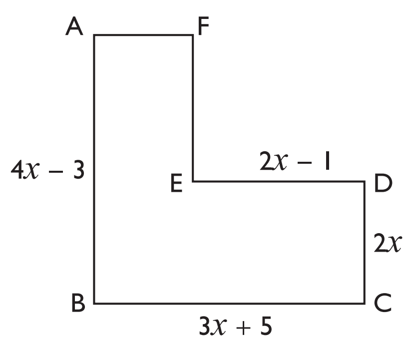
D *work out the amount of profit Ed wants*

a Calculate **A**, **B**, **C** and **D**.

b What would you do next? Help with this is given in Problem Solving Chapter 2.

An algebra problem

The perimeter of the shape shown is 39 cm.
Work out the value of x.
All measurements are in centimetres.



❶ Make a guess

Example: I guess $x = 3$.

Based on this guess:

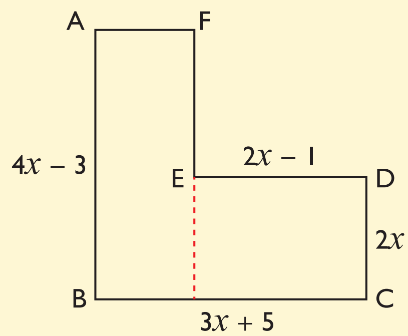
a What length is side BC?

b What could you work out next?

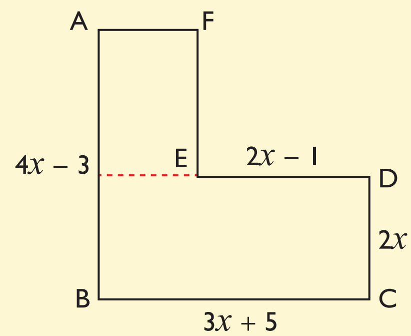
❷ Draw on a diagram

Examples: I could add any of the markings in red

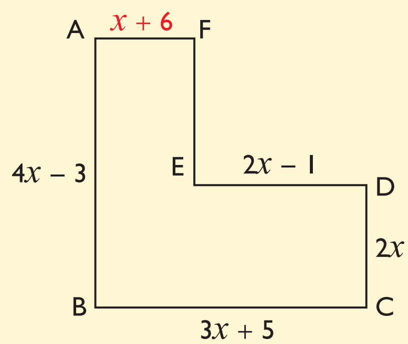
A



B



C



a The length of AF is $x + 6$. Show how this was calculated.

b What else could you add to the diagram?

c Which diagram, A, B or C, is most useful? Why?

❸ Ask yourself 'What can I do?'

Example: I can

A *work out the length of AF in terms of x (diagram C)*

B *work out the length of EF in terms of x*

C *write an expression in terms of x for the area of the shape*

D *write an expression in terms of x for the perimeter of the shape.*

a Which of the above would you have thought of doing?

b Which of the suggestions is the easiest to find? Explain why.

c Which of the suggestions is the hardest to find? Explain why.

A best buy problem

A TV normally costs £348 plus VAT (20%).
Which is the best buy? Justify your answer.

TVs 2 Go	Tikkas	Asteroid
20% off the recommended price	$\frac{1}{4}$ off the price including VAT	'Pay no VAT'

❶ Make a guess

Example: Tikkas is the best buy.

a Here are some possible reasons that could have led to this guess. They may not all be true. Which are valid reasons?

A $\frac{1}{4}$ is 25% which is more than 20%.

B The other two include VAT.

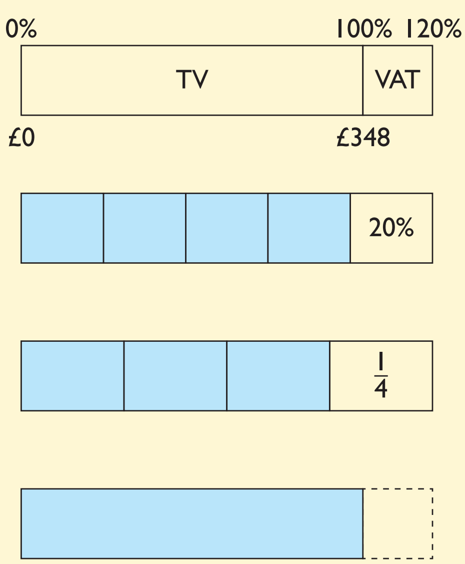
C 20% is the same as $\frac{1}{20}$ which is larger than $\frac{1}{4}$.

D The discount is applied before the VAT is added.

b How useful is this guess?

❷ Draw a diagram

Example: Here is a bar diagram.



a Label the diagrams to show what each part represents.

b Which diagram would you have drawn to represent the information?

❸ Ask yourself 'What can I do?'

Example: I can

A *work out the cost of the TV with VAT*

B *work out 20% of £348*

C *work out $\frac{1}{4}$ of £348 and $\frac{1}{4}$ of 20%.*

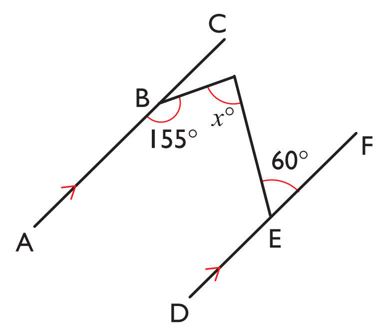
a Calculate **A**, **B** and **C**.

b Which of these will be most useful? Explain your reasoning.

c What would you do next? Help with this is given in Problem Solving Chapter 2.

An angle problem

In the diagram, the straight lines ABC and DEF are parallel.
What is the size of angle x? Give reasons for your answer.



❶ Make a guess

Example: I guess 100°.

a Here are some possible reasons that could have led to this guess. They may not all be true. Which are valid reasons?

A The angle x looks about 100°.

B Angle x looks smaller than the 155° angle and larger than the 60° angle.

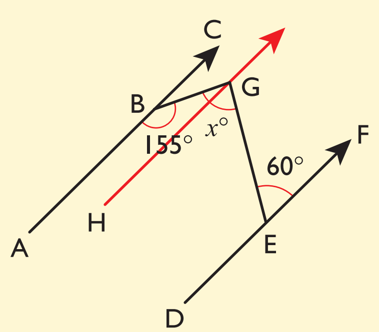
C If you join BE with a straight line to make a triangle, the angles must add up to 180°.

b How useful do you think this guess is?

❷ Draw on a diagram

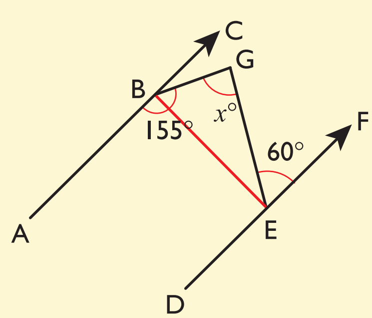
Example: I could add any of the lines in red.

Diagram A



or

Diagram B



a What angle rules do you know? Make a list.

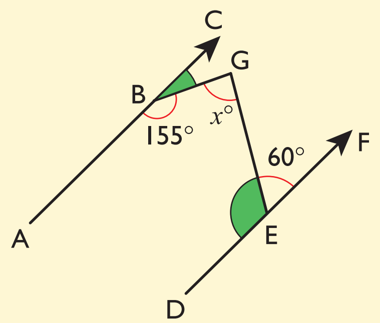
b Which of them might be helpful in Diagram A?

c Which of them might be helpful in Diagram B?

d Which diagram will be more helpful to find the value of x?

❸ Ask yourself 'What can I do?'

Example:



I can work out the size of angle CBG and angle DEG marked in green.

a Work out

i angle CBG

ii angle DEG.

b What other angles can you calculate from Diagram A or Diagram B?

c What would you do next? Help with this is given in Problem Solving Chapter 2.

Reflecting

Notice that, depending on the problem, some of the strategies work better than others. You will also find some strategies easier to use than others.

Even if making a guess does not help you solve the problem, the process will help you verify the answer once you have found it.

The next step for each problem is covered in Problem Solving Chapter 2.

Practice problems

Use the three strategies in this chapter to get started on the problems below. Go on to solve them completely if you can.

For each problem:

a Make a guess.

b Draw (on) a diagram.

c Work out what you can do.

d Try to go on and solve the problem.

e State which of the strategies was most useful.

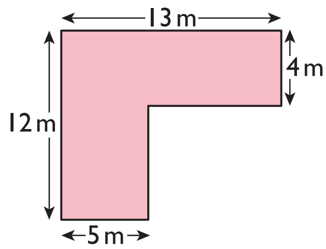
1 There are 31 rows of Economy class seats in an aeroplane.
There are 6 seats in each row.
In addition, there are 36 seats in Business class.
On one flight to Montego Bay, all the seats are full.
Work out the total cost of all the tickets sold.

2 Georgina is on holiday in Portugal.
In Portugal a pair of sunglasses costs €54.
In England the same pair of sunglasses costs £48.
The exchange rate is £1 = €1.20.
In which country are the sunglasses cheaper?

3 Helen buys a laptop priced at £180.
She pays a deposit of 30% and then makes 12 monthly payments of £13.
How much more than £180 does she pay for the laptop?

4 Work out the angle between the hour hand and the minute hand of a clock at 03:30.

5 The diagram shows the plan of a room.



Andy wants to put skirting boards along all the edges of the room.
He will leave two gaps of 1 m each for the doors.
Each skirting board is 4 m long.
He can cut the skirting board to fit.
Andy buys the skirting board in packs of four.
How many packs will he need to buy?

6 A train has wheels of diameter 140 cm. The train runs 1 km along a track.
Work out how many complete turns one wheel makes.

CHAPTER 2

PROBLEM SOLVING

Making a plan

Chapter 1 introduced three strategies that are useful when you begin to solve a problem. They may have given you enough ideas to solve the problems. If not, this chapter shows you how you can combine the strategies until you can see what you need to do and make a plan.

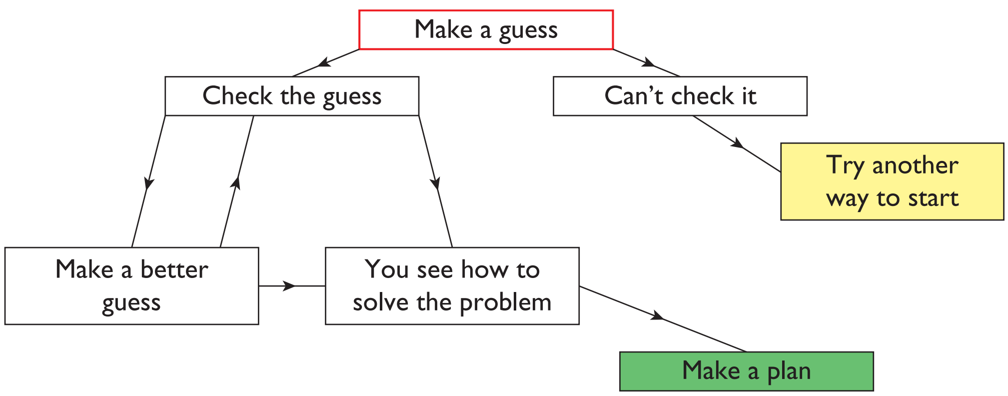
❶ Continuing from Make a guess

Once you have made a guess, you need to check it.

If there is no way to check the guess, then choose another strategy to start.

Once you have checked the guess, you will know if it was correct, too high or too low. You can use this information to make another guess.

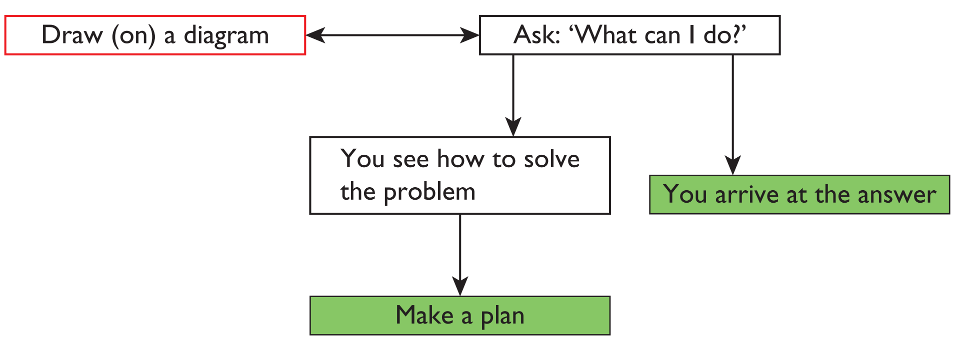
You may need to make several guesses before you can make a plan or you might need to draw (on) a diagram as well.



❷ Continuing from Draw (on) a diagram

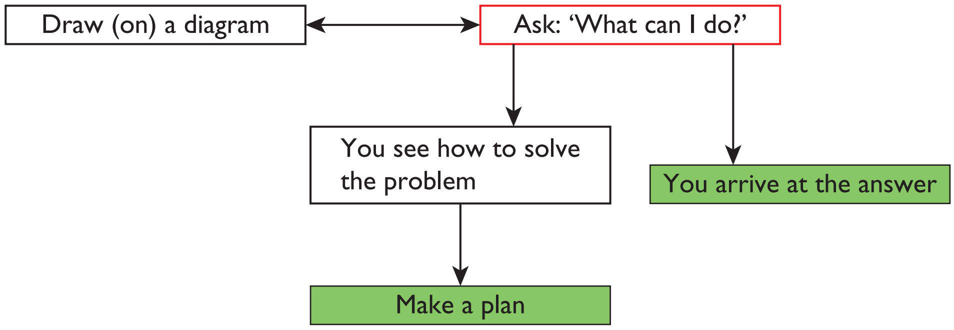
If you are still unsure how to solve the problem, ask yourself 'What **can** I do?'.

Continue to add new information to the diagram, asking yourself 'What **can** I do?' until you can make a plan or find the final answer.



❸ Continuing from Ask yourself 'What can I do?'

If you are still unsure how to solve the problem, try drawing (on) a diagram. Continue to add new information to the diagram, asking yourself 'What **can** I do?' until you can make a plan or find the final answer.



Continue the four worked examples from Chapter 1 by answering the questions within each problem.

The profit problem

Ed buys 200 bottles of lemonade for £160.
He sells 120 of them for £1.20 each.
Ed wants to make a 30% profit overall.
How much should he charge for the remaining bottles?

❶ Continuing from Make a guess

Example: I guess the remaining bottles are sold for £1 each.

Check:

$$80 \times £1 = £80$$
$$120 \times £1.20 = £144$$
$$£144 + £80 = £224$$
$$30\% \text{ of } £160 = 3 \times £16 = £48$$
$$£160 + £48 = £208$$

a Explain what is being calculated in each line above.

b Would your next guess be higher or lower than £1?

Can you see how to solve the problem now? Could you make a plan?

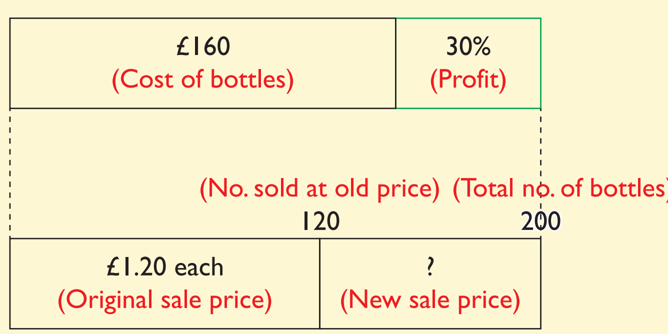
❷ Continuing from Draw a diagram

Example: Here is a bar diagram to show the information.

What can I work out?

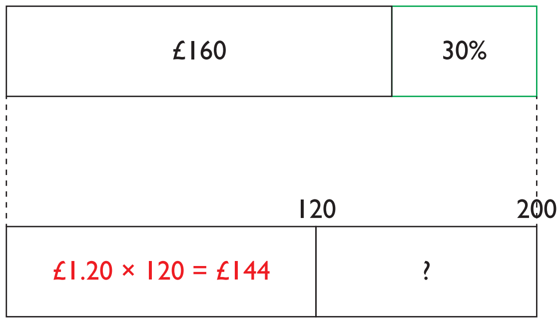
A *I can work out 30% of £160.*

B *I can work out Ed's income from the 120 bottles.*



a The diagram below has been updated with the result of **B**.

Calculate **A** and add this information to the diagram.



b What could you work out next?

Can you see how to solve the problem now? Could you make a plan?

❸ Continuing from Ask yourself 'What can I do?'

Example: I can

A *work out the total sum he received from the 120 bottles he has already sold:*
120 × £1.20 = £144

B *work out how many bottles are left to sell:*
200 − 120 = 80

C *work out how much each bottle cost:*
£160 ÷ 200 = 80p

D *work out the amount of profit Ed wants:*
30% of £160 = £48

a Add **B**, **C** and **D** to a copy of the diagram from *Continuing from Draw a diagram* above.

b What could you work out next?

Can you see how to solve the problem now? Could you make a plan?

Making a plan

There are often many different approaches to solve a problem. Whichever one you take, you will begin to see the steps you need to take to solve the problem.

At this stage a plan might be:

Step 1: Work out how much more money Ed needs to give him his required profit.

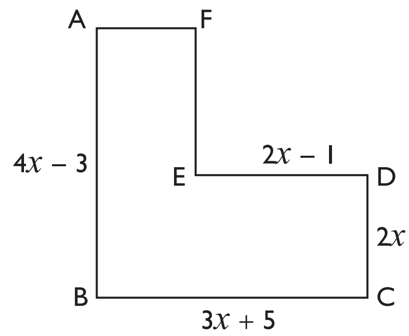
Step 2: Work out how much that is per remaining bottle.

Label a copy of the diagram above to show where the answers to Steps 1 and 2 belong on the diagram.

See Problem Solving Chapter 3 to continue solving this problem.

The algebra problem

The perimeter of the shape shown is 39 cm.
Work out the value of x.
All measurements are in centimetres.

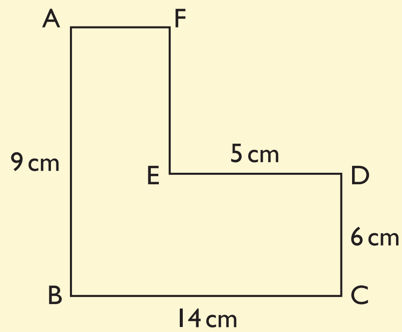


❶ Continuing from Make a Guess

Example: I guess $x = 3$.

Check:

BC = 14 cm
CD = 6 cm
DE = 5 cm
AB = 9 cm



a Work out the lengths of EF and AF when $x = 3$.

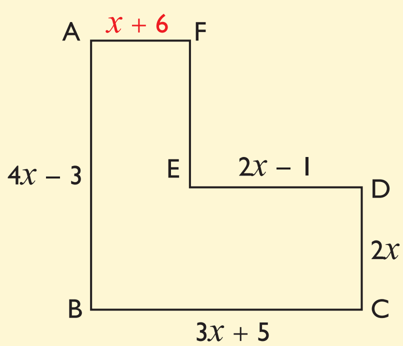
b What is the perimeter of the shape when $x = 3$?

c Would your next guess for x be higher or lower than 3?

Can you see how to solve the problem now? Could you make a plan?

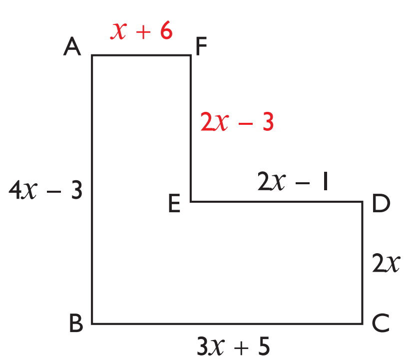
❷ Continuing from Draw on a diagram

Example: I could add the markings in red.



I can also work out EF.

a EF $= 2x - 3$. Show how this was calculated.



b What could you work out next?

Can you see how to solve the problem now? Could you make a plan?

❸ Continuing from Ask yourself 'What can I do?'

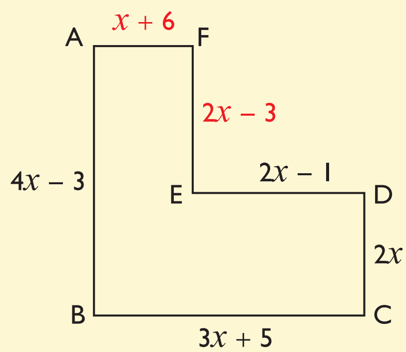
Example: I can

A *work out the length of AF in terms of x:*
$AF = x + 6$

B *work out the length of EF in terms of x:*
$EF = 2x - 3$

C *write an expression in terms of x for the area of the shape:*
Not needed because the question is about perimeter.

D *write an expression in terms of x for the perimeter of the shape:*
First, add the expressions for AF and EF to the diagram.



So perimeter = $x + 6 + 2x - 3 + 2x - 1 + 2x + 3x + 5 + 4x - 3$

a What is the definition of perimeter?

Can you see how to solve the problem now? Could you make a plan?

Making a plan

Whichever route you take, you will begin to see the steps you need to take to solve the problem.

At this stage a plan might be:

Step 1: Simplify the expression for the perimeter.
Step 2: Make the expression equal to 39.
Step 3: Solve the equation.

a What would Step 3 be?

See Problem Solving Chapter 3 to continue solving this problem.

The best buy problem

A TV normally costs £348 plus VAT (20%).
Which is the best buy? Justify your answer.

TVs 2 Go 20% off the recommended price	**Tikkas** $\frac{1}{4}$ off the price including VAT	**Asteroid** 'Pay no VAT'

❶ Continuing from Make a guess

Example: Tikkas is the best buy.

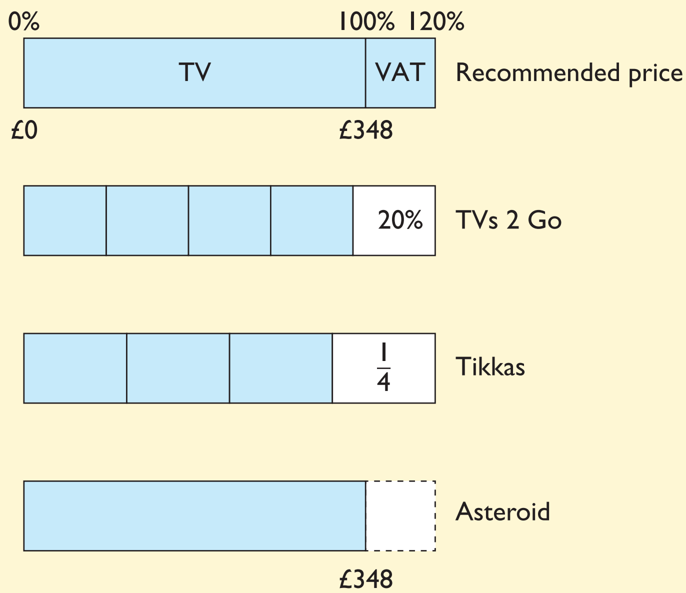
Check:

The only way to check this is by working out the cost in each case. I'll try the other strategies to help with this.

This is an example where a guess is not very helpful, but your guess can still be used to help check your final answer.

❷ Continuing from Draw a diagram

Example: Here is a bar diagram.



Now I can work out the full price of the TV and add it to the diagram.

a Show how the amounts in red could have been calculated.

b Can you add anything to the bar diagrams now?

Can you see how to solve the problem now? Could you make a plan?

❸ Continuing from Ask yourself 'What can I do?'

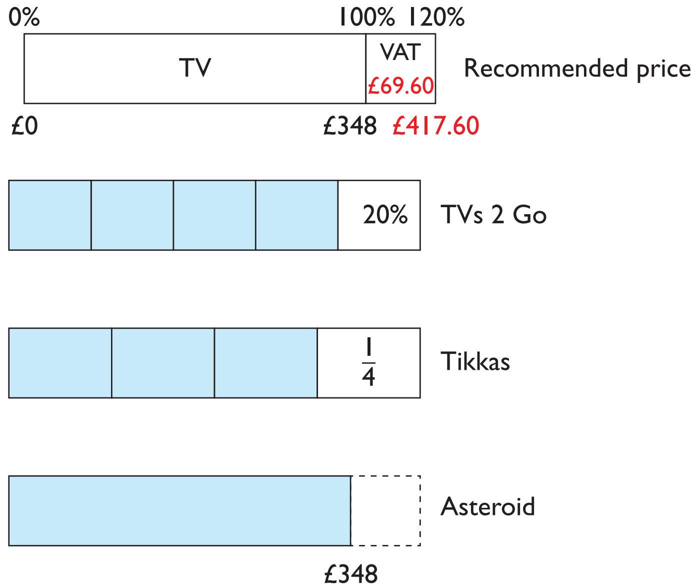
Example: I can

A *work out the cost of the TV with VAT:*
120% of £348 = £417.60

B *work out 20% of £348:*
£69.60

C *work out $\frac{1}{4}$ of £348 and $\frac{1}{4}$ of 20%:*
£348 ÷ 4 = £87
20% ÷ 4 = 5%

a The amount calculated in **A** is already on the diagram. Do the amounts calculated in **B** and **C** fit on the diagram? Add any that you think are useful.



b What else can you find using the diagram?

Can you see how to solve the problem now? Could you make a plan?

Making a plan

Whichever route you take, you will begin to see the steps you need to take to solve the problem.

At this stage a plan might be:

Step 1: Work out 20% of £417.60 and then find the cost of the TV in TVs 2 Go.

Step 2: Work out the cost of the TV in Tikkas.

Step 3: Compare the prices in all three stores to find the cheapest.

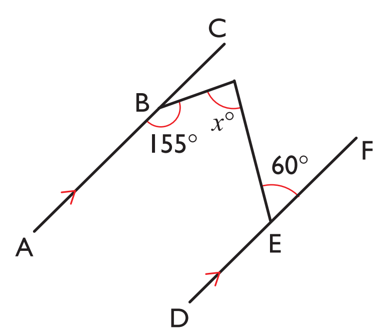
a What calculation do you need to do for Step 2?

See Problem Solving Chapter 3 to continue solving this problem.

The angle problem

In the diagram, the straight lines ABC and DEF are parallel.

What is the size of angle x? Give reasons for your answer.



❶ Continuing from Make a guess

Example: I guess 100°.

Check:

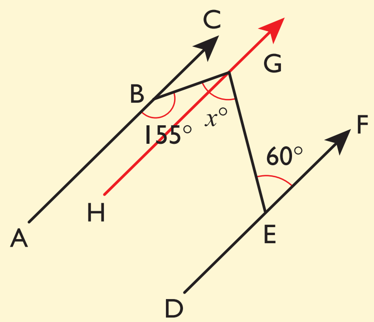
I can't check this. I'll try the other strategies instead.

This is an example where a guess is not very helpful, but your guess can still be used to help check your final answer.

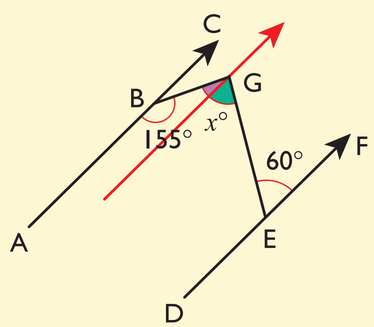
❷ Continuing from Draw on a diagram

Example: I could add the line in red.

Diagram A



This diagram is better than diagram B because you can use the parallel line facts.



The green angle is 60° (alternate angles).

a Work out the size of the purple angle.

b Which angle fact did you use?

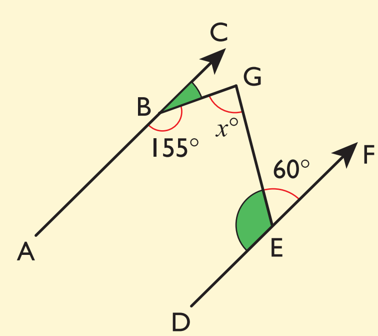
Can you see how to solve the problem now? Could you make a plan?

❸ Continuing from Ask yourself 'What can I do?'

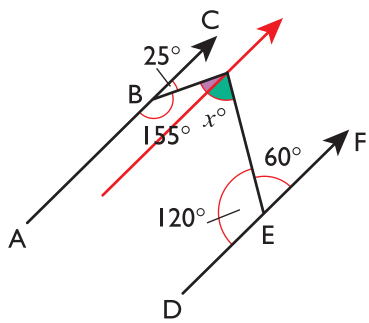
Example: I can work out the size of angle CBG and angle DEG marked in green:

Angle CBG = 25° (angles on a straight line)

Angle DEG = 120° (angles on a straight line)



a Add the size of angles CBG and DEG to the diagram.



b By adding the red line to the diagram, work out the size of both the purple and green angles in two different ways.

Can you see how to solve the problem now? Could you make a plan?

Making a plan

Whichever route you take, you will begin to see the steps you need to take to solve the problem.

At this stage a plan might be:

Step 1: Add the green and purple angles together.

Reflecting

Notice that, depending on the problem, some of the strategies work better than others. You will also find some strategies easier to use than others.

Often using them all together will give a better understanding of how to solve the problem.

The next step for each problem is covered in Problem Solving Chapter 3.

Practice problems

Use the strategies you have met so far to work on the problems below.

Go on to solve the problems if you can.

For each problem:

a Make a guess.

b Draw (on) a diagram.

c Work out what you can do.

d Try to go on by combining the strategies until you can see what to do to solve the problem.

e Make a plan or continue as far as you can with the strategies.

f State which of the strategies was most useful.

1 Steve is driving on a motorway.

There are three service stations before he reaches his destination.

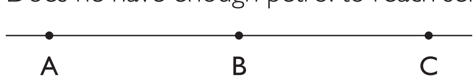
The next service station, A, is 3.5 miles away.

Service station C is 27.9 miles away.

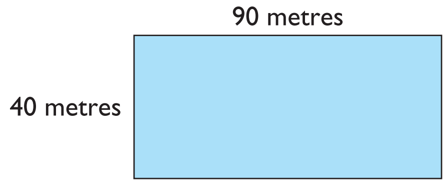
Service station B is halfway between A and C.

Steve's in-car computer tells him that he only has enough petrol to drive for 16 miles.

Does he have enough petrol to reach service station B?



2 The diagram shows a rectangular field.
Bill has 28 goats.
He wants the goats to be able to graze in this field.
Each goat needs 125 square metres of field in which to graze.
Show that the field is large enough for Bill's 28 goats.
Is there enough space for another goat?

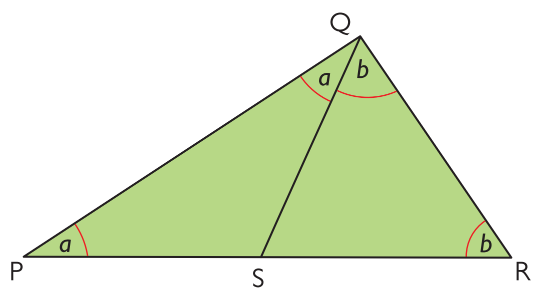


Not to scale

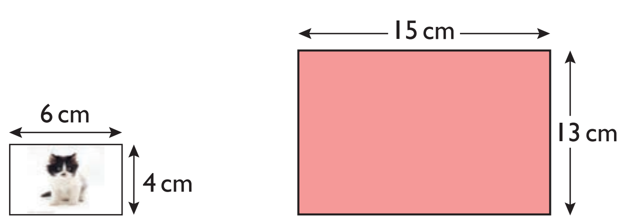
3 Sam is 5 feet 3 inches tall.
Penny is 163 cm tall.
Who is taller and by how much?
Use 2.54 cm = 1 inch and 12 inches = 1 foot.

4 Model trains are made for a 00 gauge track.
The scale is 1 : 76.
Pete has a model of the Mallard locomotive.
The Mallard is 70 feet long.
What is the length of Pete's model in centimetres (to 1 decimal place)?
Use 2.54 cm = 1 inch and 12 inches = 1 foot.

5 Here is a triangle PQR.
S is a point on the side PR.
Show that angle PQR is a right angle.



6 Hannah has this picture of her cat.
She wants to enlarge the picture to fit exactly into the frame.
The dimensions of the picture and the frame are given.
Will she be able to do this? Give a reason for your answer.



CHAPTER 3

PROBLEM SOLVING

Carrying out the plan

In the previous two chapters, you have learned how to use different strategies to solve problems. Once you can see how to solve a problem you can make a plan.

When you implement your plan:

- Be sure to write out clearly what you are doing. This will help you make progress and should prevent you making mistakes.
- It is quite common to discover errors in your original plan. Explaining clearly what you are doing will help you to identify and then correct them.
- Make it clear when you have found the answer and show how you know it is correct. This is dealt with further in Chapter 4.

Continue the four worked examples from Chapters 1 and 2 by answering the questions within each problem.

The profit problem

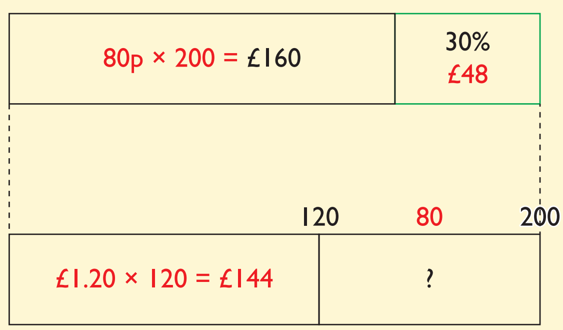
Ed buys 200 bottles of lemonade for £160.

He sells 120 of them for £1.20 each.

Ed wants to make a 30% profit overall.

How much should he charge for the remaining bottles?

What I know from the previous chapters



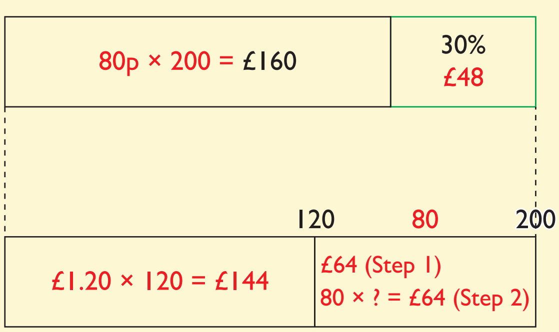
Carrying out the plan

Step 1: Work out how much more money Ed needs to give him his required profit.

£160 + £48 − £144 = £64

cost + profit − current income

Step 2: Work out how much that is per remaining bottle.



£64 ÷ 80 = 80p

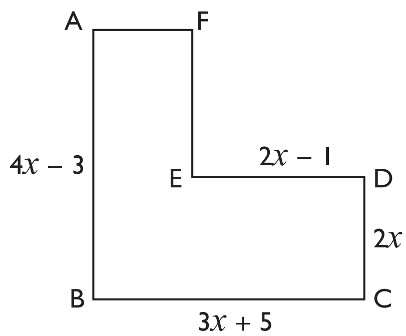
amount needed ÷ number of bottles remaining

a Does this answer seem correct?

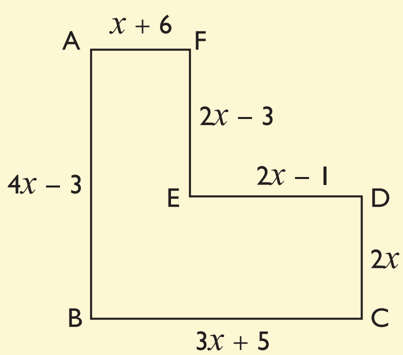
b How can you check your answer? *Help with this is given in Problem Solving Chapter 4.*

The algebra problem

The perimeter of the shape shown is 39 cm.
Work out the value of x.
All measurements are in centimetres.



What I know from the previous chapters



Perimeter $= x + 6 + 2x - 3 + 2x - 1 + 2x + 3x + 5 + 4x - 3$
When $x = 3$, the perimeter is $9 + 3 + 5 + 6 + 14 + 9 = 46$ cm.

This is too large, so x must be less than 3.

Carrying out the plan

Step 1: Simplify the expression for the perimeter.

Perimeter $= 14x + 4$

$x + 2x + 2x + 2x + 3x + 4x = 14x$
$6 - 3 - 1 + 5 - 3 = 4$

Step 2: Make the expression equal to 39.

$14x + 4 = 39$

The perimeter must be 39.

Step 3: Solve the equation.

$$14x + 4 = 39$$
$$14x = 35$$
$$x = 35 \div 14$$
$$= 2.5$$

a Does this answer seem correct?

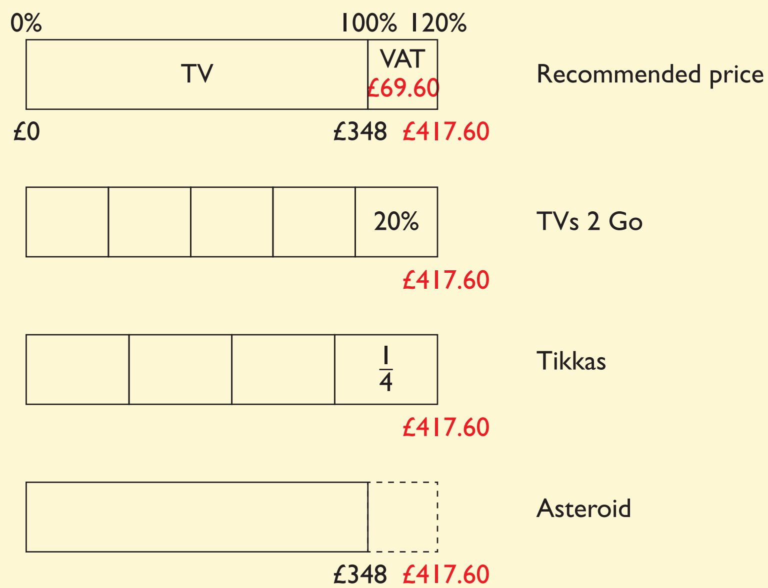
b How can you check your answer? *Help with this is given in Problem Solving Chapter 4.*

The best buy problem

A TV normally costs £348 plus VAT (20%).
Which is the best buy? Justify your answer.

TVs 2 Go	Tikkas	Asteroid
20% off the recommended price	$\frac{1}{4}$ off the price including VAT	'Pay no VAT'

What I know from the previous chapters



Carrying out the plan

Step 1: Work out 20% of £417.60 and then find the cost of the TV in TVs 2 Go.

0.20 × 417.60 = 83.52

Subtract £83.52 from £417.60 to find the cost of the TV in TVs 2 Go.

£417.60 − £83.52 = £334.08

Step 2: Work out the cost of the TV in Tikkas.

To do this, I need to calculate $\frac{1}{4}$ of £417.60.

$\frac{1}{4}$ *× £417.60 = £104.40*

And then subtract this from £417.60.

£417.60 − £104.40 = £313.20

Step 3: Compare the prices in all three stores to find the cheapest.

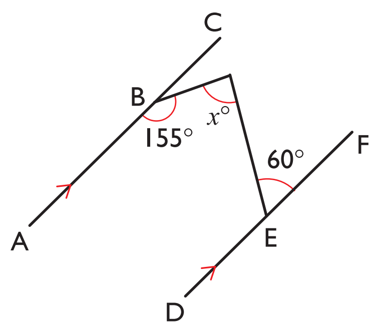
TVs 2 Go: £334.08
Tikkas: £313.20
Asteroid: £348
So Tikkas is the best buy.

a Does this answer seem correct?

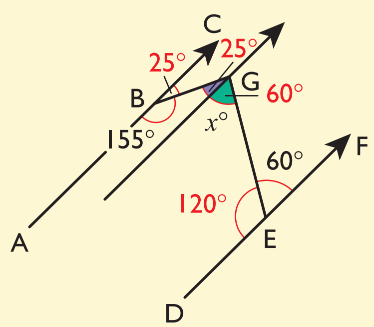
b How can you check your answer? *Help with this is given in Problem Solving Chapter 4.*

The angle problem

In the diagram, the straight lines ABC and DEF are parallel.
What is the size of angle x? Give reasons for your answer.



What I know from the previous chapters



Carrying out the plan

Step 1: Add the green and purple angles together.

$x = 60 + 25 = 85°$

a Does this answer seem correct?

b How can you check your answer? *Help with this is given in Problem Solving Chapter 4*.

Reflecting

You now have a solution, but how do you know your plan was the right one and that you have solved the problem correctly? By working carefully, you may have identified some errors in your plan. However the next crucial step is checking for errors and this is covered in Problem Solving Chapter 4.

Practice problems

Use the strategies you have met so far to solve the problems below.

For each problem:

a Make a guess.

b Draw (on) a diagram.

c Work out what you can do.

d Combine the strategies until you can make a plan or solve the problem.

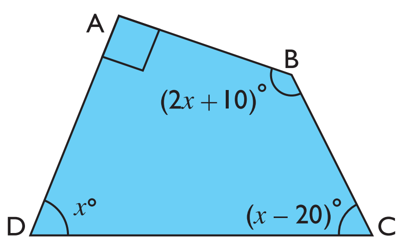
e Carry out your plan and find a solution.

f Say why you think your solution is correct.

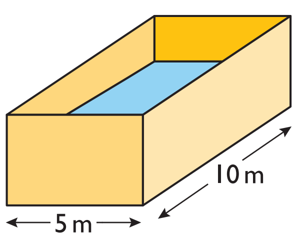
1. Aisha gets £5 pocket money each week from her parents and £3 each week from her brother.
   Aisha spends $\frac{1}{4}$ of her pocket money on magazines.
   She spends $\frac{1}{3}$ of the remaining amount on make-up.
   She spends 50p per week on sweets.
   She saves the rest of her money.
   After 12 weeks, Aisha checks her savings.
   How much should she have?

2. Marcus owns a house and an apartment.
   He bought his house for £160 000.
   He bought his apartment for £90 000.
   Marcus decides to sell both properties.
   He makes a 20% loss in selling the house and a 40% profit in selling the apartment.
   a Show that overall Marcus makes a profit.
   b Is his profit greater than 2%?

3. ABCD is a quadrilateral.
   Find the value of the largest interior angle of the quadrilateral.

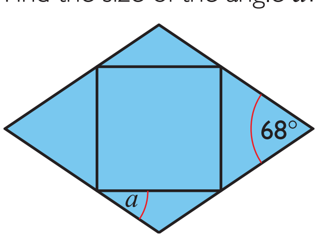


4. Jenny has an empty pool.
   She is going to pump water into the pool at the rate of 20 litres per second.
   The pool is in the shape of a cuboid 10 m long by 5 m wide.
   Work out the length of time it will take to fill the pool to a depth of 150 cm.



5. A, B, C, D and E are points.
   ABC is a straight line.
   Angle EBD is twice the size of angle DBA.
   Angle EBC is three times the size of angle EBD.
   Work out the size of angle EBC.

6. The diagram shows a square inside a rhombus. One of the angles of the rhombus is 68°.
   The corners of the square are the midpoints of the sides of the rhombus.
   Find the size of the angle a.



CHAPTER 4

PROBLEM SOLVING

Reviewing the solution

In Chapters 1 to 3, you learned and implemented strategies to solve a range of problems.

In this chapter, you will use three strategies for improving your problem solving skills and checking that your solution is correct.

1 Have I answered the question?

When you concentrate on finding a key quantity correctly, it is easy to overlook the final step of the solution. Reread the problem and check that you have answered the question fully.

Always check that your answer is written clearly.

2 Is my answer sensible?

If you made an initial guess, you can sometimes use this to help you decide if your answer is sensible.

- Check your method by using rounded numbers that are easier to work with. This means that you can check your answer against an estimated solution.
- Start with the solution and work backwards to double check each step of the solution.
- Make sure the solution makes sense given the context. For example, it is unlikely that a dog weighs 20 g or 200 kg.

3 Reflection

Reflecting on how you solved the problem will help you to develop your skills for the next problem you encounter.

Answer these questions and then discuss them with other people if you can.

- What went well? Which strategies worked best?
- What was difficult? How did you overcome those difficulties?
- Which strategies did you use?
- Did you use an efficient method? Can you see a better way to do it now?
- Was it easy to think what to do?
- What were the advantages and disadvantages of each strategy?

Now work through each of the four problems again, checking your answers and reflecting on your strategies and solutions.

The profit problem

Ed buys 200 bottles of lemonade for £160.

He sells 120 of them for £1.20 each.

Ed wants to make a 30% profit overall.

How much should he charge for the remaining bottles?

❶ Have I answered the question?

The question asks how much to charge for the rest of the bottles.
I have worked out that the charge should be 80p per bottle, so I have answered the question.

❷ Is my answer sensible?

I made a guess of £1 and that gave me a profit that was a bit too high. 80p is less than £1, so this is sensible.

❸ Reflection

- *Checking a guess involves working out quite a lot of things. Once I had checked my guess, it was easier to make a plan.*

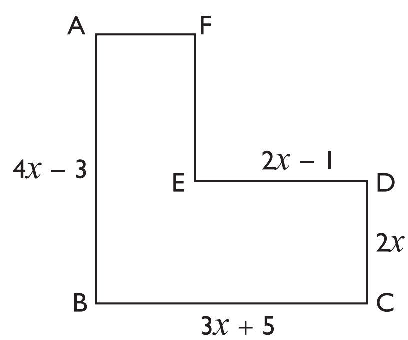
It is usually easier to check subsequent guesses.

- *It was difficult to draw a diagram that shows all of the parts of the problem. However, trying to draw it made me think harder about the problem.*
- *Working out what I could gave me the information I needed to solve the problem. It took me through the steps of the problem without me realising.*

Starting with the solution 80p, work backwards through the problem as you did with £1 in Chapter 1.

The algebra problem

The perimeter of the shape shown is 39 cm.
Work out the value of x.
All measurements are in centimetres.



❶ Have I answered the question?

The question asks for the value of x.
I found that $x = 2.5$, so I have answered the question.

Questions like this often ask you to find the longest side or the shortest side. This is straightforward to do once you have calculated the value of x, but students frequently forget the final step.

❷ Is my answer sensible?

My guess of $x = 3$ gave me a perimeter that was a bit too large.
2.5 is less than 3, so $x = 2.5$ is sensible.

❸ Reflection

- *Checking a guess was hard work but it helped me to understand the problem.*
- *Adding the missing lengths to the diagram helped me to see what to do.*
- *Working out what I could meant I went through the steps of the problem without realising. It also made me show my working.*

Starting with the solution $x = 2.5$, work backwards through the problem as you did with $x = 3$ in Chapter 1.

The best buy problem

A TV normally costs £348 plus VAT (20%).
Which is the best buy? Justify your answer.

TVs 2 Go	**Tikkas**	**Asteroid**
20% off the recommended price	$\frac{1}{4}$ off the price including VAT	'Pay no VAT'

1. Have I answered the question?

   The question asks which shop offered the best buy. I found this was Tikkas. I have shown all the calculations that justify my solution.

2. Is my answer sensible?

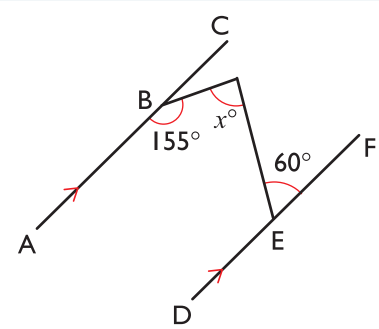
   I found all the shops were selling the TV at similar prices so there are unlikely to be any errors. Looking at the diagram also helps me to see that Tikkas will give me the cheapest price.

3. Reflection

- *Making a guess did not help as there was no way to check it.*
- *It was difficult to draw a diagram that would help.*
- *Working out 'what you can' is a step-by-step process. This means I showed my working and I could justify my answer.*

The angle problem

In the diagram, the straight lines ABC and DEF are parallel.
What is the size of angle x? Give reasons for your answer.



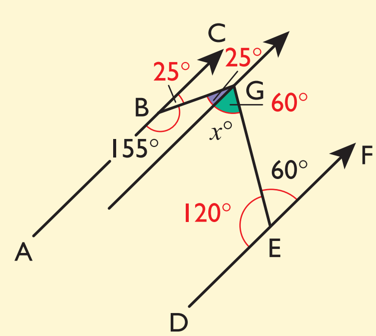
1. Have I answered the question?

   The question asks me to find the size of angle x. I found this to be 85°. I explained all my steps so this means I have given reasons for the answer.

2. Is my answer sensible?

   The diagram is very useful. I have easily worked backwards to check each angle I found is correct.

   Diagrams given in problems are often not drawn accurately so trying to measure lengths and angles can be misleading.



3. Reflection

- *Checking a guess is hard.*
- *Drawing an extra line helped a lot, then I found some more angles by doing what I could. I found the angles I needed for the answer without realising.*

Advantages and disadvantages of the main strategies

Here is a list of some advantages and disadvantages of each of the main strategies introduced in these chapters.

Strategy	Advantages	Disadvantages
Making a guess	• Easy to do • Easy to see if it's not going to work • Helps you understand the problem • Helps you to make a plan • Can lead to the answer	• May not be possible to check • Can be time-consuming
Using a diagram	• Makes you think about the problem • Easy to add to a diagram • Helps you to see what to do • Helps you to make a plan	• Deciding on what diagram may be difficult • May not find it helpful
What can I do?	• Easy to do • Gives you the steps to build up to an answer • Usually leads to an answer • Helps you understand the problem • Helps you to make a plan	• May do unnecessary work • May not be the quickest route to a solution

- A combination of strategies is likely to be most effective when you have a limit on your time.
- Adding to a diagram plus 'What can I do?' is particularly effective.

Draw your own table like the one above. Do not copy any statements that you do not agree with. Add other advantages and disadvantages that you found.

Practice problems

Use the strategies you have met so far to solve the problems below.

It is important to try and solve these problems before moving on to Chapter 5.
Chapter 5 demonstrates how all the strategies come together.

1. Richard is the owner and chief executive of a company.

   Each of the three directors works for Richard.

   Mary is one of the directors and Keith is her personal assistant.

   Each year the company makes a profit, Richard gives himself and his directors a bonus.

   $\frac{2}{3}$ of the profits are reinvested in the company.

   Of the remaining profits, Richard gives himself a bonus of $\frac{1}{2}$ and shares the rest equally between his three directors.

   Mary always gives $\frac{1}{10}$ of her bonus to Keith.

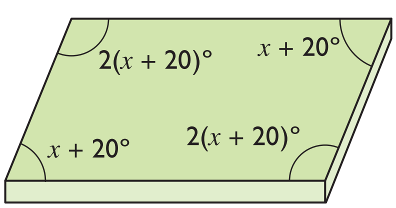
   Last year the company made a profit of £$3\frac{3}{5}$ million.

   How much was Keith's bonus payment?

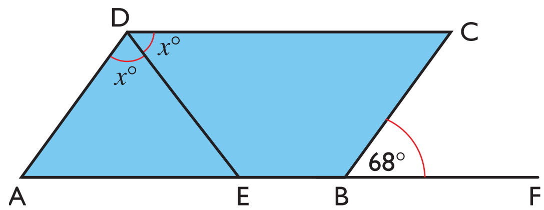
2. The tile is used in a child's toy.

   It is in the shape of a parallelogram.

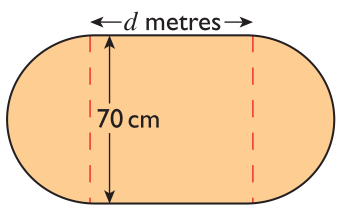
   Find the value of the smallest angle(s) in the parallelogram.



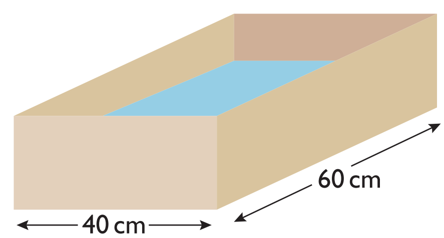
3 ABCD is a parallelogram. Angle CBF = 68°.
E is a point on the line ABF.
Angle ADE = angle EDC = $x°$
Find the value of x.



4 Alice is making a mat for her bathroom.
The mat has two ends which are semicircles.
Each semicircle has a diameter of 70 cm.
The mat will have a perimeter of 400 cm.
The length of the straight part of the mat is d metres.
Work out the value of d.



5 The diagram shows a fish tank partly filled with water.
The tank is 40 cm wide and 60 cm long.
The water level is 28 cm from the bottom.
A stone is put in the tank and the water completely covers the stone.
The water level in the tank is now 30 cm from the bottom.
Work out the volume of the stone.



CHAPTER 5

PROBLEM SOLVING

Bringing it all together

This chapter gives a possible journey to the solution for each practice problem in Chapter 4 using all the strategies you have met.

Do not look at this chapter until you have attempted these practice problems in Chapter 4.

For those questions you solved correctly, look at the solution and comments to deepen your understanding of the strategies.

For those questions you were unable to solve, look at the first step and then try again to solve the problem. If you need more help, look at the second step and try to solve the problem from there, and so on.

Practice problems

1. Richard is the owner and chief executive of a company.

   Each of the three directors works for Richard.

   Mary is one of the directors and Keith is her personal assistant.

   Each year the company makes a profit, Richard gives himself and his directors a bonus.

   $\frac{2}{3}$ of the profits are reinvested in the company.

   Of the remaining profits, Richard gives himself a bonus of $\frac{1}{2}$ and shares the rest equally between his three directors.

   Mary always gives $\frac{1}{10}$ of her bonus to Keith.

   Last year the company made a profit of £$3\frac{3}{5}$ million.

   How much was Keith's bonus payment?

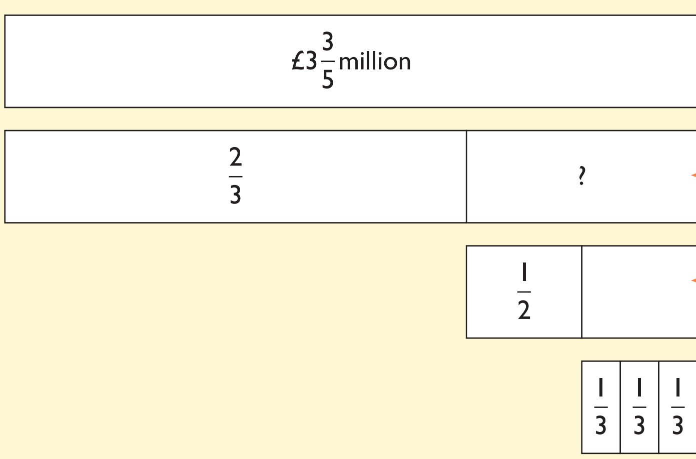
Possible solution

There is no point guessing as it will be hard to check the answer.

Strategy: Make a guess

A bar diagram might help.

Strategy: Draw a diagram



This is the profit that is shared out.

This is the profit that is shared out between the directors.

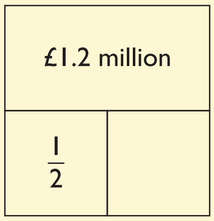
Strategy: What can I work out?

Fraction shared $= 1 - \frac{2}{3}$

$= \frac{1}{3}$

Amount shared $= \frac{1}{3}$ *of £3.6 million*

= £1.2 million

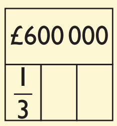


Strategy: Adding to the diagram

The directors receive $= \frac{1}{2}$ *of £1.2 million*

= £600 000

Strategy: What can I work out?

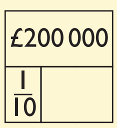


Strategy: Adding to the diagram

Strategy: What can I work out?

Mary's bonus = £600 000 ÷ 3

= £200 000



Strategy: Adding to the diagram

Strategy: What can I work out?

Keith's bonus = £200 000 ÷ 10

= £20 000

Check

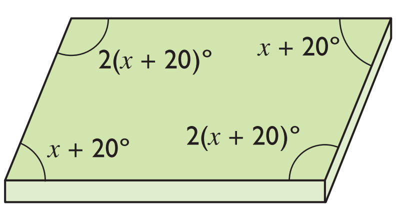
I have found how much Keith gets and shown my answer clearly.

- *Working backwards*
  $20\,000 \times 10 \times 3 \times 2 \times 3 =$ *£3.6 million*
- *The diagrams made the question clearer. I realise now I didn't have to work out all the interim amounts; I could have just completed one calculation:*
  $3.6 \times 10^6 \div 3 \div 2 \div 3 \div 10$

2 The tile is used in a child's toy.

It is in the shape of a parallelogram.

Find the value of the smallest angle(s) in the parallelogram.



Possible solution

A diagram with all the known information is given.

Strategy: Draw a diagram

I could make a guess but I don't think this is necessary.

Strategy: Make a guess

I know the angles in a parallelogram add up to 360°. I need to add up the angles, write an equation and then solve it.

Strategy: What can I work out?

$$x + 20° + 2(x + 20°) + x + 20° + 2(x + 20°) = x + 20° + 2x + 40° + x + 20° + 2x + 40°$$
$$= 6x + 120°$$

$$6x + 120° = 360°$$
$$6x = 240°$$
$$x = 40°$$

Check

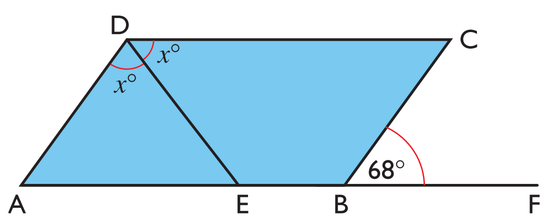
- *The question asks me to find the smallest angle. Oh! I can see the smallest angles are labelled $x + 20°$. So the smallest angles are $40 + 20 = 60°$.*
- *The remaining angles will be $2(40 + 20) = 120°$. $120 + 120 + 60 + 60 = 360$, so my answer makes sense.*
- *The diagram was very useful. I think my method was quite efficient, but I must remember to make sure I answer the question!*

3 ABCD is a parallelogram. Angle CBF = 68°.

E is a point on the line ABF.

Angle ADE = angle EDC = x°

Find the value of x.



Possible solution

Guessing doesn't help because I can't see a way to check it.

Strategy: Make a guess

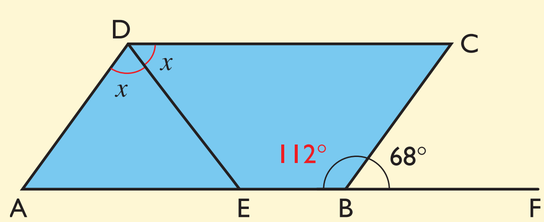
A diagram with all the known information is given.

Strategy: Draw a diagram

I can work out ∠EBC.

Strategy: What can I work out?

$\angle EBC = 180° - 68°$
$= 112°$



Strategy: Adding to the diagram

ABCD is a parallelogram, so opposite angles are equal to each other.
∠ADC is also 112°.
Now I know $x = 112 \div 2$
$= 56°$

Strategy: What can I work out?

Check

The question asks me to find the value of x. I have done that.

$56 \times 2 = 112$ and $112 + 68 = 180$, so my calculations seem to be correct.

Strategy: Working backwards

I have used a combination of 'Adding to the diagram' and 'What can I work out?' to answer the question. This seems a good strategy.

Working on the diagram made things clearer.

4 Alice is making a mat for her bathroom.

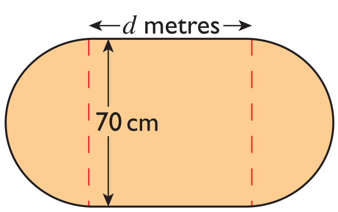
The mat has two ends which are semicircles.

Each semicircle has a diameter of 70 cm.

The mat will have a perimeter of 400 cm.

The length of the straight part of the mat is d metres.

Work out the value of d.



Possible solution

Guessing doesn't help because I can't see a way to check it.

Strategy: Make a guess

The diagram given contains most of the information.

Strategy: Draw a diagram

I can work out the radius of the semicircular sections:

$70 \div 2 = 35$ cm

Strategy: What can I work out?

I can work out the circumference of the circle with radius 35 cm:

circumference $= 2 \times \pi \times 35$ cm

$= 219.8$ cm (using $\pi = 3.14$)

Each end is a semicircle, so the length of the two semicircular parts is 219.8 cm.

I can see what to do now.

Strategy: Make a plan

I need to work out the total length of the two straight sections and then divide by 2.

The total length of the straight sections $= 400 - 219.8$ cm

Strategy: Carry out the plan

$= 180.2$ cm

$d = 180.2 \div 2$

$= 90.1$ cm

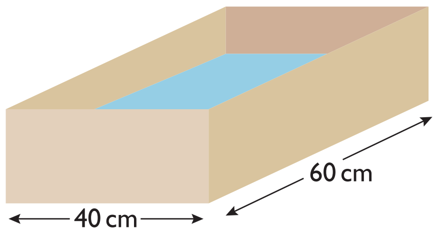
$= 0.901$ m

Check

- *The question asked me to find d in metres which I have done.*
- *I can check by working back to the perimeter:*

  $2 \times 90.1 + 219.8 = 400$
- *Just working out what I could led me to the answer quickly.*
- *The circumference should be a bit more than three times the diameter. 219.8 cm looks about right for the circular part.*

5 The diagram shows a fish tank partly filled with water.



The tank is 40 cm wide and 60 cm long.

The water level is 28 cm from the bottom.

A stone is put in the tank and the water completely covers the stone.

The water level in the tank is now 30 cm from the bottom.

Work out the volume of the stone.

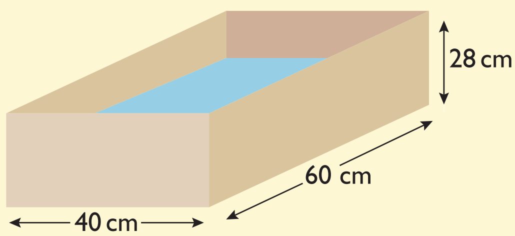
Possible solution

Guessing doesn't help because I can't see a way to check it.

Strategy: Make a guess

I can add to the diagram.

Strategy: Draw a diagram



I can work out the original volume of water in the tank.

Strategy: What can I work out?

Volume = $40 \times 60 \times 28 = 67\,200\,\text{cm}^3$

I know that the depth increases to 30 cm when the stone is placed in the tank and I can see what else I need to do.

Strategy: Make a plan

I can work out the new volume of water. I can then subtract the old volume from the new volume to find the volume of the stone.

Volume with stone in tank = $40 \times 60 \times 30 = 72\,000\,\text{cm}^3$

Strategy: Carry out the plan

Volume of stone = $72\,000 - 67\,200$

$= 4800\,\text{cm}^3$

Check

- *The question asked me to find the volume of the stone. I've done that.*
- *I can now see that I could have done this more quickly by calculating only the increased volume: $40 \times 60 \times 2 = 4800\,\text{cm}^3$.*
- *I have calculated the same answer using two different methods, so I know that my answer must be correct.*

Practice problems

Solve these problems using all the strategies you have met in these problem solving chapters.

1. There are 28 children in Mrs Davies' class.
   She is going to buy a pen, a pencil and an eraser for each of the children.
   She sees the following information in a stationery catalogue.

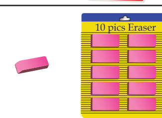
   Pens: 30p each or a pack of 5 for £1.20

   Pencils: 20p each or a pack of 3 for 50p

   Eraser: 12p each or a pack of 10 for £1

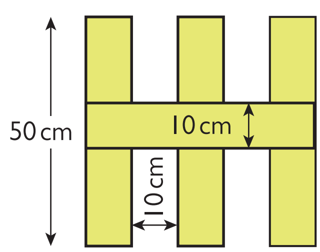
   Mrs Davies has just £15 to spend.
   Has she got enough money to buy a pen, a pencil and an eraser for each child?

2. Here is a list of ingredients for making a cheese soufflé.
   Frances is going to make 12 cheese soufflés.
   She has 150 grams of flour and 2 litres of milk.
   Does she have enough flour and milk?
   1 oz = 28 g and 1 pint = 568 ml

   Cheese soufflé
   3 eggs
   1 oz of butter
   $\frac{1}{2}$ oz of flour
   $\frac{1}{4}$ pint of milk
   3 oz of grated cheese

3. Ric is x years old.
   Steph is six years older than Ric.
   Tom is three times Steph's age.
   The total of their ages is 84.
   How old is Tom?

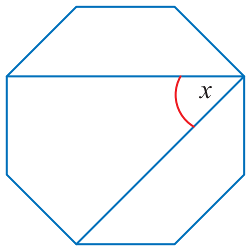
4. The diagram shows part of the fence that Jim is going to put along the length of both sides of a path.

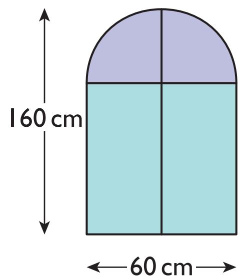
   The fence is made of repeating blocks of three vertical planks of wood and one horizontal plank.
   The vertical planks have height 50 cm and width 10 cm with a space of 10 cm between each plank.
   The horizontal plank has width 10 cm and length 50 cm.
   Jim buys the planks in packs of 10.
   Each pack costs £6.99.

The length of the path is 8 m.
Work out the total cost of the packs Jim needs.

5 Here is a regular octagon.
Work out the size of angle x.

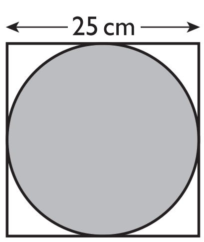


6 Here is a diagram of a stained glass window.



The window is made of two identical rectangles and two identical quarter circles.
The width of the window is 60 cm.
The height of the window is 160 cm.
Each pane of glass is surrounded by lead supports. These are shown by the lines in the diagram.
Work out the total length of the lead supports.

7 A factory makes metal discs by punching out a circle from a piece of metal.
The piece of metal is a square of side 25 cm.
Any metal left over when the circle is removed is sent for recycling.
The factory makes 800 discs each hour.
Work out the total area of metal sent for recycling each hour.
Give your answer in square metres.



CHAPTER 2

NEXT STEPS – NUMBER

Using our number system

2.3 Calculating with standard form

What you need to know

Did you know?

Scientists use numbers in standard form because they use up less memory in a computer. It makes it easier to work out the number of stars in the Milky Way galaxy.

Adding or subtracting numbers in standard form is straightforward if the powers of ten are the same.

Five million added to three million is eight million.

This can be written as $5 \times 10^6 + 3 \times 10^6 = 8 \times 10^6$.

If the powers of ten are not equal, rewrite them so they are. Then you can use the same strategy.

$6 \times 10^9 + 5 \times 10^8 = 60 \times 10^8 + 5 \times 10^8$ ← Make the powers of 10 the same.

$= 65 \times 10^8$ ← Add the numbers.

$= 6.5 \times 10^9$ ← Rewrite the number in standard form.

When multiplying (or dividing) two numbers in standard form, work with each part of the number separately.

$5 \times 10^7 \times 3 \times 10^3 = 5 \times 3 \times 10^7 \times 10^3$ ← $5 \times 3 = 15$, $10^7 \times 10^3 = 10^{10}$

$= 15 \times 10^{10}$ ← Note: This is not standard form.

$= 1.5 \times 10^{11}$ ← The number must be between 1 and 10.

How to do it

➤ Multiplying large and small numbers

The mass of a grain of sand is about 3.5×10^{-10} kg.
It is thought that there are about 7.5×10^{18} grains of sand on the Earth.
Use the figures above to calculate the mass of all of the sand on Earth. Give your answer in standard form.

Solution

$3.5 \times 10^{-10} \times 7.5 \times 10^{18} = 26.25 \times 10^8$ ← $3.5 \times 7.5 = 26.25$, $10^{-10} \times 10^{18} = 10^8$

$= 2.625 \times 10^9$ kg ← Make the number between 1 and 10.

➤ Subtraction and division of small numbers

A loaf of bread contains 5×10^{-3} kg of yeast and 1×10^{-2} kg of salt.

a What is the total mass of the salt and the yeast?

b How much greater is the mass of the salt than the mass of the yeast? Give your answer in kilograms using standard form.

c How many times greater is the mass of the salt than the mass of the yeast?

Solution

a $1 \times 10^{-2} + 5 \times 10^{-3} = 10 \times 10^{-3} + 5 \times 10^{-3}$

Convert to the same power of ten.

$= 15 \times 10^{-3}$ kg

$= 1.5 \times 10^{-2}$ kg

Rewrite in standard form.

b $1 \times 10^{-2} - 5 \times 10^{-3} = 10 \times 10^{-3} - 5 \times 10^{-3}$

$= 5 \times 10^{-3}$ kg

The salt weighs 5×10^{-3} kg more than the yeast.

c $\dfrac{\text{mass of salt}}{\text{mass of yeast}} = \dfrac{1 \times 10^{-2}}{5 \times 10^{-3}}$

$= 0.2 \times 10^{-2-(-3)}$

$= 0.2 \times 10^{1}$

$= 2\ (\times 10^{0})$

There is twice as much salt as yeast.

Answer the questions in the following exercises *without* using a calculator. Then check your answers using a calculator.

Learning exercise

① Work out the values of the following, giving your answers in standard form.

a $3.2 \times 10^{5} + 4.6 \times 10^{5}$

b $6.8 \times 10^{-2} - 5.1 \times 10^{-2}$

c $8000 + 700$

d $6.4 \times 10^{3} + 2000$

e $1.8 \times 10^{-3} + 2.2 \times 10^{-3}$

f $6.4 \times 10^{-2} - 0.033$

② Work out the following, giving your answers in standard form.

a $7.2 \times 10^{5} + 4.6 \times 10^{5}$

b $7.2 \times 10^{5} + 4.6 \times 10^{4}$

c $7.2 \times 10^{5} + 4.6 \times 10^{6}$

d $7.2 \times 10^{5} - 4.6 \times 10^{5}$

e $7.2 \times 10^{6} - 4.6 \times 10^{5}$

f $7.2 \times 10^{5} - 4.6 \times 10^{6}$

③ Work out the values. Give your answers in standard form.

a $3 \times 10^{5} \times 2 \times 10^{7}$

b $2 \times 10^{3} \times 4 \times 10^{5}$

c $2 \times 10^{5} \times 5 \times 10^{2}$

d $3 \times 10^{-5} \times 3 \times 10^{7}$

e $5 \times 10^{-7} \times 2 \times 10^{5}$

f $9 \times 10^{-6} \times 7 \times 10^{-4}$

④ Work out the values of these calculations.

a $6 \times 10^5 \div (2 \times 10^3)$ **b** $8 \times 10^9 \div (4 \times 10^8)$

c $6 \times 10^7 \div (2 \times 10^3)$ **d** $3 \times 10^7 \div (2 \times 10^3)$

e $2 \times 10^5 \div (4 \times 10^3)$ **f** $2 \times 10^6 \div (8 \times 10^8)$

⑤ Write down a number in standard form that is between:

a 6×10^5 and 6×10^4 **b** 6×10^{-3} and 6×10^{-2}

c 7.1×10^2 and 7.1×10^3 **d** 7.1×10^{-6} and 7.1×10^{-7}

⑥ Coley says:

When you're multiplying numbers in standard form, you have to multiply the two numbers at the front together, write down what that comes to, then write '× 10' and finally add the two powers together and write that down.

Explain why Coley's method won't always give the correct answer in standard form.

⑦ Work out the following, giving your answers in standard form.

a $3.204 \times 10^2 + 4 \times 10^{-1}$ **b** $3.204 \times 10^2 - 4 \times 10^{-1}$

c $3.204 \times 10^2 \times 4 \times 10^{-1}$ **d** $3.204 \times 10^2 \div (4 \times 10^{-1})$

⑧ The speed of light is 3×10^8 metres per second and there are roughly 3×10^7 seconds in a year.

A light year is the distance travelled by light in one year.

Approximately how many metres is a light year?

Give your answer in standard form.

⑨ The masses of some of the planets in our Solar System are given in the box.

Jupiter 1.9×10^{27} kg	Saturn 5.7×10^{26} kg
Mercury 3.3×10^{23} kg	Earth 6×10^{24} kg

a Write the names of the planets in order of their mass, starting with the largest.

b How many times greater is the mass of Jupiter than the mass of the Earth (to 2 d.p.)?

c How many times greater is the mass of Jupiter than the mass of Mercury (to 2 d.p.)?

d How many times greater is the mass of the Earth than the mass of Mercury (to 2 d.p.)?

⑩ The approximate masses of some items are listed in the box below.

caffeine molecule 3.2×10^{-25} kg	eyebrow hair 7×10^{-8} kg
average human cell 1×10^{-12} kg	water molecule 3×10^{-26} kg

a How many water molecules have the same mass as an eyebrow hair?

b How many water molecules have the same mass as one caffeine molecule?

c How many times greater than the mass of a water molecule is the mass of an average human cell?

⑪ A hydrogen atom has a mass of 1.67×10^{-27} kg.

An oxygen atom has a mass of 2.67×10^{-26} kg.

Show that the mass of a molecule of water is 3×10^{-26} kg.

Problem solving exercise

① The mass of a spacecraft is 7.8×10^4 kg.

The spacecraft is carrying equipment with a total mass of 2.4×10^3 kg.

The spacecraft docks with a space station.

The mass of the space station is 4.62×10^5 kg.

The commander of the space station does not want the total mass on docking to be greater than 5.43×10^5 kg.

Is the total mass within this limit?

② Jenny is making a scale model of the Solar System.

She wants the distance from Earth to Saturn to be 20 cm on her scale model.

The real distance from Earth to Saturn is 1.25×10^9 km.

a Find the scale of the model in the form $1 : n$, where n is written in standard form.

Jenny wants to put the position of a spacecraft on the scale model.

The real distance of the spacecraft from Earth, correct to 2 significant figures, is 8.5×10^8 km.

b Work out the distance of the spacecraft from Earth on the scale model.

③ Karim is trying to find out the thickness of a piece of paper.

He has a box of paper which contains 3000 sheets of paper positioned on top of each other.

The height of the paper is 0.3 m.

a Work out the thickness of each sheet of paper.

Give your answer in metres, in standard form.

Karim also wants to know the mass of each sheet of paper.

He weighs the box containing the paper, then he weighs the box when it is empty.

The mass of the box and paper is 54 kg.

The mass of the empty box is 500 g.

b Work out the mass of each piece of paper.

Give your answer in kilograms, in standard form.

④ Rod is a keen physicist interested in the wavelengths of sound waves.

Rod wants to find the difference between the wavelength of his favourite radio station and the wavelength of his dad's favourite radio station.

Rod listens to FM Capital Radio which has a frequency of 102 MHz.

Rod's dad listens to AM Radio 5 Live which has a frequency of 909 kHz.

1 MHz = 10^6 waves per second. 1 kHz = 10^3 waves per second.

To find the wavelength (in m), Rod uses the formula:

wavelength = 3×10^8 ÷ frequency (in waves per second)

Work out the difference between the wavelength of Rod's favourite radio station and the wavelength of his dad's favourite radio station.

Do I know it now?

① **a** Work out $8.48 \times 10^4 + 8.4 \times 10^3 - 3 \times 10^2$.
Give your answer in standard form.

b Write the following as ordinary numbers.

i 8.48×10^4 **ii** 8.4×10^3 **iii** 3×10^2

c Use your answers to part **b** to check your answer to part **a**.

② Work out the values of the following, giving your answers in standard form.

a $6000 \times 1.5 \times 10^9$

b $1.6 \times 10^{-4} \times 2 \times 10^{-3}$

c $2.3 \times 10^6 + 3$ million

d $0.0052 - 3.2 \times 10^{-3}$

e $7.6 \times 10^2 \times 2 \times 10^{-1}$

f $7.6 \times 10^2 \div (2 \times 10^{-1})$

Can I apply it now?

① Elaine is estimating how far away a thunderstorm is from her home. The speed of sound is estimated at 3.3×10^2 metres per second. The speed of light is estimated at 3.0×10^8 metres per second.

a The thunderstorm is 6 km away and Elaine sees a flash of lightning. She hears the clap of thunder x seconds later. Work out the value of x to the nearest whole number.

b The length of time between seeing the next flash of lightning and hearing the clap of thunder is 3 seconds. How far away, to the nearest whole kilometre, is the thunderstorm now? State any assumptions that you have made.

CHAPTER 3

NEXT STEPS – NUMBER

Accuracy

3.3 Limits of accuracy

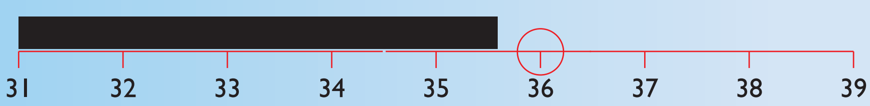
What you need to know

Did you know?

Each of these bags of coffee is advertised with a mass of 25 kg to the nearest kilogram. A bag could weigh 24.5 kg.

To say how accurate a measurement is, give its lower and upper bounds.

The length of a line is l, recorded as 36 cm to the nearest centimetre.



All lines between 35.5 cm and 36.499 999… cm are rounded to 36 cm.

35.5 cm is called the lower bound.

36.499 999 is effectively 36.5 cm, so 36.5 cm is called the upper bound.

This is written as $35.5 \leq l < 36.5$

It can also be written as $l = 35 \pm 0.5$ cm

How to do it

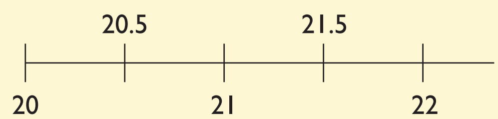
Recognising the effect of bounds on a result

Stella draws a square with a side length of 21 cm to the nearest centimetre.

a What are the upper and lower bounds of the side length of the square?

b What are the upper and lower bounds of the perimeter of the square she has drawn?

Solution



a Upper bound is 21.5 cm

Lower bound is 20.5 cm

b Upper bound of perimeter $= 4 \times 21.5 = 86$ cm

Lower bound of perimeter $= 4 \times 20.5 = 82$ cm

Learning exercise

1. Write down the lower and upper bounds for each measurement.

a 80 cm (to the nearest cm) **b** 80 cm (to the nearest 10 cm)
c 300 g (to the nearest g) **d** 300 g (to the nearest 100 g)

2. Write down the lower and upper bounds for each measurement.

a 5000 m (to the nearest m) **b** 5000 m (to the nearest 10 m)
c 5000 m (to the nearest 100 m) **d** 5000 m (to the nearest 1000 m)

3. Write down the lower and upper bounds for each measurement.

a 600 m (to the nearest 10 m) **b** 600 m (to the nearest 5 m)
c 600 m (to the nearest 100 m) **d** 600 m (to the nearest 50 m)

4. The mass of a bag of potatoes is m kg. To the nearest kilogram, the mass is 6 kg.

Copy and complete this statement.

$\square \leqslant m < \square$

5. The length of a pencil is l cm. To the nearest centimetre, the length is 9 cm.

Copy and complete this statement.

$\square \leqslant l < \square$

6. Each of these measurements is rounded to 1 significant figure. Write down the lower and upper bounds for each measurement.

a 3 m **b** 60 m **c** 0.4 mg **d** 0.07 km

7. Each of these measurements is rounded to 2 significant figures. Write down the lower and upper bounds for each measurement.

a 24 ml **b** 360 g **c** 0.83 kg **d** 0.019 m

8. Copy and complete this table.

	Number or quantity	Lower bound	Upper bound
a	4 (to nearest whole number)		
b	70 (to nearest 10)		
c	600 (to nearest 10)		
d	0.3 (to 1 decimal place)		
e	0.06 (to 2 decimal places)		
f	80 km (to 1 significant figure)		
g	68 mg (to 2 significant figures)		
h	0.032 (to 2 significant figures)		

9. The capacity of a pot of paint is 300 ml, to the nearest 10 ml. Thabo buys five pots. The total volume of paint is V ml.

Copy and complete this statement.

$\square \leqslant V < \square$

10. Mel runs 8 km, to the nearest kilometre, every day. What is the least possible distance she runs in a week?

(11) Ann draws a square with a side length of 14 cm, to the nearest centimetre. The area of the square is A cm^2.

Copy and complete this statement.

$\square \leqslant A < \square$

(12) A bag of flour weighs 250 g, to the nearest 10 g. Val needs 740 g of flour for a recipe. Will three bags of flour definitely be enough? Explain your answer.

(13) A rectangle has length 80 m, to the nearest 10 m, and width 40 m, to the nearest 5 m. Its perimeter is p m and its area is A m^2.

Copy and complete these statements.

a $\square \leqslant p < \square$

b $\square \leqslant A < \square$

(14) Which is the odd one out among these statements about a length l m?

A $l = 50$ to the nearest 5	**B** $47.5 \leqslant l < 52.5$
C $l = 50 \pm 5$	**D** The upper and lower bounds of l are 52.5 and 47.5.

(15) The number n is 680, correct to 2 significant figures.

Copy and complete these statements about n.

a $n = 680 \pm \square$

b $\square \leqslant n < \square$

c The upper and lower bounds of n are $\square$ and $\square$ respectively.

d n is 680 to the nearest $\square$.

Problem solving exercise

(1) The measurements of this photograph are accurate to the nearest centimetre.

Jo has 100 photographs of this size to stick in her photograph album. The measurements of each page of the album are exactly 38 cm by 18.5 cm. There are nine empty pages in Jo's album.

Is there definitely enough space in Jo's photograph album for these 100 photographs without overlapping?

② The following people want to travel in a lift.

David 65 kg	Brian 92 kg	Bronwen 74 kg
Pat 54 kg	Peter 86 kg	Bruce 95 kg
Ahmed 89 kg	Mark 93 kg	

Lift
Maximum safe load
8 persons or
650 kg

a Explain why it might not be safe for these people to travel together in the lift.

b Eight different people get in the lift. They all have the same mass to the nearest kilogram. What is the largest that this mass can safely be?

③ A particular paperback book is 2.6 cm thick, measured to the nearest tenth of a centimetre. Tom has 50 of these paperback books. His bookshelf is 1.30 m in length to the nearest centimetre.

What is the greatest possible number of these books Tom can definitely put on his bookshelf?

Do I know it now?

① Write down the lower and upper bounds of each measurement.

a 40 ml (to the nearest ml) **b** 40 ml (to the nearest 10 ml)

c 700 kg (to the nearest 100 kg) **d** 700 kg (to the nearest 10 kg)

② Write down the lower and upper bounds of each measurement.

a 650 m (to the nearest m) **b** 650 m (to the nearest 10 m)

c 650 m (to the nearest 50 m) **d** 650 m (to the nearest 5 m)

③ The mass of a fish is m g. To the nearest 10 g, the mass is 320 g.

Copy and complete this statement.

$\square \leqslant m < \square$

④ Each of these measurements is rounded to 1 significant figure. Write down the lower and upper bounds for each measurement.

a 900 cm **b** 2000 km **c** 0.2 g **d** 0.005 m

⑤ Each of these measurements is rounded to 2 significant figures. Write down the lower and upper bounds for each one.

a 7100 m **b** 49 cm **c** 520 mm **d** 0.0028 km

Can I apply it now?

① Eve has made a square cake with sides of length 20 cm to the nearest centimetre. She wants to put a ribbon round the sides. Her piece of ribbon is 80 cm to the nearest centimetre. Does she definitely have enough ribbon for the cake? Explain your answer.

CHAPTER 6

NEXT STEPS – NUMBER

Ratio and proportion

6.3 The constant of proportionality

What you need to know

A **conversion graph** shows how one value is related to another.

It can help you to convert between two units easily.

Direct proportion means there is a connection between two variables: as one variable increases/decreases the other increases/decreases at the same rate.

For example, if one variable is trebled, the other will also be trebled.

The sign $\propto$ means 'is proportional to' and can be used to help create a formula.

If $a \propto b \quad a = kb$, where k is the constant of proportionality.

So $k = a \div b$

How to do it

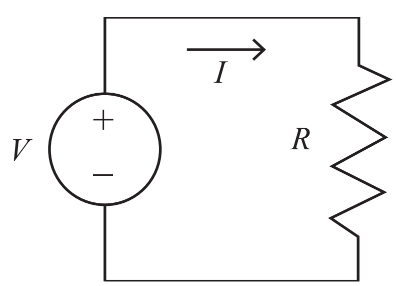
Creating a formula

In physics, the current (I) in an electric circuit is directly proportional to the voltage (V) when the resistance (R) is constant.

In one circuit, when $I = 0.2$, $V = 10$.

a Write a formula connecting I and V.

b Find the value of I when $V = 100$.



Solution

a $I \propto V$ so $I = kV$

k is the constant of proportionality

When $I = 0.2$, $V = 10$

$0.2 = k \times 10$

$k = 0.2 \div 10$

$k = 0.02$

So the formula is $I = 0.02V$.

b When $V = 100$,

$I = 0.02V$

$I = 0.02 \times 100$

$I = 2$

➤ Drawing a conversion graph

One day, £6 is worth the same as €9.

a Draw a conversion graph between pounds (£) and euros (€).

b An ice cream costs £1.40.

How much is this in euros?

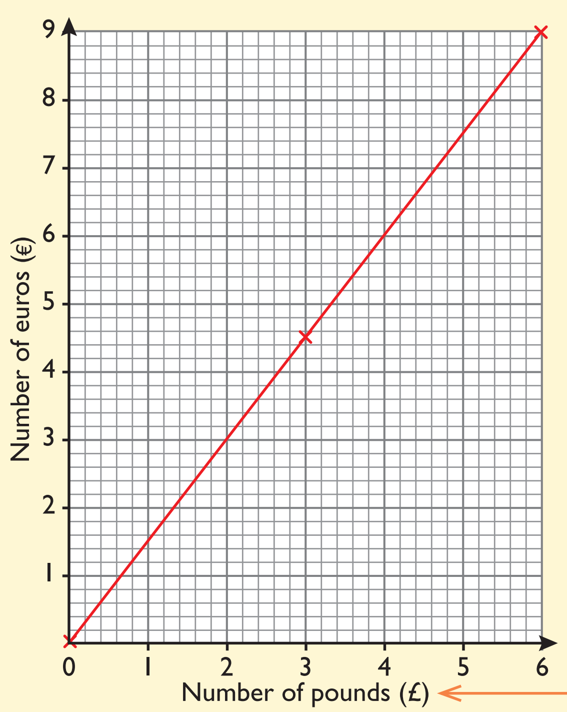
Solution

a You need two points to plot the graph. However, it is good practice to use at least one more point as a check.

Draw up and complete a table.

£	6	0	3
€	9	0	4.5

This is the check point.



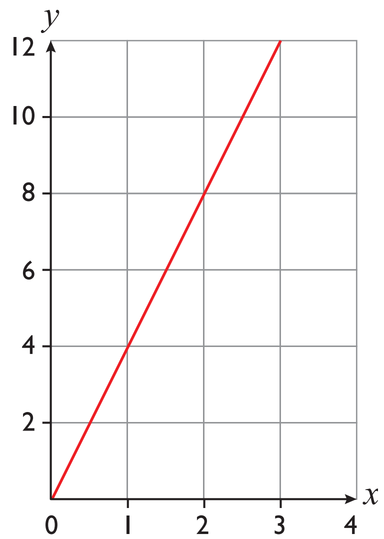
Label the axes.

b Reading from the graph, £1.40 = €2.10.

Learning exercise

① **a** Copy and complete the table of values. Read the values from the graph.

x	1		2.5	
y		12		6



b Find the value of k in the equation $y = kx$ connecting y and x.

c Copy and complete these statements.

i The straight-line graph shows that y is directly ____ to x.

ii k is called the ____ of ____.

② For each part, write down a formula involving a constant k.

a m is directly proportional to n.

b $y \propto x$

c A is directly proportional to B.

d P varies as Q.

e $T \propto d$

f C is directly proportional to d.

Note that people often shorten statements such as 'y is directly proportional to x' to 'y is proportional to x'.

③ y is (directly) proportional to x. When $x = 2$, $y = 10$.

a Write down the value of y when

i $x = 4$ **ii** $x = 6$ **iii** $x = 10$ **iv** $x = 1$.

b Write down the constant of proportionality.

c Write down a formula connecting y and x.

④ D is proportional to w. When $w = 4$, $D = 12$.

a Write down the value of D when

i $w = 8$ **ii** $w = 12$ **iii** $w = 1$ **iv** $w = \frac{1}{3}$.

b Write down the constant of proportionality.

c Write down a formula connecting D and w.

⑤ V is proportional to n.

a Write down a formula involving the constant k.

When $n = 6$, $V = 3$.

b Write down the value of k.

c Work out the value of V when $n = 2$.

⑥ y is proportional to x.

When $x = 3$, $y = 30$.

a Write down a formula connecting y and x.

b Find the value of y when $x = 5$.

c Find the value of x when $y = 12$.

⑦ C is proportional to d.

When $d = 4$, $C = 10$.

a Write down a formula connecting C and d.

b Find the value of C when $d = 6$.

c Find the value of d when $C = 25$.

⑧ Write down the odd one out in each set.

a $P = 3L$ $P = 15$ when $L = 5$ $P = 27$ when $L = 7$
P is directly proportional to L $P \propto L$

b $A = 4B$ $A \propto B$ $A = 12$ when $B = 2$ A varies as B
$A = 3$ when $B = 0.5$

c $M \propto N$ $M = 8$ when $N = 4$ $M = 4$ when $N = 8$
M is proportional to N $M = 0.5N$

⑨ In each part, decide if the statement is true or false. Explain how you know.

a $V \propto d$ means V is proportional to d.

b y is proportional to x means x is proportional to y.

c $T = 2n$ is the same as $n = 2T$.

⑩ M is proportional to e.

$M = 2$ when $e = 8$.

Find the value of e when $M = 1.5$.

⑪ x and y are variables and are directly proportional to each other. Blaise and Bobby are given data to work out the formula connecting x and y.

Blaise works it out to be $y = 2x$. Bobby works it out to be $x = \frac{1}{2}y$.

Can they both be correct? Explain your answer.

⑫ V is proportional to x. Copy and complete the table of values.

x	8	12	
V	10		45

⑬ T is proportional to n. There is one error in the table of values. What is it?

n	14	6	10
T	60	27	45

⑭ Seth wants to know if h and t are proportional. He draws a table for some values of h and t.

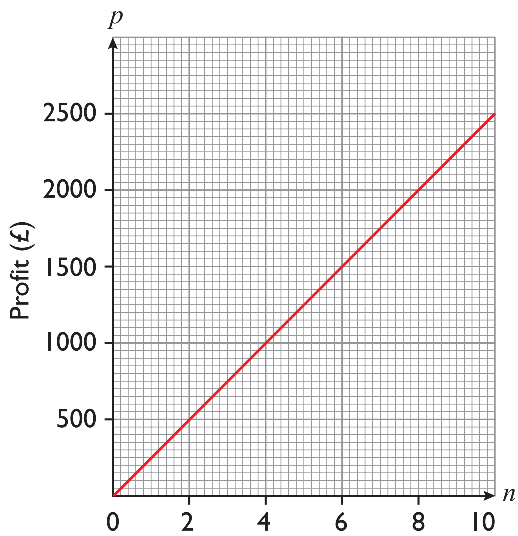
h	51	96	105
t	34	64	70

Are h and t proportional? Explain how you know.

Problem solving exercise

① Tracey sells paintings in an art exhibition.

The graph shows the profit, £p, that she makes when she sells n paintings.



a Write down a formula connecting p and n.

b Tracey has just booked a holiday costing £3500.

How many paintings does she need to sell to pay for the holiday?

② A ship travels s kilometres in t hours.

s is directly proportional to t.

a What does this tell you about the movement of the ship?

After 30 minutes, the ship has travelled 10 km.

b Find the value of the constant of proportionality.

c How far has the ship travelled after 75 minutes?

③ Clear plastic bottles can be recycled. The value, £V, of the bottles is in direct proportion to their mass, m g.

1 tonne of clear plastic has a value of £260.

Ann collects clear plastic bottles that each weigh 40 g.

She wants to buy a game for her computer that costs £25.99.

How many bottles does Ann need to collect and recycle before she has enough money to buy this game?

Do I know it now?

① t is proportional to d.

When $d = 5$, $t = 35$.

a Find the value of t when

i $d = 10$ **ii** $d = 15$ **iii** $d = 20$.

b Find the constant of proportionality.

c Write down the formula connecting t and d.

② y is proportional to x.

When $x = 4$, $y = 24$.

a Write down a formula connecting y and x.

b Find the value of y when $x = 7$.

c Find the value of x when $y = 30$.

③ s is directly proportional to t. The constant of proportionality is 70.

Copy and complete this table.

s		210	35		
t	1			8	$\frac{1}{10}$

Can I apply it now?

① Michaela wants to paint the walls in her bedroom.

Their area is 75 m². The paint costs £6.80 a litre and 2 litres of paint cover 30 m².

Write down a formula connecting the area of wall, A m², and the number of litres of paint needed, L. Use it to work out the cost of painting the walls in Michaela's bedroom.

6.4 Working with inversely proportional quantities

What you need to know

Inverse proportion means there is a connection between two variables: as one variable increases, the other decreases by the same proportion.

For example, if one variable doubles, the other will halve.

We represent inverse proportion using the **reciprocal**. Where a and b are inversely proportional, ab is a constant.

$a \propto \frac{1}{b}$ means a is inversely proportional to b.

So $ab = k$ where k is a constant.

Did you know?

It takes three men four hours to repair this road. Doubling the number of workers should halve the amount of time it takes to complete the job.

How to do it

Solving problems involving inverse proportion

A company is testing four electric cars on a journey from Edinburgh to London. All the cars travel the same route.

a Car A travels for 6 hours at an average speed of 60 miles per hour.

How long is the route from Edinburgh to London?

b Car B travels at an average speed of 30 miles per hour.

How long does the journey take Car B?

A formula connecting the speed (s) and the time (t) taken for this journey is

$st = 360$

c Car C took 9 hours to complete the journey.

Use the formula to find its average speed.

d The average speed of Car D was 50 miles per hour.

Use the formula to calculate the time taken.

Solution

a Car A travels 60 miles in 1 hour so in 6 hours the car travels $60 \times 6 = 360$ miles.

The route from Edinburgh to London is 360 miles.

b At 30 mph the journey takes $360 \div 30 = 12$ hours. Notice that this is a case of inverse proportion. Travelling at half the speed takes twice as long. ← 2×6 hours $= 12$ hours

c $st = 360$

$s \times 9 = 360$ ← Car C takes 9 hours.

$s = 360 \div 9$ ← Dividing both sides by 9.

$s = 40$

Car C travelled at an average speed of 40 miles per hour.

d $st = 360$

$50 \times t = 360$ ← Average speed was 50 mph.

$t = 360 \div 50$ ← Dividing both sides by 50.

$t = 7.2$

0.2 hours $= 0.2 \times 60 = 12$ minutes

Car D takes 7 hours 12 minutes to complete the journey.

➤ Looking for inversely proportional relationships

The table shows values for two variables, c and d.

c	8	12	24
d	30	20	10

Martin thinks that $c \propto \frac{1}{d}$.

Is Martin correct?

Explain how you know.

Solution

If c and d are inversely proportional, then $cd = k$. ← This is the same as $c = k \times \frac{1}{d}$ or $d = k \times \frac{1}{c}$.

$8 \times 30 = 240$ $12 \times 20 = 240$ $24 \times 10 = 240$

The answer is 240 in each case, so c and d are indeed inversely proportional.

Learning exercise

① Write down a formula involving the constant k for each of the following.

a y is proportional to x.

b y is inversely proportional to m.

c T varies as d.

d M varies inversely as t.

e $W \propto \frac{1}{x}$

f $C \propto d$

② Look at the formulae in the box. In each formula, the letter k is a constant and the rest of the letters used are variables.

$A = kd$	$M = \frac{k}{n}$	$Ct = k$	$\frac{w}{r} = k$

a Write down the formulae that show direct proportion.

b Write down the formulae that show inverse proportion.

③ y is inversely proportional to x.

a Write down a formula involving the constant k that connects y and x.

When $x = 2$, $y = 5$.

b Work out the value of k.

c Find the value of y when

i $x = 5$ **ii** $x = 10$.

④ C is inversely proportional to d.

a Write down a formula involving the constant k that connects C and d.

When $d = 5$, $C = 5$.

b Work out the value of k.

c Write down the value of C when

i $d = 4$ **ii** $d = \frac{1}{4}$ **iii** $d = 100$ **iv** $d = \frac{1}{100}$.

d Write down the value of d when

i $C = 25$ **ii** $C = \frac{1}{25}$ **iii** $C = 0.1$ **iv** $C = 10$.

⑤ M is inversely proportional to t.

$M = 8$ when $t = 3$.

a Write down a formula connecting M and t.

b Use your formula to find the value of M when $t = 10$.

c Use your formula to find the value of t when $M = 6$.

⑥ E is inversely proportional to h.

$E = 12$ when $h = 3$.

a Write down a formula connecting E and h.

b Find the value of E when $h = 12$.

c Find the value of h when $E = 150$.

⑦ Say whether each statement is true or false.

a y is inversely proportional to x means $y \propto \frac{1}{x}$.

b m is inversely proportional to d means $m = kd$, where k is a constant.

c T is inversely proportional to C means as T increases, C also increases at the same rate.

d W is inversely proportional to g means $W = \frac{k}{g}$, where k is a constant.

e P is inversely proportional to Q means $PQ = k$, where k is a constant.

8 y is inversely proportional to x.

$y = 12$ when $x = 3$.

a Write down a formula connecting y and x.

b Find the value of y when $x = 4$.

c Is it possible for x and y to have the same value?

9 In an electrical circuit the current, I amps, is inversely proportional to the resistance, R ohms. When $R = 24$, $I = 0.5$.

a Write down a formula connecting I and R.

b Find the value of I when $R = 6$.

c Find the value of R when $I = 0.1$.

10 A group of m people share the chocolates in a box. Each person gets n chocolates.

a Copy and complete this table showing some possible values of m and n.

m	6		16
n	8	2	

b Write down a formula connecting m and n.

c Describe the relationship between m and n in words.

11 C is inversely proportional to m. There is one error in the table of values. What is it?

C	8	400	10
m	40	0.6	32

12 Levi thinks h and d are directly proportional. Luke thinks h and d are inversely proportional.

Who is correct? Explain your answer.

h	4	0.8	32
d	10	2	80

13 y is directly proportional to x. $y = 40$ when $x = 5$.

x is inversely proportional to z. $z = 2$ when $x = 5$.

Find the value of y when $z = 40$.

Problem solving exercise

1 A farmer needs to build a rectangular fence around part of his field.

The length is l metres and the width is w metres.

The fenced-off section needs to have an area of 100 square metres.

a Make a table of some possible values for l and w.

b What is the minimum length of fencing needed?

② **a** Copy and complete the table of values for the curve $y = \frac{10}{x}$.

x	1	2	2.5	4	5	10
y	10					

b Draw the graph. Use equal scales for x and y.

c What are the values of y when

i $x = 0.5$ **ii** $x = 0.1$?

d Describe the shape of the graph.

e Do all inverse proportion graphs look like this?

Do I know it now?

① E is inversely proportional to T.

a Write down a formula involving the constant k that connects E and T.

When $T = 2.5$, $E = 4$.

b Work out the value of k.

c Work out the value of E when $T = 2$.

d Work out the value of T when $E = 20$.

② v is inversely proportional to t.

When $t = 4$, $v = 60$.

a Find a formula connecting v and t.

b Use your formula to find the value of v when $t = 3$.

c Use your formula to find the value of t when $v = 50$.

Can I apply it now?

① Scott is planning a journey between two places in the Antarctic. He knows it will be slow, but he doesn't know how slow. He says, 'It will take me 600 hours if I travel at $\frac{1}{2}$ a kilometre per hour.'

a Write down a formula that gives Scott's journey time, t hours, when his speed is v kilometres per hour.

b Use your formula to find how long the journey will take if Scott's speed is

i 2 km per hour **ii** $\frac{1}{3}$ km per hour.

c The journey actually takes Scott 200 hours. At what speed does he travel?

CHAPTER 7

NEXT STEPS – NUMBER

Number properties

7.3 Rules of indices

What you need to know

You can use the rules of indices when the base number is the same for all the numbers.

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$(a^m)^n = a^{mn}$

Powers are **repeated multiplication**.

Looking at the pattern helps you to understand what happens if the power is negative.

Multiply by 4 each time

$4^3 = 4 \times 4 \times 4 = 64$

$4^2 = 4 \times 4 = 16$

$4^1 = 4$

$4^0 = 1$

$4^{-1} = \left(\frac{1}{4}\right)^1 = \frac{1}{4}$

$4^{-2} = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$

$4^{-3} = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$

Divide by 4 each time

It is helpful to know common square and cube numbers.

$1^2 = 1$	$11^2 = 121$	$1^3 = 1$
$2^2 = 4$	$12^2 = 144$	$2^3 = 8$
$3^2 = 9$	$13^2 = 169$	$3^3 = 27$
$4^2 = 16$	$14^2 = 196$	$4^3 = 64$
$5^2 = 25$	$15^2 = 225$	$5^3 = 125$
$6^2 = 36$		$6^3 = 216$
$7^2 = 49$		$7^3 = 343$
$8^2 = 64$		$8^3 = 512$
$9^2 = 81$		$9^3 = 729$
$10^2 = 100$		$10^3 = 1000$

How to do it

➤ Using the rules of indices

Work out each of these. Give your answers using indices.

a $6^4 \times 6^9$ **b** $6^9 \div 6^4$ **c** $6^4 \div 6^9$ **d** $(6^4)^9$

e How many times bigger is your answer to **b** than your answer to **c**?

Solution

a $6^4 \times 6^9 = 6^{(4+9)}$
$= 6^{13}$

b $6^9 \div 6^4 = 6^{(9-4)}$
$= 6^5$

c $6^4 \div 6^9 = 6^{(4-9)}$
$= 6^{-5}$

d $(6^4)^9 = 6^{(4\times 9)}$
$= 6^{36}$

e $6^5 \div 6^{-5} = 6^{(5-(-5))}$
$= 6^{10}$

6^5 is 6^{10} times bigger than 6^{-5}.

Learning exercise

① **a** Copy and complete this table.

b Write down the value of

i 2^7 **ii** 2^{-4}.

Index form	In full	Ordinary number
2^5	$2 \times 2 \times 2 \times 2 \times 2$	32
2^4	$2 \times 2 \times 2 \times 2$	16
2^3		
2^2		
2^1		
2^0		1
2^{-1}	$\frac{1}{2}$	$\frac{1}{2}$
2^{-2}	$\frac{1}{2 \times 2}$	
2^{-3}	$\frac{1}{2 \times 2 \times 2}$	

② **a** Copy and complete this table.

b Write 10^6 as a number and in words.

c Write 10^{-6} as a decimal and in words.

Index form	In full	Ordinary number	In words
10^3	$10 \times 10 \times 10$	1000	One thousand
10^2			
10^1			
10^0		1	
10^{-1}	$\frac{1}{10}$	$\frac{1}{10}$	
10^{-2}	$\frac{1}{10 \times 10}$		
10^{-3}			One thousandth

③ Multiply the following. Give your answers both in index form and as ordinary numbers.

a $2^3 \times 2^3$ **b** $3^4 \times 3^2$ **c** 5×5^2 **d** $5^5 \times 5^{-2}$

④ Niamh is working out $3^5 \div 3^2$.

This is what she writes.

$3^5 \div 3^2$

$\dfrac{3\times3\times3\times\cancel{3}\times\cancel{3}}{\cancel{3}\times\cancel{3}}$

3^3 Answer

Check

$243 \div 9 = 27$

$3^3 = 27$ ✓

a Use Niamh's style to write out the answers to these calculations.

i $3^6 \div 3^4$
ii $5^4 \div 5^3$
iii $10^5 \div 10^2$

b Copy and complete the rule for dividing numbers in index form.

$a^m \div a^n = \square$

⑤ Work out these divisions. Give your answers both in index form and as ordinary numbers.

a $2^6 \div 2^3$ **b** $3^4 \div 3^3$ **c** $10^6 \div 10^3$ **d** $5 \div 5^{-2}$

⑥ Sanjay is working out $(7^2)^3$.

This is what he writes.

$(7^2)^3$

$7\times7 \times 7\times7 \times 7\times7$

7^6 Answer

I used my calculator to check.
Both 49^3 and 7^6 are the same.
They are 117649 ✓

a Use Sanjay's style to work these out.

i $(2^4)^3$ **ii** $(3^2)^5$
iii $(10^3)^2$

b State a rule for simplifying numbers in the form $(a^m)^n$.

⑦ Simplify the following. Give your answers both in index form and as ordinary numbers.

a $(2^2)^5$ **b** $(2^5)^2$ **c** $(10^3)^4$ **d** $(10^2)^6$

⑧ Work out the following. Give your answers in index form.

a $3^2 \times 3^4 \times 3^7$ **b** $2^3 \times 2^4 \times 2^5$ **c** $10 \times 10^2 \times 10^3$ **d** $5^4 \times 5 \times 5$

⑨ Work out the following. Give your answers in index form.

a $6^7 \times 6^4 \div 6^2$ **b** $6^7 \times 6^4 \times 6^{-2}$ **c** $(6^7 \times 6^4)^2$ **d** $(6^{-7} \times 6^{-4} \div 6^{-2})^{-1}$

⑩ Work out the following. Give your answers in index form.

a $\dfrac{3^6 \times 3^7}{3^8}$ **b** $3^6 \times 3^7 \div 3^8$ **c** $\dfrac{3^3 \times 3^{10}}{3^4 \times 3^4}$ **d** $\dfrac{3^3 \times (3^2)^5}{(3^2)^4}$

Do I know it now?

① Work out the following. Give your answers in index form.

a $5^2 \times 5^3 \div 5^4$ **b** $\dfrac{5^2 \times 5^3}{5^4}$ **c** $5^2 \times 5^3 \times 5^{-4}$

② Work out the following. Give your answers in index form.

a $\dfrac{10^3 \times 10^3}{10^8}$ **b** $10^2 \times 10^2 \times 10^3 \times 10^{-8}$ **c** $\dfrac{(10^2)^3}{(10^2)^4}$

③ Work out the following. Give your answers in index form.

a $(2^3)^4 \div 2^{10}$ **b** $(7^3)^5 \times (7^{-2})^7$ **c** $\dfrac{(10^{-6})^2}{(10^{-3})^4}$

④ Use the rules of indices to show that $23^0 = 1$.

CHAPTER 8

NEXT STEPS – ALGEBRA

Starting algebra

8.3 Simplifying harder expressions

What you need to know

Did you know?

Computer game designers use formulae to control movement within the game. A formula is used to guide the path of everything from kicking a ball to the flow of water.

These are the laws of indices.

$a^n = \underbrace{a \times a \times a \times \ldots \times a}_{n \text{ factors of } a}$ So $a^5 = \underbrace{a \times a \times a \times a \times a}_{5 \text{ factors of } a}$

$a^1 = a$

$a^0 = 1$

$a^x \times a^y = a^{x+y}$ So $a^5 \times a^7 = a^{5+7} = a^{12}$

$a^x \div a^y = a^{x-y}$ So $a^9 \div a^4 = a^{9-4} = a^5$

$(a^x)^y = a^{x \times y}$ So $(a^5)^3 = a^{5 \times 3} = a^{15}$

When you expand a pair of brackets, you multiply every term in the second bracket by every term in the first bracket then simplify your answer.

$$\begin{aligned}(x + 5)(x - 3) &= x \times x + x \times (-3) + 5 \times x + 5 \times (-3) \\ &= x^2 + (-3x) + (+5x) + (-15) \\ &= x^2 + \quad 2x \quad - 15 \\ &= x^2 + 2x - 15\end{aligned}$$

Alternatively, you can use a table.

	x	$+5$
x	x^2	$5x$
-3	$-3x$	-15

$x^2 + 5x - 3x - 15$

$= x^2 + 2x - 15$

How to do it

Simplify harder expressions

Simplify these expressions.

a $2a^3b^7 \times 4ab^5$ **b** $\dfrac{12a^4b^3}{4ab^2}$

Solution

a Use the rules of indices: $a^x \times a^y = a^{x+y}$

$2 \times 4 = 8$

$a^3 \times a = a^3 \times a^1 = a^{3+1} = a^4$

$b^7 \times b^5 = b^{7+5} = b^{12}$

So, $2a^3b^7 \times 4ab^5 = 8a^4b^{12}$

b Use the rules of indices: $a^x \div a^y = a^{x-y}$

$12 \div 4 = 3$

$a^4 \div a = a^4 \div a^1 = a^{4-1} = a^3$

$b^3 \div b^2 = b^{3-2} = b^1 = b$

So, $\dfrac{12a^4b^3}{4ab^2} = 3a^3b$

Expanding a pair of brackets

Expand $(x - 4)(x - 2)$.

Solution

You can use a grid to help you.

	x	-4
x	x^2	$-4x$
-2	$-2x$	8

Write the contents of one bracket along the top and the other down the side, then multiply at each cross-section.

$(x - 4)(x - 2) = x^2 - 4x - 2x + 8$ Then add the results.

$= x^2 - 6x + 8$

Learning exercise

① Write these as single powers.

a $f \times f$

b $g \times g \times g \times g$

c $(d \times d \times d)^2$

d $(a \times a \times a \times a \times a \times a \times a \times a)^3$

② Write each of these in full and work out its value.

a $x^2 \times x^3$ **b** $x^3 \times x^2$ **c** $x^3 \div x^2$

d $x^2 \div x^3$ **e** $(x^2)^3$ **f** $(x^3)^2$

③ Simplify these expressions.

a $5a^3 \times 4a^2$ **b** $6b^5 \div 3b^4$ **c** $2c^8 \times 3c^6$

d $10d^7 \times 3d$ **e** $(2e^2)^3 \div 4e^5$ **f** $4f^3g^2 \times 2f^2g^6$

g $8m^3p \times 3mp^4$ **h** $10s^8t^5 \div 5s^5t^5$

④ Simplify these expressions.

a $\dfrac{12a^5}{4a^3}$ **b** $20c^{12} \div (5 \times c^5 \times c)$

c $\dfrac{15(d^3)^3}{3d}$ **d** $6(ef^2)^3 \div (2 \times e^2 \times ef)$

⑤ **a** **i** Copy and complete for the expansion of $(x + 2)(x + 4)$.

$\times$	x	4
x	x^2	
2		

ii Now add the four terms together and simplify your answer.

b Copy and complete these grids and add the terms together. Simplify your answers.

i $(x + 3)(x + 5)$

$\times$	x	5
x		
3		

ii $(x + 6)(x - 4)$

$\times$	x	-4
x		
6		

iii $(x - 5)(x - 7)$

$\times$	x	-7
x		
-5		

⑥ Match to give six pairs of equivalent expressions.

n^2	$n^4 \times n^2$	n^8	$n^3 \times n^2$	$n^3 \div n$	$(n^5)^2$
$n^{12} \div n^4$	n^3	$n^6 \div n^3$	n^5	n^{10}	n^6

⑦ Expand these expressions and simplify your answers.

a $(x + 2)(x + 9)$ **b** $(x + 5)(x - 3)$ **c** $(x - 4)(x + 1)$ **d** $(x - 2)(x - 5)$

⑧ **a** Tim and Harry have expanded the brackets $(x + 5)^2$.

Tim says the answer is $x^2 + 10x + 25$ and Harry says it is $x^2 + 25$.

Who is correct? What has the other one done wrong?

b Expand these brackets and simplify the answers.

i $(a + 3)^2$ **ii** $(b + 6)^2$ **iii** $(c - 4)^2$

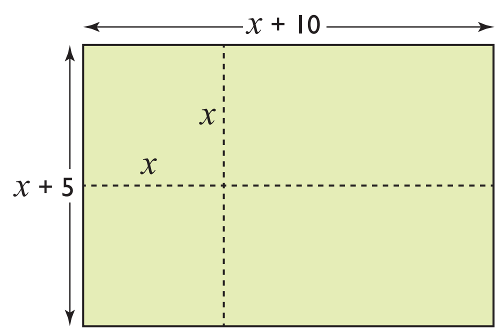
⑨ **a** Multiply $(x + 5)$ by $(x + 10)$.

b This rectangle has sides $(x + 5)$ and $(x + 10)$.

It is divided into four parts. One of them is a square of side x.

Copy the rectangle and mark the areas of the four parts on the rectangle.

c Explain the connection between your answers to parts **a** and **b**.



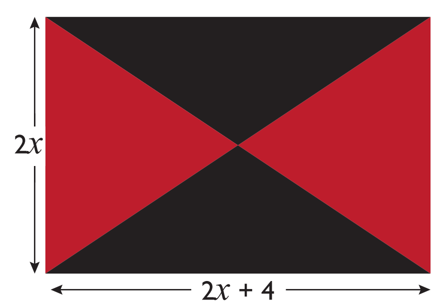
⑩ The sides of this rectangle are $2x + 4$ cm and $2x$ cm.

a Find, in terms of x

i the area of the whole rectangle

ii the area that is coloured red.

b Show that the red and black regions have the same area.

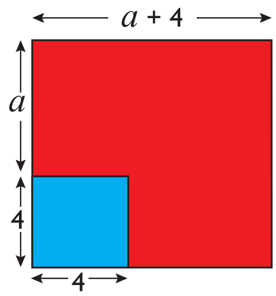


Problem solving exercise

① A farmer has a field in the shape of a rectangle.

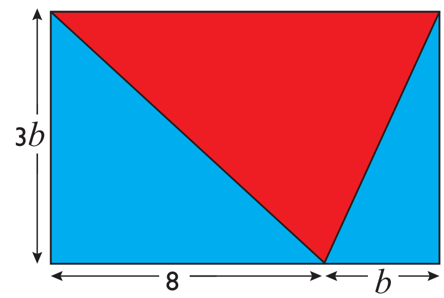
The length of the field is 20 m greater than the width. Write an expression, in terms of w, for the area of the field.

② Write down an expression, in terms of a, for the area shaded red.



③ Here is a rectangle.

Write down an expression, in terms of b, for the area shaded red.



④ Find the value of n to make these statements true.

a $a^4 \times a^n = a^{12}$ **b** $\dfrac{12p^9}{3p^n} = 4p^3$ **c** $(y^n)^4 = y^{12}$

Do I know it now?

① Write these expressions as single powers.

a $a^6 \times a^8$ **b** $b^{12} \div b^6$ **c** $c^{16} \div (c^4 \times c^{12})$ **d** $(d^9)^2$

② Simplify these expressions.

a $6a^4 \times 2a^3$ **b** $\dfrac{8b^{10}}{2b^5}$ **c** $\dfrac{3a^2b \times 2ab^2}{6(ab)^3}$ **d** $\dfrac{30g^{12}h^4}{6g^4h^4}$

③ Expand these expressions and simplify your answers.

a $(a + 5)(a + 6)$ **b** $(b - 3)(b + 7)$ **c** $(c - 5)^2$ **d** $(d - 3)(d - 5)$

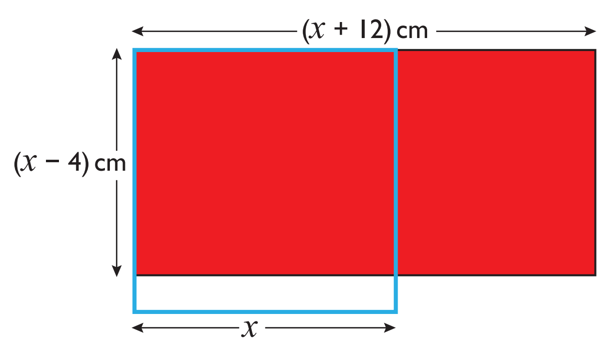
Can I apply it now?

① **a** Find the area of the red rectangle in terms of x.

The blue square has sides of x cm.

b Find the areas of each of the three regions in terms of x.

c Find the area of the whole figure in terms of x.



8.4 Using complex formulae

What you need to know

You can **substitute** numbers into formulae. This includes negative numbers and decimals.

$v^2 = u^2 + 2as$

Find the value of v when $u = -3$, $a = 9.8$ and $s = 20$.

$v^2 = (-3)^2 + 2 \times 9.8 \times 20$ ← Substitute the values given into the formula.

$v^2 = 9 + 392$

$v^2 = 401$ ← Square root both sides of the formula to find the value of v.

$v = +20.02$ or -20.02 (2 d.p.) ← Remember that there are two answers when you square root a number, one positive and one negative.

v^2 is the subject of the formula $v^2 = u^2 + 2as$.

You can **rearrange a formula** to make another letter the subject.

Use a method similar to solving an equation.

You must do the same thing to both sides of the formula to get the variable you want by itself on one side of the formula.

Make s the subject.

$$\begin{aligned} u^2 + 2as &= v^2 && -u^2 \\ 2as &= v^2 - u^2 && \div 2a \\ s &= \frac{v^2 - u^2}{2a} \end{aligned}$$

Make u the subject.

$$\begin{aligned} u^2 + 2as &= v^2 && -2as \\ u^2 &= v^2 - 2as && \text{square root} \\ u &= \pm\sqrt{v^2 - 2as} \end{aligned}$$

How to do it

Substituting negative numbers into a formula

$a = \dfrac{3b^2 - 6}{b + 4}$ Work out the value of a when $b = -2$.

Solution

$a = \dfrac{3b^2 - 6}{b + 4}$ ← Substitute $b = -2$ into the expression.

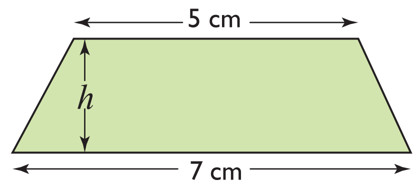
$= \dfrac{3 \times (-2)^2 - 6}{(-2) + 4}$ ← Note that $(-2)^2$ means -2×-2 which is equal to 4.

$= \dfrac{3 \times 4 - 6}{2}$

$= \dfrac{6}{2} = 3$

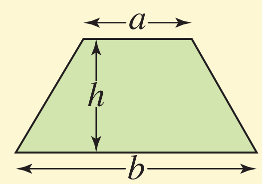
Working with formulae

The area of this trapezium is 48 cm^2.
Work out the height of the trapezium.



Solution

The formula for the area of a trapezium is



$A = \frac{1}{2}h(a + b)$

So $48 = \frac{1}{2}h(5 + 7)$ ← Substitute $A = 48$, $a = 5$ and $b = 7$ into the formula.

$\frac{1}{2}h(5 + 7) = 48$ ← It is usual to work with the variable on the LHS of the equation.

$\frac{1}{2}h \times 12 = 48$

$6h = 48$

$h = 8$

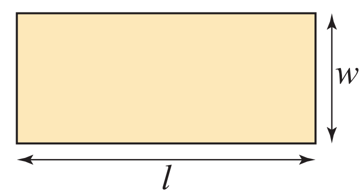
So the height of the trapezium is 8 cm.

Rearranging a formula

The formula for the perimeter of a rectangle is

$P = 2(l + w)$

Rearrange the formula to make l the subject.



Solution

Look at how Beth and David rearrange the formula:

Beth

$P = 2(l + w)$

It helps to swap the formula around first.

$2(l + w) = P$

Expand the brackets

$2l + 2w = P$

$-2w$ $\quad 2l = P - 2w \quad$ $-2w$

$\div 2$ $\quad l = \dfrac{P - 2w}{2} \quad$ $\div 2$

David

$P = 2(l + w)$

$2(l + w) = P$

$\div 2$ $\quad 2l + 2w = P \quad$ $\div 2$

$l + w = \dfrac{P}{2}$

$-w$ $\quad l = \dfrac{P}{2} - w \quad$ $-w$

You must do the same thing to both sides to keep the formula balanced.

Learning exercise

1. Find the value of $5a - 3b + 2c$ when

   a $a = 4, b = 6, c = 5$ **b** $a = 8, b = -3, c = -2$

   c $a = 5, b = -1, c = 6$ **d** $a = 0.5, b = 1.5, c = 2.5$.

2. Find the value of $3x^2 - 6x$ when

   a $x = 4$ **b** $x = 5$ **c** $x = -2$ **d** $x = -1$.

3. Make x the subject of each formula.

   a $y = x - 8$ **b** $y = 3x$ **c** $y = \frac{x}{5}$ **d** $y = 2x + 1$

4. Make the bold letter the subject of each formula.

   a $y = \boldsymbol{x} + 4$ **b** $y = 4\boldsymbol{x} - 3$

   c $a = 6\boldsymbol{b}$ **d** $p = \boldsymbol{mt}$

5. Work out the value of $4t^2 - 3w$ when

   a $t = 3, w = 4$ **b** $t = 5, w = -2$

   c $t = -4, w = 5$ **d** $t = -3, w = -6$.

6. Football teams use the formula $p = 3w + d$ to work out the number of points they have, where w is the number of wins, d is the number of draws and p is the number of points.

   a Work out the points teams with these results have.

      i 8 wins and 3 draws
      ii 10 wins and 2 draws
      iii 7 wins and 5 draws

   b Make w the subject of the formula.

   c A team has 20 points. It has drawn five matches. How many matches has it won?

7. A taxi driver uses the formula $c = 2p + 1.5m$ to work out the fares, where £c is the fare, p is the number of passengers and m miles is the distance travelled.

   a Work out the fares for these journeys.

      i 2 passengers and 6 miles
      ii 1 passenger and 8 miles
      iii 3 passengers and 10 miles

   b Make m the subject of this formula.

   c Find the length of a journey for three passengers that costs £10.50.

8. A plumber charges £35 per hour plus £18 call-out charge.

   a Write a formula for the cost of a job, £C, lasting h hours.

   b How much does he charge for a job lasting

      i 2 hours **ii** 6 hours **iii** 10 hours?

   c Rearrange your formula to make h the subject.

   d How long did the job last if it cost

      i £158 **ii** £438 **iii** £963?

⑨ The surface area of a sphere is given by the formula $S = 4\pi r^2$, where r is the radius of the sphere. Use the π key on your calculator for these calculations.

a Calculate the surface area of the spheres with radius

i 6 cm **ii** 12 m **iii** 400 km.

b The radius of the Earth is approximately 6400 km. Calculate its surface area.

c Rearrange the formula to make r the subject.

d Work out the radius of a marble with surface area 50.265 cm². Give your answer to 2 decimal places.

⑩ The formula to convert temperatures in Fahrenheit, °F, to degrees Celsius, °C, is $C = \frac{5}{9}(F - 32)$.

a Convert these Fahrenheit temperatures to degrees Celsius.

i 32 °F **ii** 95 °F **iii** 212 °F **iv** −40 °F

b Make F the subject of the formula.

c Find the value of F when

i $C = 0$ **ii** $C = 35$ **iii** $C = 300$ **iv** $C = -40$.

⑪ Make the bold letter the subject of each formula.

a $y = 5\boldsymbol{x} - 6$ **b** $y = 5(\boldsymbol{x} - 6)$ **c** $T = 4m\boldsymbol{p}$ **d** $T = m^2 + 4\boldsymbol{p}r$

⑫ Tara uses the formula $C = 30h + 15$ to work out how much she is going to charge a customer, £C, for working on a car for h hours.

a How much does she charge for working on a car for 5 hours?

b Make h the subject of the formula.

c Tara charges Tom £52.50 for working on his car.

For how long does Tara work on Tom's car?

⑬ The cost, £C, of hiring a cement mixer for d days is given by the formula $C = 12d + 20$. Rob hires a cement mixer for 7 days.

a How much does it cost?

b Make d the subject of the formula.

c Celia hires a cement mixer and pays £200.

For how many days did Celia hire the mixer?

⑭ The density, d, of a solid object is given by the formula $d = \frac{m}{V}$, where m is the mass of the object and V is its volume.

a Make m the subject of the formula.

b A packet of butter in the shape of a cuboid has dimensions 10 cm by 6 cm by 4 cm.

The density of the butter is 1.05 g/cm³.

Calculate the mass of the butter.

⑮ The volume, V, of tomato soup in a can is given by the formula $V = \pi r^2 h$, where r is the radius of the can and h is the height of the can.

a Make h the subject of the formula.

b The radius of the can is 3.5 cm and the volume of the soup in the can is 400 cm³. Find the height of the can.

Give your answer to the nearest 0.1 cm.

Do I know it now?

① Make the bold letter the subject of each formula.

a $d = \boldsymbol{e} + f$ **b** $s = \boldsymbol{a}t + b$ **c** $f = \frac{\boldsymbol{g}}{h}$ **d** $v = 4\boldsymbol{w} - 3$

e $C = 2\pi \boldsymbol{r}$ **f** $A = \pi \boldsymbol{r}^2$ **g** $P = \frac{Mtr}{100}$ **h** $S = 3\boldsymbol{t}^2$

② The final velocity, v, of particles in an experiment is given by the formula $v = u + at$, where a is the acceleration, u is the initial velocity and t is the time of travel.

a Find the value of v when

i $u = 4, a = 2, t = 3$ **ii** $u = -10, a = 5, t = 2$

iii $u = 20, a = -10, t = 4$ **iv** $u = 30, a = 0, t = 7$.

b Make a the subject of the formula.

c A particle accelerates from 10 m/s to 25 m/s in 5 seconds. Calculate its acceleration.

8.5 Identities

What you need to know

Did you know?

Almost every business uses spreadsheets to do some of its work. It may be for ordering goods, keeping track of stock or handling accounts. Formulae are used throughout to determine what calculations are done in the cells and which cells are accessed for the required data.

Here are two formulae: $F = \frac{9}{5}C + 32$ $u^2 = v^2 - 2as$

A **formula** is a rule for working something out. It shows the relationship between the variables.

These formulae both have three terms. A **term** is a single number or variable or a product of numbers and variables.

The six terms here are: F $\frac{9}{5}C$ 32 u^2 v^2 $2as$

An **expression** is a collection of terms, or a single term.

So each of the six terms is an expression, but so are $\frac{9}{5}C + 32$ and $v^2 - 2as$.

Expressions and terms also occur in **equations** and **identities**.

An **equation** can be solved to find an unknown quantity.

The equation $95 = \frac{9}{5}C + 32$ can be solved to find the value of C.

An **identity** is **always true**. In other words, it is true for all possible values of the variable.

$3(x + 2) \equiv 3x + 6$ is an identity.

Conventions for whole numbers

By convention, $\boldsymbol{n}$ represents a whole number. This means that:

$\mathbf{2}\boldsymbol{n}$ represents multiples of 2, or even numbers, and so does $\mathbf{2}\boldsymbol{n} + \mathbf{2}$.

$\mathbf{2}\boldsymbol{n} + \mathbf{1}$ or $\mathbf{2}\boldsymbol{n} - \mathbf{1}$ stands for one more or one less than a multiple of 2, so an odd number.

$\mathbf{3}\boldsymbol{n}$ represents a multiple of 3, and so on.

$\boldsymbol{n} + \mathbf{1}$ represents one more than n, so n and $n + 1$ are consecutive integers.

These expressions are useful when making an argument to prove a result.

How to do it

Using the vocabulary, understanding the concept

n stands for an integer (whole number) in each of these expressions.

$5n \qquad 3n + 2 \qquad 3n + 2n \qquad 2n + 2 \qquad 10n + 15 \qquad 4n + 1$

a Which of these expressions are always a multiple of 5?

b How do you know which expressions are a multiple of 5 and which expressions are not a multiple of 5?

Solution

a $5n$, $3n + 2n$ and $10n + 15$ are always multiples of 5.

b $5n = 5 \times n$ so is a multiple of 5 by definition.

$3n + 2n = 5n$ so is a multiple of 5.

$10n + 15 = 5(2n + 3)$ so is a multiple of 5.

It doesn't matter which value of n you use, the result will always be a multiple of 5.

When $n = 5$:

$3n + 2 = 3 \times 5 + 2 = 17$

$2n + 2 = 2 \times 5 + 2 = 12$

$4n + 1 = 4 \times 5 + 1 = 21$

To show something is not true, you only have to find one example that is not true.

So these expressions may equal multiples of 5 for some values of n, but not all.

Proof

a Write expressions for three consecutive integers.

b Prove that the sum of three consecutive integers is always a multiple of 3.

Solution

Consecutive means 'following each other' so 4, 5 and 6 are three consecutive numbers.

a There are several possible answers:

$n, n + 1, n + 2$ is the most often used.

$n - 1, n, n + 1$ is another possibility.

Call the first number n, the next number is 1 more than n and the third number is 2 more than n.

b Using the answers from **a**

sum $= n + n + 1 + n + 2$

$= 3n + 3$

$= 3(n + 1)$, which is a multiple of 3.

Factorising.

or

sum $= n - 1 + n + n + 1$

$= 3n$, which is a multiple of 3.

Learning exercise

1. Copy each statement below and complete it using the correct word from the box.

an expression	an identity	a term	an equation	a formula	the coefficient

a $5x$ is ____ of $x^2 + 5x + 7$.
b 5 is ____ of x in $x^2 + 5x + 7$.
c $5w - 7$ is ____.
d $2n$ can be ____ and ____.
e $3t - 5 = t + 7$ is ____.
f $s = 4t - 4.9t^2$ is ____.
g $a + b = b + a$ is ____.

2. Given that n is an integer, which term in the expression $n^2 + 2n - 1$ is always an even number? How do you know?

3. One of these is a formula, one is an equation and one is an identity.

$h = x^2 - 6x + 5$	$0 = x^2 - 6x + 5$	$(x - 5)(x - 1) = x^2 - 6x + 5$

Which is the equation and which is the identity? How do you know?

4. Show that each statement is true.

a $x + 5 + 3x - 6 = 4x - 1$
b $x(x + 1) - x - 1 = x^2 - 1$
c $4(x - 3) + 3(x + 7) = 7x + 9$

5. Given that n is an integer, match each expression to its description.

an odd number	a multiple of 3	a multiple of 7	a square number	an even number

a $2n$ **b** n^2 **c** $7n$ **d** $2n - 1$ **e** $4n - n$

6. $n^2 + 5$ is even for some values of n.

Choose the values from the list that make $n^2 + 5$ even.

1	6	9	15	24	99

7. Given that n is an integer, say whether these expressions are always odd, never odd or sometimes odd. Give examples to justify your answers.

a $n + 1$ **b** $2n – 1$ **c** $3n + 5$ **d** $2n + 1$ **e** $6n$

8. **a** Are the following statements true for all values of x, some values of x or no values of x?

i $5x - 2 = 2 - 5x$
ii $5x + 2 = 2 + 5x$
iii $x^2 + 5 = 9 + x^2$
iv $x \div 2 = \frac{1}{2}x$
v $x \div \frac{1}{2} = \frac{1}{2}x$
vi $x \div \frac{1}{2} = 2x$

b Which of the statements above are identities? Justify your answer.

9. Show that each statement is true.

a $5(x - 2) - 2(x + 3) = 3(2 - x) + 2(3x - 11)$
b $(x + 1)(x - 1) = x^2 - 1$
c $(3x + 1)(5 - 2x) = 1 - 6(x - 4)(x + 1) - 5(x + 4)$

10. Prove that each statement is true.

a The sum of two consecutive integers is always odd.
b The sum of five consecutive integers is always a multiple of 5.
c The sum of four consecutive integers is always a multiple of 2.

11 Three piles of stones are made up of a total of 45 stones.

The first pile has n stones.

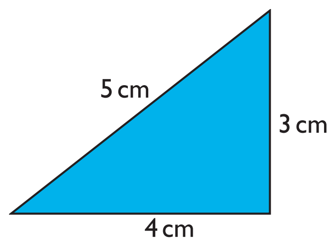
The second pile has twice as many stones as the first one.

The third pile has 5 more stones than the second pile.

a Write expressions for the number of stones in each pile.

b Use them to work out how many stones there are in each pile.

12 Look at this triangle.



Pythagoras' theorem tells you that the triangle is right-angled.

$3^2 + 4^2 = 5^2$

$(9 + 16 = 25)$

Here is a way to find other right-angled triangles with whole-number sides.

Step	**How to do it**
Write down an even number	4
Square it	16
Write down the numbers just above and below it	17 and 15
Double your original number	8

The last three numbers you wrote down are the sides of a right-angled triangle.

$8^2 + 15^2 = 17^2$

a Check that this works with the following starting numbers.

i 2 **ii** 6 **iii** 10 **iv** 1000

b Prove that it works with $2n$ as a starting number (where n is any positive whole number).

Do I know it now?

1 Some of the following statements are equations; others are identities.

i $2x + 6x = 2x + 24$

ii $2(x + 7) + 3(x + 2) = 5(x + 4)$

iii $x^2 - 8x - 1 = (x - 2)(x - 6) - 13$

iv $x(x - 2) - x^2 + 1 = (x - 1)^2 - x^2$

a Say if each statement is an equation or an identity.

b Solve the equations and show that the identities are true.

2 Say whether each statement is true or false and explain your answer.

a $(n + 1)^2 > n^2$ for all values of n (positive, zero or negative).

b An identity in x is true for any value of x.

c No square number ends in 2.

d $4n^2 = (2n)^2$

CHAPTER 9

NEXT STEPS – ALGEBRA

Sequences

9.3 Quadratic sequences

What you need to know

The sequence of **square numbers** begins 1, 4, 9, 16, 25, …

The differences between these terms are 3, 5, 7, 9, … and the difference between these differences is constant. In this case, it is 2.

Sequence	1		4		9		16		25 …
1st difference		+3		+5		+7		+9	
2nd difference			+2		+2		+2		

Such sequences are known as **quadratic sequences** and the expressions for the ***n*th term** always contain a term in n^2.

For example, the formula for the nth term of a simple quadratic sequence rule might be $n^2 + 1$, which would give 2, 5, 10, 17, 26, …

To find the rule for a quadratic sequence, look at the relationship between the square numbers (1, 4, 9, 16, 25) and the sequence itself.

Did you know?

The curve of a bridge like the one shown is described by a quadratic equation. The lengths of the vertical wires are terms in a quadratic sequence.

How to do it

Generating terms of a quadratic sequence

Find the fourth and sixth terms of the sequence whose nth term is given by $3n^2 - 8$.

Solution

When $n = 4$, $3n^2 - 8 = 3 \times 4^2 - 8$ ← Substitute 4 into the formula.

$= 3 \times 16 - 8$ ← Squaring first.

$= 48 - 8$ ← Then multiplication.

$= 40$

When $n = 6$, $3n^2 - 8 = 3 \times 6^2 - 8$ ← Substitute 6 into the formula.

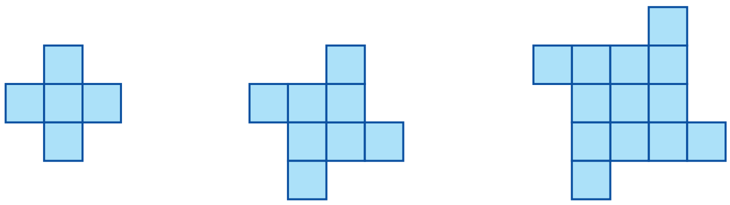
$= 3 \times 36 - 8$ ← Squaring first.

$= 108 - 8$ ← Then multiplication.

$= 100$

➤ Finding the nth term of a quadratic sequence

Here is a sequence of patterns.



Pattern 1 Pattern 2 Pattern 3

a Draw pattern 4.

b The number of squares in the patterns form a sequence.

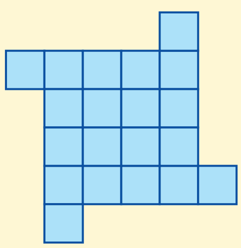
What are the first five terms of the sequence?

c Find the nth term of this sequence.

Explain how the patterns help you to find the nth term of the sequence.

Solution

a



Pattern 4

b From the diagrams, the first four terms are 5, 8, 13, 20.

Sequence	5		8		13		20 …
1st difference		+3		+5		+7	
2nd difference			+2		+2		

The next first difference will be 9, so the fifth term is 29.

c The first differences are not the same so the sequence is not linear.

The second differences are the same so this is a quadratic sequence.

Square numbers	1	4	9	16	25	← Start with the square numbers.
	↓+4	↓+4	↓+4	↓+4	↓+4	
This sequence	5	8	13	20	29	← To get the required sequence, add 4.

The nth term is therefore $n^2 + 4$.

Look at the pattern.

- The first pattern is a 1×1 square plus four single squares.
- The second pattern is a 2×2 square plus four single squares.
- The third pattern is a 3×3 square plus four single squares.

So the nth pattern will be an $n \times n$ square plus four single squares, giving $n^2 + 4$ squares in total.

Learning exercise

① Match each sequence with its position-to-term formula.

a	9, 8, 7, 6, 5, …	**i**	$n^2 + 10$
b	90, 80, 70, 60, 50, …	**ii**	$10 - n$
c	10, 40, 90, 160, 250, …	**iii**	$100 - 10n$
d	11, 14, 19, 26, 35, …	**iv**	$10n^2$

② For each sequence, write down the missing term and the position-to-term formula.

a 1, 4, 9, $\boxed{}$, 25

b 2, 5, 10, $\boxed{}$, 26

c 2, 8, $\boxed{}$, 32, 50

d 5, 11, 21, $\boxed{}$, 53

③ Match each sequence with its position-to-term formula.

a	10, 22, 42, 70, 106, …	**i**	$2n^2 + 2$
b	98, 92, 82, 68, 50, …	**ii**	$n^2 + 2$
c	4, 10, 20, 34, 52, …	**iii**	$4n^2 + 6$
d	3, 6, 11, 18, 27, …	**iv**	$5n^2 + 5$
e	10, 25, 50, 85, 130, …	**v**	$100 - 2n^2$

④ **a** Which of these sequences are quadratic?

i 5, 8, 13, 20, 29, …

ii 10, 20, 30, 40, 50, …

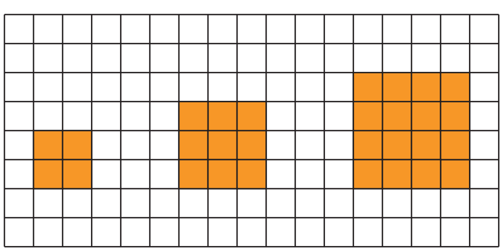
iii 3, 12, 27, 48, 75, …

iv 2, 16, 54, 128, 250, …

v 5, 11, 21, 35, 53, …

b For the quadratic sequences, write down the position-to-term formula.

⑤ Katrina is making a sequence of patterns from square tiles.



Pattern 1 Pattern 2 Pattern 3

a Draw the next pattern in the sequence.

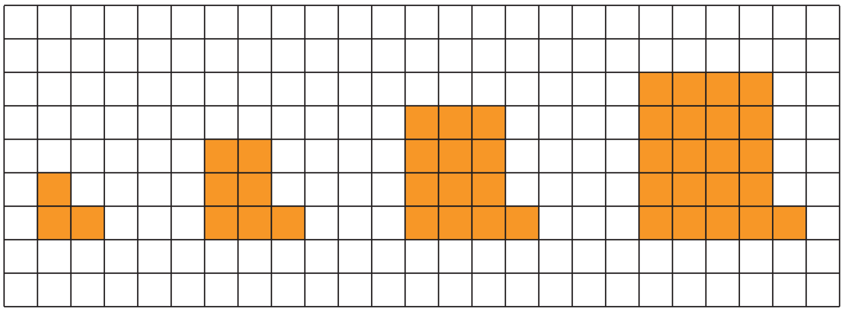
b Copy and complete the table.

Pattern	1	2	3	4	5
Number of tiles					

c Which pattern uses 100 tiles?

d How many tiles are there in the nth pattern?

Katrina removes some tiles from each pattern to make the following sequence of patterns.



Pattern 1 Pattern 2 Pattern 3 Pattern 4

e **i** How many tiles does Katrina remove from the 50th pattern?

ii How many tiles are left in the 50th pattern?

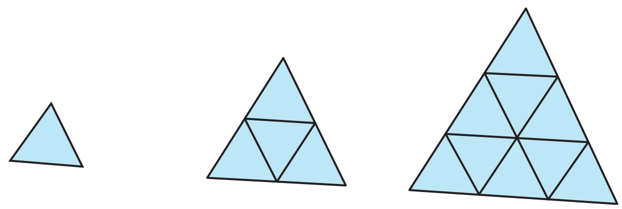
iii How many tiles are left in the nth pattern?

⑥ Here is a pattern made from triangular tiles.

a How many tiles would be needed for pattern number 8?

b How many tiles are needed for pattern number n?

c Which pattern has 100 tiles?



Pattern 1 Pattern 2 Pattern 3

⑦ Ben is stacking tins of baked beans.

a How many tins are in stack 5?

The 20th stack needs 210 tins.

b How many tins are needed for the 21st stack?

Ben has 120 tins to stack.

c How many tins should he place in the bottom row?

The nth stack has $\frac{1}{2}n(n + 1)$ tins.

d How many tins are in the 100th stack?

Comfort has 169 tins to stack.

e **i** Can she make one stack out of these tins?
Explain your answer fully.

ii Comfort uses all of the 169 tins to make 2 stacks.
How many tins are in each stack?

Stack 1 Stack 2 Stack 3

⑧ This Rubik's cube is made up of smaller cubes around a central mechanism.

a Write down how many of the smaller cubes have

i 1 sticker **ii** 2 stickers **iii** 3 stickers.

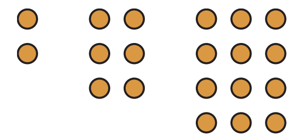
b How many of the smaller cubes have at least one sticker on them?

c You can get different-sized Rubik's cubes. The smallest is a 2 by 2 by 2 cube. You can also get larger cubes, such as a 5 by 5 by 5 cube.

Copy and complete this table for different-sized cubes.

Cube size	2	3	4	5	10	n
1 sticker						
2 stickers						
3 stickers						
Total number of stickered cubes						

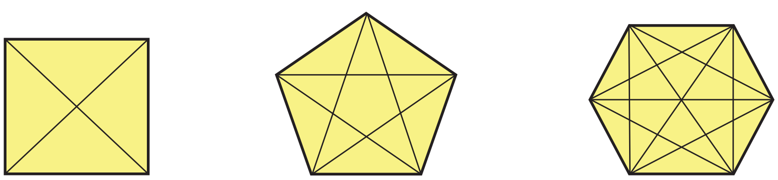
Problem solving exercise



1. Here are the first three shapes in a rectangular pattern made from dots.
   - **a** How many dots are there in pattern number 6?
   - **b** How many dots are in the nth pattern?
   - **c** Write an expression, in terms of n, for the difference in the number of dots between the pattern n and pattern $(n + 1)$.
   - **d** Between which two consecutive patterns is the difference in the number of dots 102?
2. The nth term of a quadratic sequence is $n^2 + 8$.

   The nth term of a different quadratic sequence is $49 - n^2$.

   Which numbers are in both sequences?
3. The diagrams show the numbers of diagonals in some regular polygons.



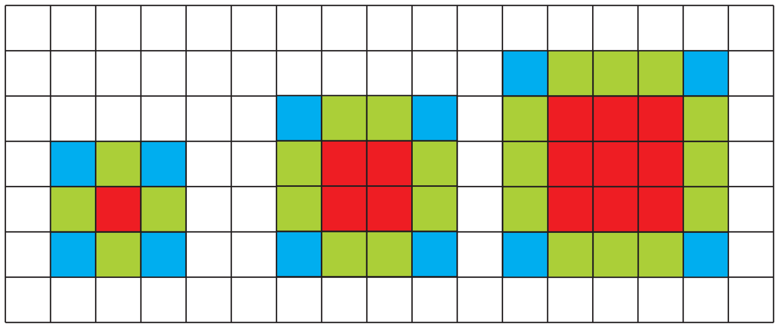
   - **a** Find the number of diagonals in a regular decagon.
   - **b** Find an expression, in terms of n, for the number of diagonals at each vertex of an n-sided polygon.
   - **c** Hence, or otherwise, find the number of diagonals in an n-sided regular polygon.
   - **d** An n-sided polygon has over 100 diagonals. What is the smallest possible value of n?

Do I know it now?

1. For each sequence, write down the missing term and the position-to-term formula.
   - **a** 20, 30, 40, 50, 60, $\boxed{}$, …
   - **b** 3, 12, 27, $\boxed{}$, 75, …
   - **c** 99, 93, 87, $\boxed{}$, 75, …
   - **d** 12, 45, 100, $\boxed{}$, 276, …
2. Copy and complete this table for the sequence with nth term $n^2 + 5$.
   (You may find compiling a spreadsheet useful for this.)

Term	$n^2 + 5$	1st difference	2nd difference
1	6		
2	9	3	
3	14	5	2
4			
5			

③ Here are some patterns made from square tiles.



Pattern 1 Pattern 2 Pattern 3

a Draw pattern number 5.

b Copy and complete the table.

Pattern number	1	2	3	4	5
Number of green squares					
Number of blue squares					
Number of red squares					
Total number of squares, S					

c Write down how many of these are in the 10th pattern.

i green squares **ii** blue squares **iii** red squares **iv** total number of squares

d Write down how many of these are in the nth pattern.

i green squares **ii** blue squares **iii** red squares

e Write down a formula for the total number of squares, S, in the nth pattern.

Write your formula in two different ways.

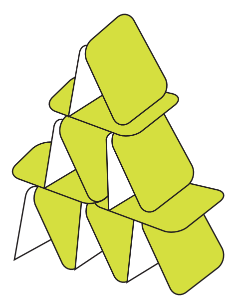
Can I apply it now?

① **a** This tower has three levels and is made from 15 playing cards.
Investigate for different numbers of levels.

b At one time the world record for a tower made from cards had 61 levels.

i How many cards are needed for 61 levels?

ii A typical playing card is 9 cm × 6 cm. Would the tower of cards fit in an ordinary room?



9.4 Geometric progressions

What you need to know

Did you know?

The population of any species over a period of years can be described using a geometric progression.

A **geometric progression** is a sequence in which each term is found by multiplying the previous term by the same amount.

The **common ratio** of a geometric progression is the number you multiply each term by to get the next term.

You need to know the **first term** and **common ratio** to work out the terms of a geometric progression.

Sometimes you are given these, but sometimes you are simply given the information to work them out.

In questions involving geometric progressions, always start by working out the first term and common ratio if these are not given.

A geometric progression has first term $\frac{1}{2}$ and common ratio 4.

First term $= \frac{1}{2}$

Second term $= \frac{1}{2} \times 4 = 2$

Third term $= 2 \times 4 = 8$

Fourth term $= 8 \times 4 = 32$

Fifth term $= 32 \times 4 = 128$

Sixth term $= 128 \times 4 = 512$

Seventh term $= 512 \times 4 = 2048$

The size of the terms increases very quickly.

Geometric progressions have terms that **increase or decrease exponentially**.

As a result they are used in many contexts, for example: compound interest, depreciation, population growth or decline, radioactive decay.

How to do it

Generating geometric progressions

A geometric progression has first term 1 and common ratio 3.
Write down the first five terms of the sequence.

Solution

First term $= 1$

Second term $= 1 \times 3 = 3$ ← To find the next term, multiply the previous term by 3.

Third term $= 3 \times 3 = 9$

Fourth term $= 9 \times 3 = 27$

Fifth term $= 27 \times 3 = 81$

They are powers of 3. $(3^0, 3^1, 3^2, 3^3, \ldots)$

Finding the common ratio

Here are two different sequences.

a 1, 4, 9, ...

b 10, 4, 1.6, ...

In each case, decide if the sequence is geometric. If it is, work out the common ratio.

Solution

a Work out the ratios between the terms.

$4 \div 1 = 1$

$9 \div 4 = 2.25$

There is no common ratio between successive terms so the sequence is not a geometric progression.

b Work out the ratios between the terms.

$4 \div 10 = 0.4$ ← $10 \times r = 4$ so $r = 4 \div 10$

$1.6 \div 4 = 0.4$ ← $4 \times r = 1.6$ so $r = 1.6 \div 4$

These are the same so the sequence is a geometric progression.

The common ratio, r, is 0.4. ← Check: $10 \times 0.4 = 4$ ✓ and $4 \times 0.4 = 1.6$ ✓

Learning exercise

① Decide whether each of the sequences is a geometric progression.

a 1, 2, 3, 4, …

b 1, 2, 4, 8, …

c 1, 2, 4, 7, …

d 80, 40, 20, 10, …

② A geometric progression has first term 3 and common ratio 3.

a Write down the first four terms of the sequence.

b What is this sequence?

③ The sequences are geometric progressions.

Write down the common ratio for each one.

a 1, 4, 16, 64, …

b 4, 12, 36, 108, …

c 4, 2, 1, $\frac{1}{2}$, …

d 10, 1, 0.1, 0.01, …

④ The sequences are geometric progressions.

Write down the next two terms for each one.

a 2, 10, 50, 250, …

b 50, 5, 0.5, 0.05, …

c 8, 12, 18, 27, …

d 6, 1.2, 0.24, 0.048, …

⑤ The second and third terms of a geometric progression are 6 and 18.

Work out

a the common ratio **b** the first term **c** the fifth term.

⑥ Here are the first three terms of some sequences.

a 200, 300, 450, …

b 150, 120, 100, …

c 24, 30, 37.5, …

In each case:

i Write down the ratios between the first two terms and the second and third terms.

ii Decide whether the numbers form a geometric progression.

⑦ In each part, the first and third terms of a geometric progression are given. Work out the required term.

a 1 and 36. Work out the second term.

b 4 and 1. Work out the second term.

c 2 and 12.5. Work out the fourth term.

⑧ The first term of a sequence is 1 and the fifth term is 81.

Write down the second, third and fourth terms of the sequence if it is a

a geometric progression **b** linear sequence.

⑨ £5000 is invested at a compound interest rate of 2%.

a Work out how much it is worth after 1 year.

b Work out how much it is worth after 2 years.

c Work out how much it is worth after 3 years.

d The amounts form a geometric progression. What is its common ratio?

Problem solving exercise

① Jason bought a new car for £10 000.

Its value depreciated by 10% each year.

So its value £V, after n years was given by

$V = 10\,000 \times 0.9^n$

a Work out the value of the car after

i 1 year

ii 2 years

iii 3 years.

b The answers to part **a** give a sequence of values of the car.

What is the term-to-term rule for this sequence?

c After how many years is the value of the car first less than £3000?

② Payday loan companies offer loans for a short period of time, typically for a few weeks.

One payday loan company charges interest at 30% per month. To work out how much you owe, you need to multiply the previous month's amount by 1.3.

a If you borrow £100, how much will you owe them after 2 months?

b The companies are not supposed to lend money to people for a long period of time. How much would you owe if you borrowed £100 and did not pay it back for a year?

c The amount of money in the world is about three trillion pounds (£3 000 000 000 000). If you borrow £100, roughly how long would it take before you needed to pay back more money than there is in the whole world?

Do I know it now?

① For each geometric progression:

a Find the common ratio.

b Write down the next two terms.

i 3, 6, 12, 24, … **ii** 64, 16, 4, 1, …

② In each part, the first and third terms of a geometric progression are given. Work out the required terms.

a 3 and 12. Work out the fifth and tenth terms.

b 5 and 0.2. Work out the second and fourth terms.

Can I apply it now?

① The half-life of fermium-253 is 3 days.

This means that, every three days, half of the remaining fermium-253 in a sample decays. How many days is it before only 6.25% of the original radioactive atoms remain?

CHAPTER 10

NEXT STEPS – ALGEBRA

Functions and graphs

10.3 Finding equations of straight lines

What you need to know

How many different straight lines can you draw through two points, for example (3, 6) and (5, 10)? The answer, of course, is one. Two points define a straight line.

In maths a line usually means a straight **line**.

If a graph produces something that is not straight, then this is usually described as a '**curve**'.

To find the equation of the line, think of the two points as part of a right-angled triangle.

The line through (3, 6) and (5, 10) goes 2 units across and 4 units up. So the gradient of the line is
$4 \div 2 = 2$.

If you know the gradient of a line and a point on that line, then the equation of the line can be found using:

$$y = mx + c$$

For example, for a line with a gradient of 2 that goes through (3, 6), we know that $y = 6$ when $x = 3$.

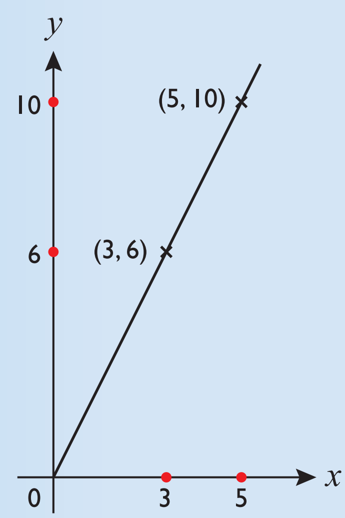
So, using $y = mx + c$ gives

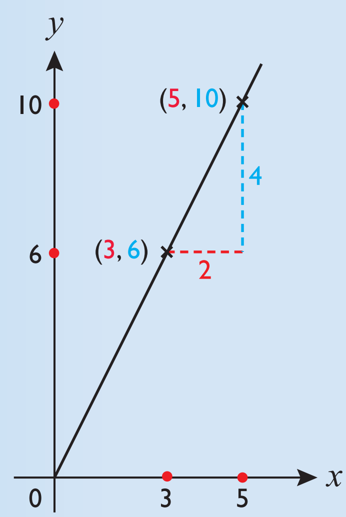
$$6 = 2 \times 3 + c$$

$$6 = 6 + c$$

$$c = 0$$

So the equation of the line with a gradient of 2 that goes through (3, 6) is $y = 2x$.





How to do it

Using one point and the gradient

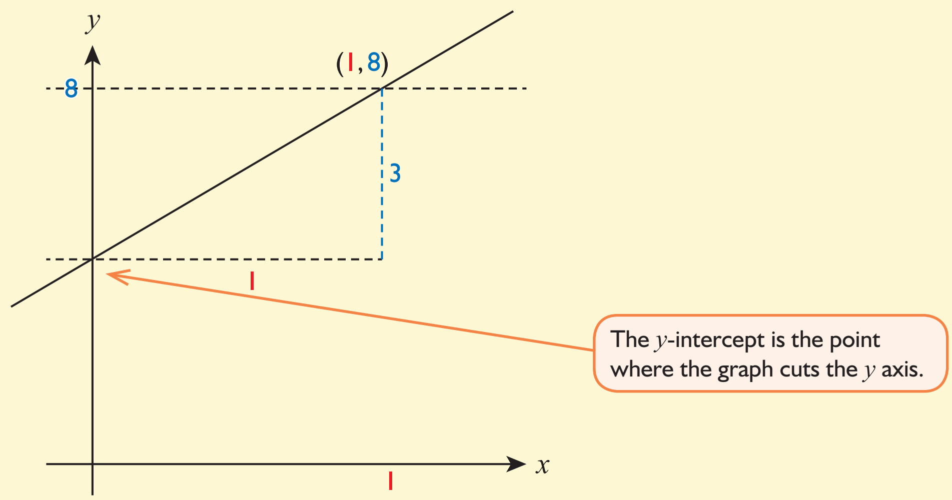
A line has a gradient of 3 and passes through the point (1, 8).

a Calculate the y-intercept of the line.

b Work out the equation of the line.

Solution

a Here is a sketch of the line.



$8 - 3 = 5$ — From $x = 0$ to $x = 1$ the graph goes up 3…

So the y-intercept is at (0, 5). — …so it goes from $y = 5$ to $y = 8$.

b The equation of a straight line is $y = mx + c$ where m is the gradient and c is the y-intercept.

The gradient is 3, so $m = 3$. — From the question.

The y-intercept is at (0, 5) so $c = 5$. — From part **a**.

So $y = 3x + 5$.

Using two points

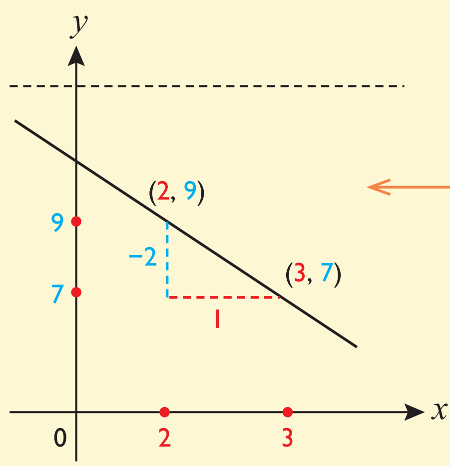
A line passes through (2, 9) and (3, 7).

a Find the gradient of the line.

b Write down the equation of the line.

Solution

a



Always start with a sketch.

$$\text{Gradient} = \frac{\text{change in } y}{\text{change in } x}$$

$$= \frac{-2}{1}$$

$$= -2$$

'Downhill lines' have a negative gradient.

b The equation of a straight line is $y = mx + c$.

The gradient is -2 so $m = -2$.

So $y = -2x + c$.

You can find the value of c by substituting the co-ordinates of one point on the line into $y = -2x + c$.

For the point (2, 9):

You could use (3, 7) instead.

Substituting $x = 2$ and $y = 9$ into $y = -2x + c$ gives

$9 = -2 \times 2 + c$

$9 = -4 + c$

$13 = c$

So $c = 13$

The equation of the line is $y = -2x + 13$.

You can write this as $y = 13 - 2x$.

Learning exercise

① All of the lines in this question have a gradient of 2.

a Draw the graphs of the lines that go through these points.

i (1, 5) **ii** (1, 12)

iii (1, 1) **iv** (5, 7)

v (−2, 1)

b Find their y-intercepts.

c Write down the equation of each line.

② A line has equation $y = 3x + k$.

a What is the gradient of the line?

The line passes through the point (2, 11).

b How does this give you the equation $11 = 6 + k$?

c Find the value of k and write down the equation of the line.

d Where does the line cross the y axis?

③ Write down the equation of the line that has a gradient of

a 3 and goes through (1, 1)

b 7 and goes through (2, 5)

c 2 and goes through (10, 1)

d $\frac{1}{2}$ and goes through (2, 2)

e $-\frac{1}{2}$ and goes through (2, 2).

④ A line goes through the points (2, 1) and (4, 9).

a Show that the gradient of the line is 4.

b Find the equation of the line with gradient 4 that passes through (2, 1).

c Check that (4, 9) lies on the line.

⑤ **a** For the following pairs of points, write down the gradient of the line that joins them and the equation of the line through them.

i (1, 1) to (5, 7)

ii (1, 3) to (5, 7)

iii (3, 4) to (5, 7)

iv (−2, 3) to (5, 7)

v (2, 9) to (5, 7)

b Check that both points lie on the lines by substituting x- and y- co-ordinates into the equation.

⑥ Write down the equation of a straight-line graph that fits into each section of this two-way table.

	Gradient is 3	**Gradient is −7**
y-intercept is 6		
y-intercept is −5		

⑦ The line **l** goes through the points (4, 0) and (0, 4).

The line **m** goes through the points (4, 2) and (0 −2).

a Write down the equations of the lines **l** and **m**.

b Draw a graph showing the lines **l** and **m**. Use the same scale for both the x axis and the y axis.

⑧ **a** Write down the equation of each line.

i A line that passes through (1, 2) and (3, 4)

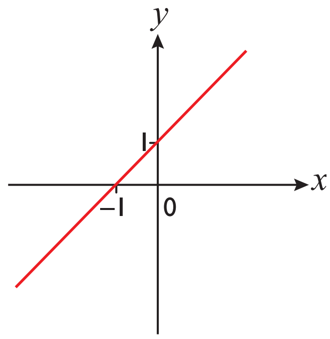
ii A line that passes through (1, 3) and (5, 7)

iii A line that passes through (1, 5) and (9, 13)

iv A line that passes through (5, 9) with gradient 1

v A line with gradient 1 and y-intercept 2

vi



b Which lines are the same as each other?

⑨ The co-ordinates of point A are (a, b) and the co-ordinates of point B are (b, a).

Is it always, sometimes or never true that the line through A and B has a negative gradient?

If you think always or never, explain how you can be so certain.

If you think sometimes, explain when the statement is and isn't true.

⑩ A triangle is drawn on a co-ordinate grid. It has vertices at (1, 1), (6, 6) and (1, 11).

a Write down the equations of the three lines that make the triangle.

b Draw the triangle on graph paper. Use the same scale for the x axis and the y axis.

c Describe the triangle.

d Work out the area of the triangle.

⑪ **l** and **m** are two straight lines.

l has a gradient of 2 and crosses the y axis at $(0, -1)$.

m has a gradient of -3 and crosses the y axis at (0, 4).

a Write down the equations of **l** and **m**.

b Draw lines **l** and **m** on a graph.

c What are the co-ordinates of their point of intersection?

⑫ **p** and **q** are two straight lines.

p has a gradient of 3 and passes through the point (3, 2).

q has a gradient of -2 and passes through the point $(1, -4)$.

a Draw lines **p** and **q** on the same pair of axes.

b Write down the equations of **p** and **q**.

c R is the point where lines **p** and **q** intersect. Write down the co-ordinates of R.

d Show that R lies on the line $y = -4x$.

⑬ A straight line **r** is parallel to the line $y = x + 3$ and passes through the point (0, 5).

Another straight line, **s**, is parallel to the line $x + y = 5$ and passes through the point (0, 1).

a Write down the equations of lines **r** and **s**.

b By drawing a graph, find and write down the co-ordinates of the point of intersection of the two straight lines.

c Substitute the x- and y-values you found in part **b** into the equations of the lines **r** and **s**. How does this check your answers?

⑭ A straight line, **l**, is parallel to the line $y = 4x - 3$ and passes through the point $(-1, 2)$.
Another straight line, **m**, is parallel to the line $4x + 3y = 2$ and passes through the point $(3, -2)$.

a Draw the lines **l** and **m** on a graph.

b Write down the equations of **l** and **m**.

c Write down the point of intersection of **l** and **m**.

d Use your answer to part **c** to check your equations for **l** and **m**.

Do I know it now?

① Write down the equation of each line.

a A line parallel to $y = 2x + 3$ with y-intercept 5.

b A line with gradient -1 through the point (5, 2).

c A line through the points $(-1, -4)$ and (2, 5).

② **a** Draw a graph showing the lines $y = 2x + 3$ and $y = -x + 6$.

b Write down the co-ordinates of the point of intersection, P.

c Show algebraically that the line joining (0, 7) to (3.5, 0) passes through P.

③ A is (0, 0), B is (3, 4), C is (9, 4) and D is (6, 0).

a Show that the quadrilateral ABCD is a parallelogram, but not a rhombus.

b Write down the equations of the lines AC and BD.

c Use algebra to show that the point E (4.5, 2) lies on both lines AC and BD.

d Show points A, B, C, D and E on a graph. Show also the parallelogram ABCD and its diagonals.

10.4 Quadratic functions

What you need to know

A **quadratic function** takes the form $y = ax^2 + bx + c$.

Its graph will always take the shape of a **parabola**.

It is **symmetrical**, and is either ∩-shaped or ∪-shaped.

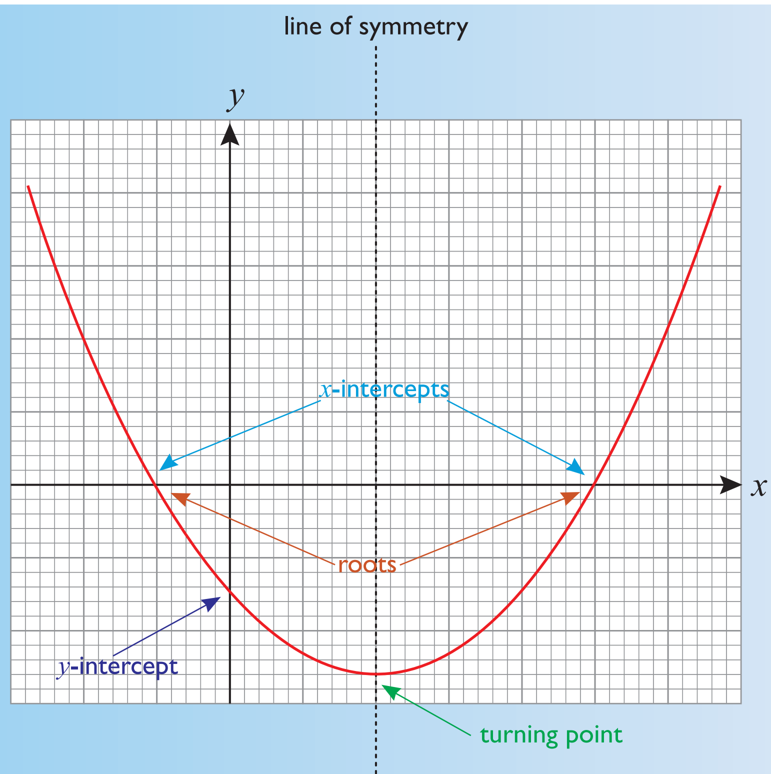
The **roots** of the equation $ax^2 + bx + c = 0$ are the values of x for which the equation is true.

They are the values of x at which the graph intersects the x axis.

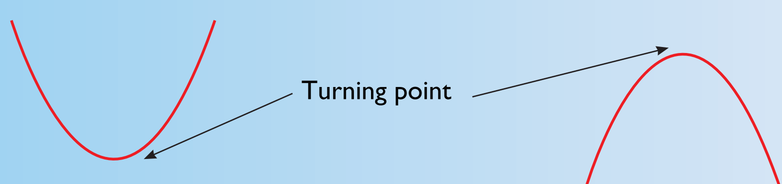
The line of symmetry of the graph of a quadratic function is always parallel to the y axis and passes through the turning point of the graph.

Did you know?

Many platform computer games feature a character that runs and jumps across various gaps and obstacles. This movement needs to be programmed so that the character follows the right path and lands in the right place for the game. Any object that moves through the air with gravity as the only force acting on it will follow a path that is a quadratic function.



The **turning point** is the name given to the **maximum** or **minimum point** on the graph.



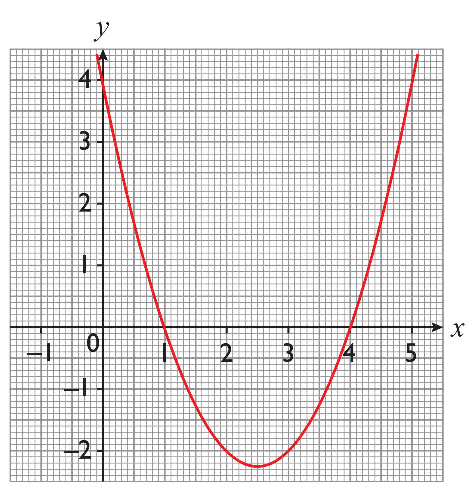
How to do it

Finding intercepts and roots

The graph shows the quadratic function $y = x^2 - 5x + 4$.

a Write down the intercepts with the axes.

b Write down the roots of the equation $x^2 - 5x + 4 = 0$.



Solution

a The intercept with the y axis is where the graph crosses the y axis.

It crosses at (0, 4).

The intercept with the x axis is where the graph crosses the x axis.

It crosses at (1, 0) and (4, 0).

Don't forget to find all three intercepts!

b The roots of the equation $x^2 - 5x + 4 = 0$ are 1 and 4.

The roots are the x co-ordinates of the points where the curve cuts the x axis.

To check, substitute the numbers into the function and verify that the answer is zero.

$x = 1$

$y = 1^2 - 5 \times 1 + 4$

$= 1 - 5 + 4$

$= 0$

$x = 4$

$y = 4^2 - 5 \times 4 + 4$

$= 16 - 20 + 4$

$= 0$

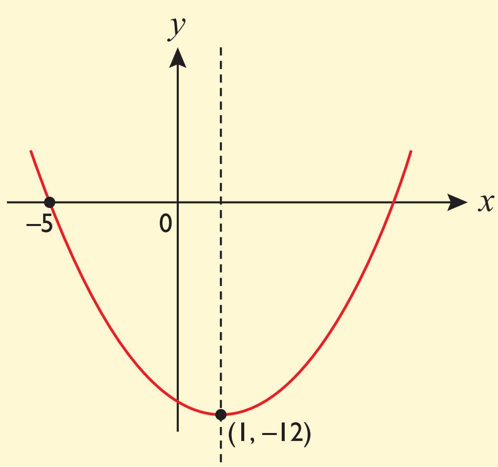
Using turning points

A quadratic function has a root when $x = -5$ and a turning point at $(1, -12)$.

a Sketch its graph. **b** Give the co-ordinate of the other root.

Solution

a



b The line of symmetry goes vertically through the turning point so the line of symmetry is $x = 1$.

Reflecting $(-5, 0)$ in the line of symmetry gives $(7, 0)$. So the second root is at $x = 7$.

Learning exercise

① The graph shows the quadratic function $y = x^2 - 8x + 12$.

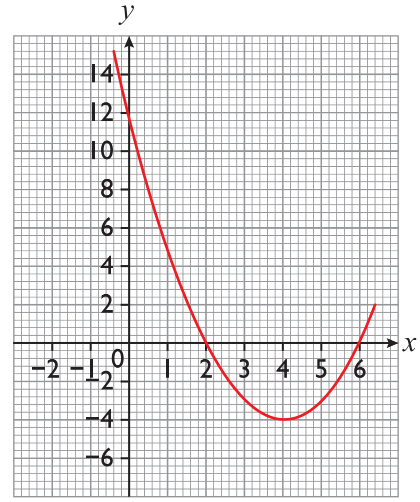
a Write down the intercept on the y axis.

b Write down the intercepts on the x axis.

c Write down the roots of $0 = x^2 - 8x + 12$.

d Write down the equation of the line of symmetry.

e Write down the co-ordinates of the turning point.



② A quadratic function has intercepts $(0, -3)$, $(-1, 0)$ and $(3, 0)$.

a Draw the function on axes with the x axis from -2 to 5 and the y axis from -5 to 10.

b Write down the roots of the function.

c What is the equation of the line of symmetry?

③ The graph of a quadratic function crosses the axes at $(0, -6)$, $(-3, 0)$ and $(2, 0)$.

a Draw the curve. Take the x axis from -3 to 2 and the y axis from -6 to 1.

b Write down the roots of the function.

c What is the equation of the line of symmetry?

④ The graph shows the quadratic function $y = -x^2 - 3x + 4$.

a Write down the intercept on the y axis.

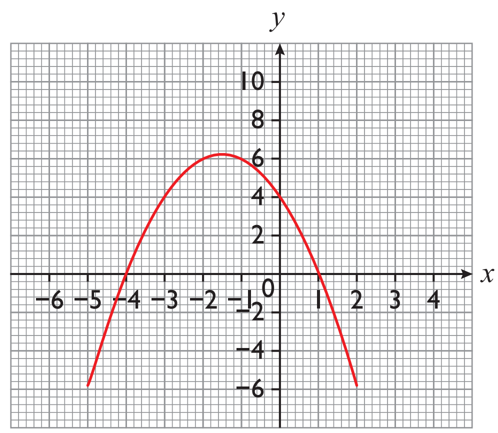
b Write down the intercepts on the x axis.

c Write down the roots of $0 = -x^2 - 3x + 4$.

d Write down the equation of the line of symmetry.

e **i** Read from the graph and write down the co-ordinates of the turning point.

ii Check the y co-ordinate of your answer by calculation.



⑤ Here is a table of values for a quadratic function.

x	-3	-2	-1	0	1	2	3	4
y	6	0	-4	-6	-6	-4	0	6

a Without drawing the graph, work out

i the intercepts with the axes **ii** the roots of the function

iii the equation of the line of symmetry.

b Estimate the co-ordinates of the turning point.

c What further information do you need to calculate the y co-ordinate of the turning point?

⑥ A quadratic function has the equation $y = x^2 - 3x + 2$.

a Copy and complete the table of values for the function.

x	-1	0	1	2	3	4
x^2						
$-3x$						
$+2$						
$y = x^2 - 3x + 2$						

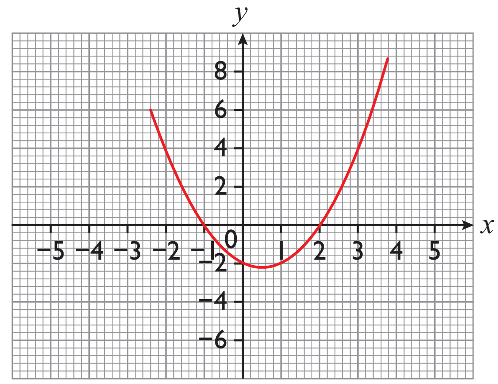
b Plot the points and join them with a smooth curve.

c Write down the intercepts with the axes.

d Write down the roots of the equation $x^2 - 3x + 2 = 0$. Call them p and q.

e Substitute for p and q in $(x - p)(x - q)$. Expand and simplify the expression. What do you notice?

⑦ The graph shows the quadratic function $y = x^2 - x - 2$.



a Write down the intercepts with the axes.

b Write down the roots of the equation $x^2 - x - 2 = 0$. Call them p and q.

c Substitute for p and q in $(x - p)(x - q)$. Expand and simplify the expression. What do you notice?

d Work out the co-ordinates of the turning point. Is it a maximum or minimum value?

⑧ **a** Draw the graph of the function that has a turning point at (3, 7) and one root when $x = -4$.

b Identify the co-ordinates of the second root.

c Draw a graph of a different function that has a turning point at (3, 7) but no roots.

d Is it possible for a function to have a turning point at (3, 7) but only one root? Explain your answer.

⑨ A quadratic function passes through the points (2, 4), (3, 15), (1, −3), (0, −6) and (−3, 9).

a Draw the graph of the function for values of x from −3 to 3.

b Estimate the turning point of the graph.

c Estimate the roots of the equation.

Problem solving exercise

① The path of a cannon ball is described by the quadratic function $y = \frac{1}{2}x - \frac{1}{300}x^2$, where the units are measured in metres.

a Make a table of values for $x = 0, 30, 60, 90, 120$ and 150.

b Draw the cannon ball's path on a graph.

c How far does the ball travel horizontally before hitting the ground?

d What is the maximum height that the cannon ball reaches?

② A quadratic function passes through the points (0, 4) and (6, 0).

Draw two possible quadratic functions passing through these points with:

a two roots and a turning point when $x = 4$

b exactly one root.

③ At time t seconds, the velocity, v metres per second, of a particle moving along a straight line was measured and recorded in the table.

t	0	1	2	3	4	5	6
v	8	1	−2	−1	4	13	26

a Plot the graph.

b **i** Use your graph to estimate the turning point.

ii Describe the movement of the particle at this point.

④ Here are five quadratic graphs.
Their equations (not in order) are:

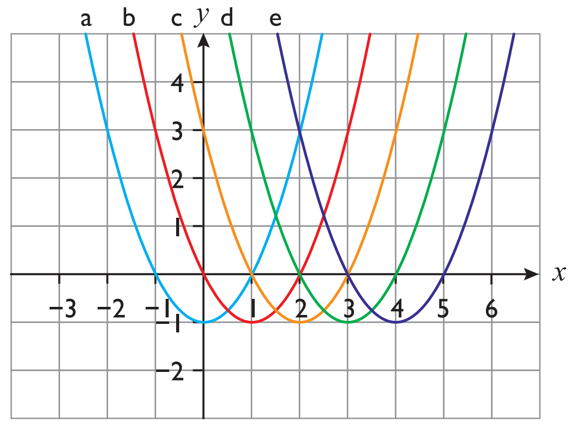
A $y = (x - 1)(x - 3)$

B $y = (x - 1)(x + 1)$

C $y = x(x - 2)$

D $y = (x - 3)(x - 5)$

E $y = (x - 2)(x - 4)$



i Match the correct graph to each equation.

ii Expand the brackets. What do you notice about the algebra?

Do I know it now?

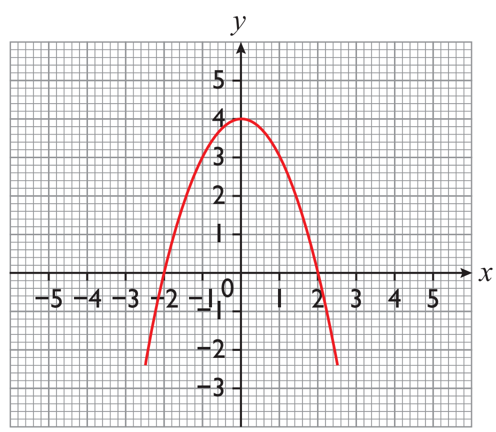
① The graph shows the quadratic function $y = 4 - x^2$.

a Write down the intercepts with the axes.

b Write down the roots of the equation $4 - x^2 = 0$. Call them p and q.

c Substitute for p and q in $(x - p)(x - q)$. Expand and simplify the expression. What do you notice?

d Work out the co-ordinates of the turning point. Is it a maximum or minimum value?



Can I apply it now?

① The path of a golf ball is described by the quadratic function $y = \frac{1}{3}x - \frac{1}{540}x^2$, where the units are measured in metres.

a Make a table of values for $x = 0, 30, 60, 90, 120, 150$ and 180.

b Draw the path of the golf ball on a graph.

c How far does the ball travel horizontally before hitting the ground?

d What is the maximum height that the ball reaches?

② The graph of a quadratic function intercepts the axes at just two points, (0, 4) and (7, 0).

a Draw the graph.

b Identify the co-ordinates of the turning point.

A second quadratic function intercepts the axes at (0, 4), (7, 0) and one other point.

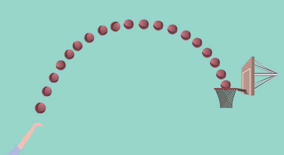
c If the graph has a line of symmetry at $x = 2$, what are the roots of the function?

10.5 Polynomial and reciprocal functions

What you need to know

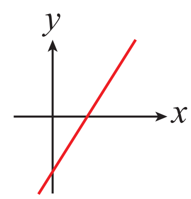
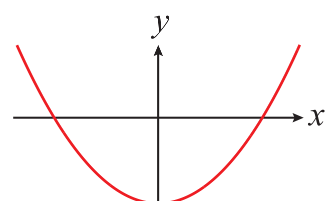
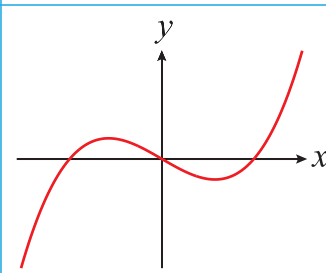
Did you know?

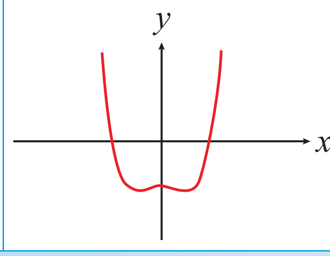
The path of this ball follows a parabola as it is thrown into the hoop.



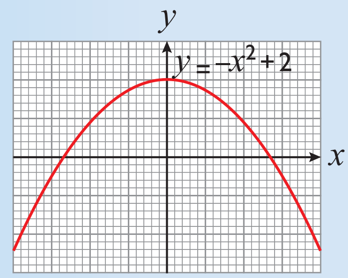
By remembering the shape of the positive graph, you can work out the shape of the negative graph by reflecting it in the x axis.

Look at these graphs for different powers of x.

Power	Equation	Curve	Turning points
1	$y = 2x - 1$		0
2	$y = x^2 - 2$		1
3	$y = x^3 - x$		2

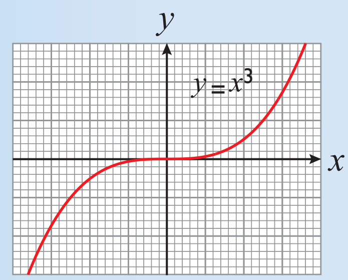
Power	Equation	Curve	Turning points
4	$y = 2x^4 - x^2 - 1$		3

The curves in this unit go up to power 3. The quartic shows how the pattern continues.



A graph tells you a lot about the equation of its function. In general, the power is 1 more than the number of turning points.

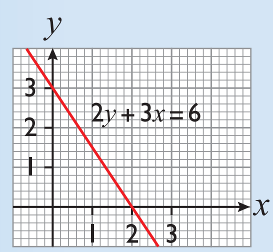
The term with the highest power of x is called the **leading term**. When the leading term is negative, the curve is turned upside down.



Look at the curve $y = -x^2 + 2$. It is still the parabola you get with all quadratics, but is shaped ⋂ rather than ⋃.

Sometimes a curve flattens out instead of having separate turning points. You can see that in the case of $y = x^3$.

Sometimes the equation of a line may not have y as the subject.

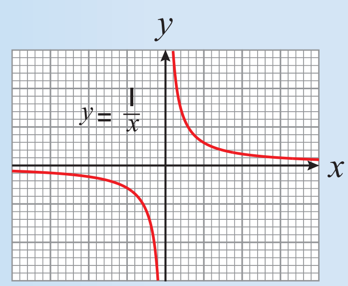


For example, the line $y = -\frac{3}{2}x + 3$ is often written in the form $2y + 3x = 6$.

Substituting $x = 0$ shows you it crosses the y axis at (0, 3).

Substituting $y = 0$ shows you it crosses the x axis at (2, 0).

Another important graph is $y = \frac{1}{x}$. This has two separate branches. The curve never reaches the y axis. The y axis is called an **asymptote**. The x axis is also an asymptote.



Note:

Plot means work out the co-ordinates of some of the points and join them up carefully to give a smooth curve (or a straight line).

Sketch means show the main features of a curve. These include where it crosses the x axis and the y axis, and any turning points.

How to do it

Identifying graphs

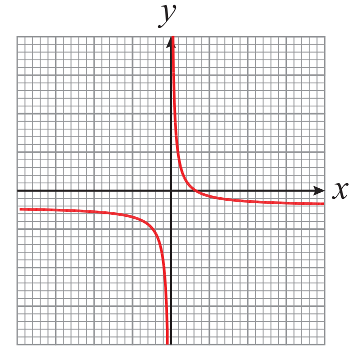
Which of the following could be the equation of this curve?

$y = 1 - x^3$

$y = -\frac{1}{x}$

$y = x^2 - 1$

$y = \frac{1}{x} - 1$



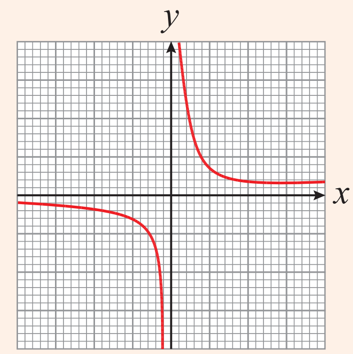
Solution

The graph is a **reciprocal graph**.

$y = 1 - x^3$ is a cubic and $y = x^2 - 1$ is a quadratic.

So the equation is either $y = -\frac{1}{x}$ or $y = \frac{1}{x} - 1$.

The curve is the same shape as $y = \frac{1}{x}$, but 'shifted' down.



So the curve is $y = \frac{1}{x} - 1$.

Note: $y = -\frac{1}{x}$ will be the 'upside down' version of $y = \frac{1}{x}$.

Plotting graphs

a Draw and complete a table of values for $y = x^3 - 6x^2 + 11x - 6$.

Use $x = 0, 1, 2, 3$ and 4.

b Draw the graph.

Solution

a

x	0	1	2	3	4
x^3	0	1	8	27	64
$-6x^2$	0	−6	−24	−54	−96
$+11x$	0	11	22	33	44
-6	−6	−6	−6	−6	−6
y	−6	0	0	0	6

When $x = 4$, $x^3 = 4^3 = 64$.

And $-6x^2 = -6 \times 4^2$
$= -6 \times 16$
$= -96$

And $+11x = +11 \times 4$
$= 44$

All the entries in this row are '−6'.

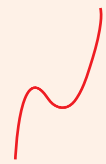
$64 - 96 + 44 - 6 = 6$
So when row $x = 4$, $y = 6$.

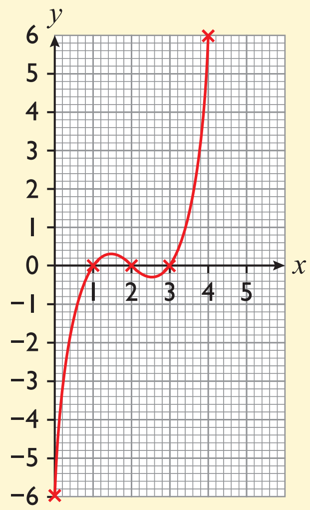
b First plot the points (0, −6), (1, 0), (2, 0), (3, 0) and (4, 6).

$y = x^3 - 6x^2 + 11x - 6$ is a cubic as the highest power is 3.

Next join the points with a smooth curve.

The sign in front of the x^3 is positive, so the graph is this way up:





Learning exercise

① These equations and their graphs have been muddled up.

Match each graph with its equation.

A $y = \frac{1}{x} + x$

B $2y - 3x = 4$

C $y = x^3 + x^2 - 2x$

D $y = x^3$

E $2y - 3x + 4 = 0$

F $y = x^2$

G $y + 3x - 2 = 0$

H $y = x^2 - 3x - 4$

I $y = -x^2$

J $y = -x^3$

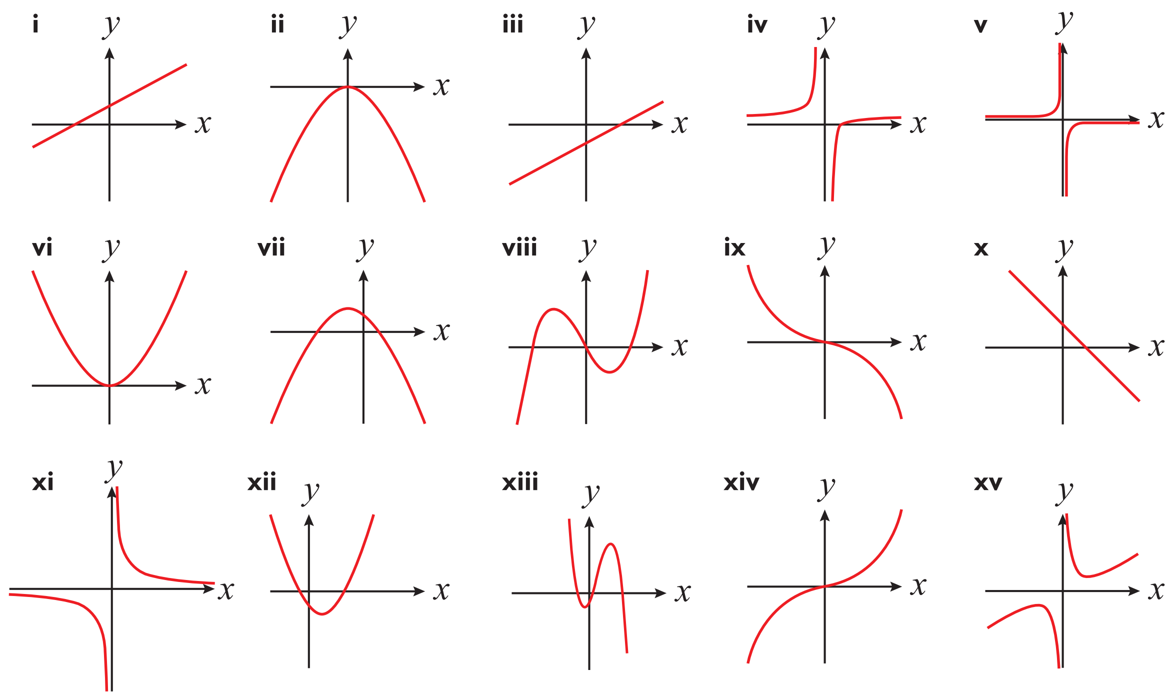
K $y = -x^3 + 5x^2 + 6x$

L $y = -\frac{1}{x}$

M $y = 1 - \frac{1}{x}$

N $y = \frac{1}{x}$

O $y = -x^2 - 3x + 4$

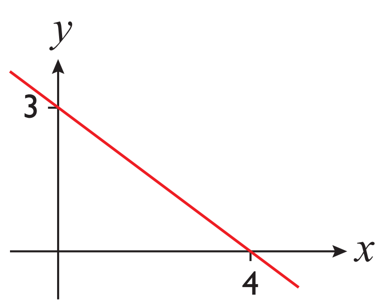


② **a** A curve has the equation $y = x^3 - 2x^2 - 3x$.

Make a table of values for x from -2 to 4.

b Draw the graph of $y = x^3 - 2x^2 - 3x$.

③ Which of the following are equations of this straight line?



$y = -\frac{3}{4}x + 3$ $\qquad$ $\frac{x}{4} + \frac{y}{3} = 1$ $\qquad$ $3x + 4y = 12$

④ **a** Draw each pair of curves (or lines) on a set of axes.

i $y = x$ and $y = -x$

ii $y = x^2$ and $y = -x^2$

iii $y = x^3$ and $y = -x^3$

iv $y = \frac{1}{x}$ and $y = -\frac{1}{x}$

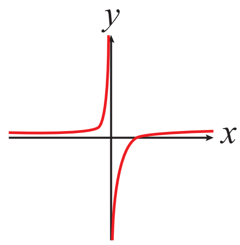
b Say what is the same and what is different about your answers to part **a**.

Problem solving exercise

① Match each graph to its equation.

a $y = x^3 + x$ **b** $y = 1 - x^3$ **c** $y = x - \frac{1}{x}$ **d** $y = \frac{4}{x}$

i



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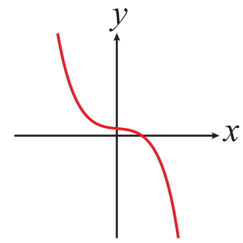

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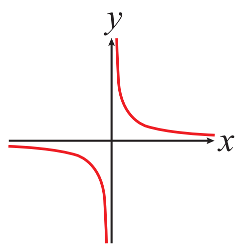

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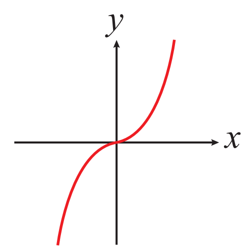


② Chloe is sketching a curve. She plots the point (2, −3).

a Copy and complete each of these to give a possible equation for Chloe's curve.

The missing term in each equation is a constant.

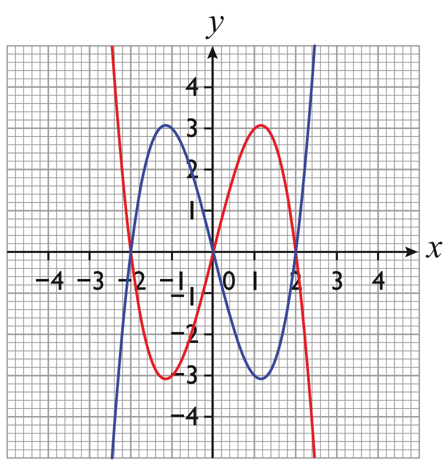
$y = \square - x^2$ $y = \square - x^3$ $y = \frac{2}{x} - \square$

b Draw the curves on the same pair of axes.

c Next, Chloe plots the point (−2, −3) and joins her points.

What is the equation of Chloe's curve?

③ Here are the graphs of $y = x^3 - 4x$ and $y = 4x - x^3$.



a For which values of x is $y = 4x - x^3$ positive?

b For which values of x is $y = x^3 - 4x$ negative?

c Find the values of x when $4x - x^3$ is greater than $x^3 - 4x$.

d Describe the relationship between the two curves.

e Describe the symmetry of $y = x^3 - 4x$.

Do I know it now?

① Plot the graph of $y = x^3 + x^2 - 2x$ for $-3 \leq x \leq 2$.

Hence solve $x^3 + x^2 - 2x = 0$.

② **a** Complete the table of values for $y = x + \frac{1}{x}$.

x	−5	−4	−3	−2	−1	−0.5	−0.2	0.2	0.5	1	2	3	4	5
$\frac{1}{x}$														
y														

b Why is $x = 0$ not in the table?

c Draw the graph.

d Use your curve to find the value of x for which $x + \frac{1}{x} = 3$.

Can I apply it now?

① **a** Draw the graph of $y = kx^2$ when $k > 0$.

b On the same pair of axes sketch $y = kx^2$ when $k < 0$.

c How do your curves relate to each other?

d Repeat parts **a** to **c** for

i $y = \frac{k}{x}$ **ii** $y = kx^3$.

CHAPTER 11

NEXT STEPS – ALGEBRA

Algebraic methods

11.4 Using graphs to solve simultaneous equations

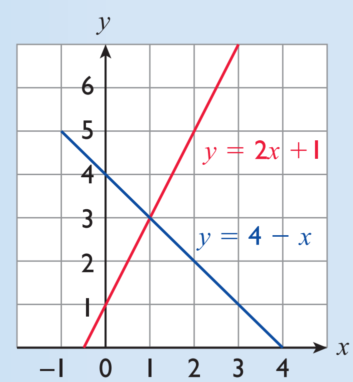
What you need to know

To solve simultaneous equations graphically:

- Plot both equations on the same axes.
- If necessary, extend the lines so that they meet.
- Find the co-ordinates of the point where the two lines meet.
- Make sure you write the solution as $x = \square$ and $y = \square$.

For example: $y = 2x + 1$ and $y = 4 - x$

The two lines meet at (1, 3), so the solution is $x = 1$ and $y = 3$.



How to do it

Solving simultaneous equations graphically

Solve these simultaneous equations using a graph.

$y = 2x$

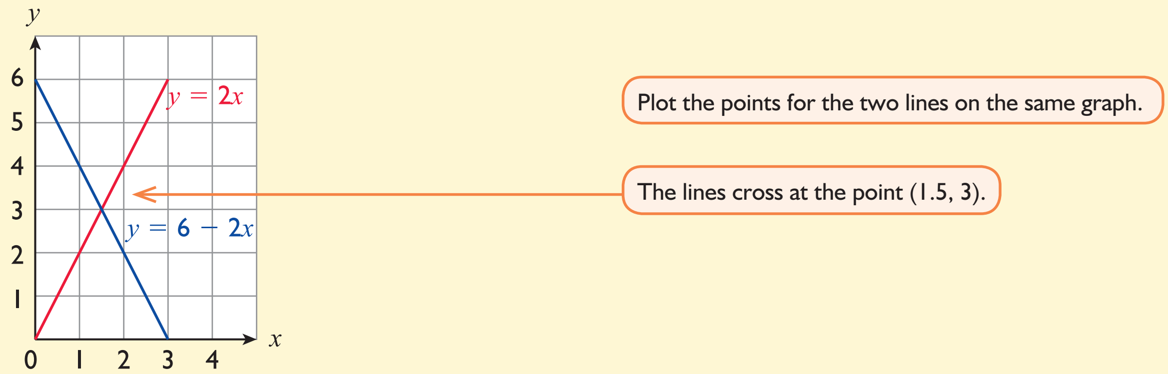
$y = 6 - 2x$

Solution

x	0	1	3
$y = 2x$	0	2	6

x	0	1	3
6	6	6	6
$-2x$	0	−2	−6
$y = 6 - 2x$	6	4	0

Calculate three points on each line.



The solution is $x = 1.5$ and $y = 3$.

Learning exercise

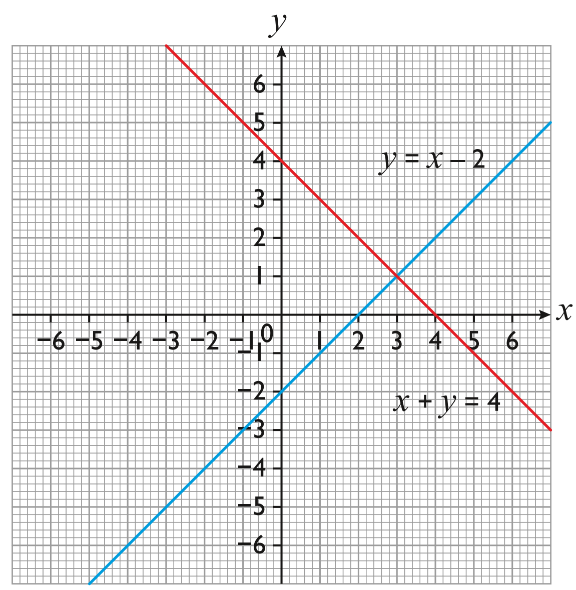
① **a** Write down the co-ordinates of the point of intersection of the two lines on this graph.

b Write down the solution of these equations.

$x + y = 4$

$y = x - 2$

c Check your answer by substituting values for x and y in the two equations.

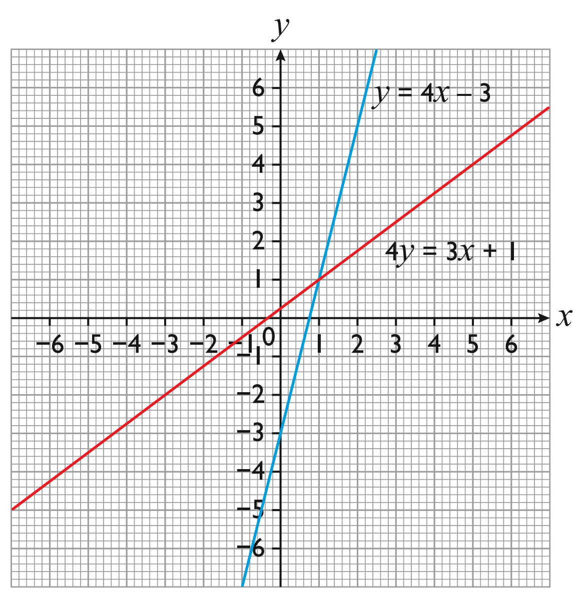


② **a** The graph shows the lines $y = 4x - 3$ and $4y = 3x + 1$. Write down the co-ordinates of their point of intersection.

b Solve these equations simultaneously by:

i looking at the graph

ii using algebra (substitution method).



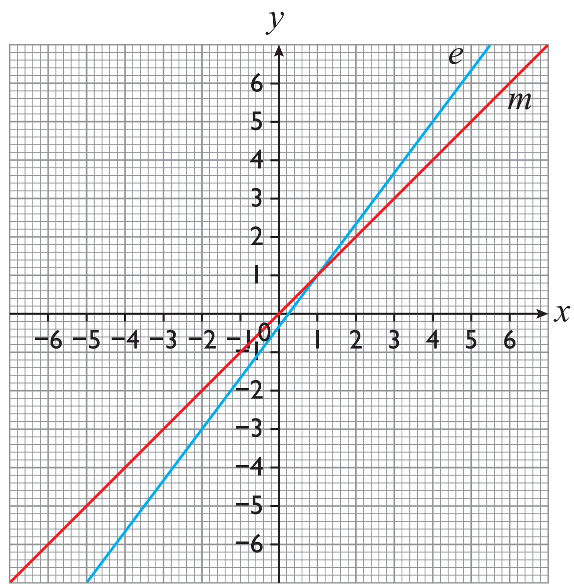
③ **a** One of the lines on this graph is $y = x$ and the other is $3y = 4x - 1$. Which is which?

b Write down the co-ordinates of the point of intersection of the two lines.

c Solve these equations simultaneously by:

i looking at the graph

ii using algebra (substitution method).

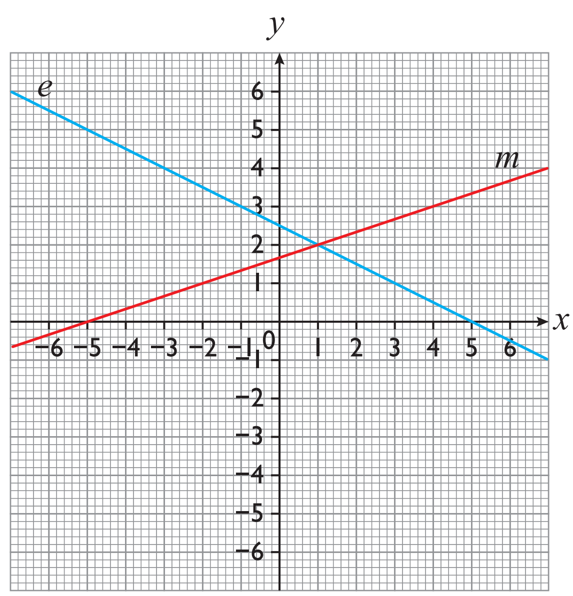


④ **a** One of the lines on this graph has equation $y = \frac{1}{3}x + 1\frac{2}{3}$. The other has equation $y = -\frac{1}{2}x + 2\frac{1}{2}$. Which is which?

b Write down the co-ordinates of the point of intersection of the two lines.

c Write down the two simultaneous equations for which this point is the solution.

d Check your solution by substituting your x- and y-values in both equations.



⑤ **a** Copy and complete this table of values for $y = 3x - 2$.

x	0	1	2	3
$3x$	0			
-2	-2			
$y = 3x - 2$	-2			

b Make a table of values for the line $y = 6 - x$.

c Draw the lines $y = 3x - 2$ and $y = 6 - x$ on the same graph.

d Use your graph to solve the simultaneous equations $y = 3x - 2$ and $y = 6 - x$.

e Use algebra to check your solution.

⑥ **a** Use algebra to solve these equations simultaneously.

$y = 4x - 3$ $\qquad\qquad$ $y = x + 3$

b Draw the lines $y = 4x - 3$ and $y = x + 3$ on the same graph.

c Use your graph to check your answers to part **a**.

d Check your answer again, this time by substituting the x- and y-values in both equations.

⑦ **a** Try to use algebra to solve these simultaneous equations.

$y = 2x + 3$ $\qquad$ $y = 2x - 1$

b What do you notice?

c Draw these two lines on the same axis.

d Use your graph to explain the result in part **a**.

⑧ Two electricity companies advertise the following rates.

Green Power	**Sparkle**
Standing charge 30p per day	Standing charge 50p per day
Cost of electricity 18p per unit	Cost of electricity 16p per unit

a Write down an equation for the daily cost, C pence, of using u units of electricity for

i Green Power $\qquad$ **ii** Sparkle.

b On the same graph, draw two lines to illustrate the daily cost of electricity from each company for values of u from 0 to 15.

c **i** Use your graph to find the number of units for which both companies charge the same amount.

ii How much do they each charge for this number of units?

iii Check your answer to **c i** by solving your equations algebraically using the substitution method.

d The Watts family uses an average of 12 units of electricity a day. Which company would you recommend?

⑨ A quadrilateral is bounded by four lines with equations:

$4y = x + 24 \qquad y + 4x = 23 \qquad y + 4x + 11 = 0 \qquad 4y = x - 10$

a Draw the quadrilateral on a graph.

b Use your graph to find the co-ordinates of the vertices of the quadrilateral.

c Show how you can check your answer to **b** by substituting the x and y co-ordinates into the appropriate pair of equations.

d What is the name of the quadrilateral?

Problem solving exercise

① Pete wants to hire a car. He wants to spend as little as possible.

He can hire it from one of two companies: U hire and Cars 2 go.

U hire	**Cars 2 go**
50p a mile	£60 plus 20p a mile

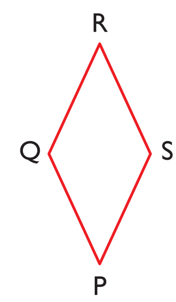
a Write down the equations for the cost, £C, of hiring a car for m miles from each company.

b Draw lines for your equations on the same pair of axes.

Use the vertical axis for C with a scale from 0 to 200 and the horizontal axis for m with a scale from 0 to 400.

c Pete expects to drive 250 miles. Which company would you advise him to use?

② Ros makes an accurate drawing of this diamond shape on graph paper. The equations of the four lines are



$y = 3x - 3 \qquad y = 3x - 9 \qquad y = -3x + 15 \qquad y = -3x + 9$

a Make tables of values for these lines, taking values of x from 0 to 5.

b Draw a graph showing the parts of the lines that make the shape.

c Write down the co-ordinates of the four vertices, P, Q, R and S.

d **i** Show how Ros could have used algebra to find the co-ordinates of the vertices.

ii Use this method to check your answers to part **c**.

e What is the mathematical name for this shape?

③ Catherine draws a triangle which is bounded by the lines $x + 2y = 12$, $2y = x + 4$ and $y = x - 3$.

a Draw the three lines on the same pair of axes.

b **i** Write the co-ordinates of the vertices of the triangle.

ii Use your diagram to solve the following pairs of simultaneous equations.

A $x + 2y = 12$, $2y = x + 4$

B $x + 2y = 12$, $y = x - 3$

C $2y = x + 4$, $y = x - 3$

Catherine adds two horizontal and two vertical lines to her diagram to form a rectangle around her triangle. Each line passes through one of the vertices of the triangle.

c **i** Add these lines to your diagram.

ii State the equations of the lines that Catherine adds.

iii Work out the area of the rectangle bounded by these four lines.

d Work out the area of Catherine's triangle.

④ Look at these pay-as-you-go tariffs for two mobile phone companies.

Q-Mobile	**Pear**
30p per minute for calls, 8p per text	20p per minute for calls, 12p per text

a Let m stand for the number of call minutes and t stand for the number of texts.

Write down an equation for m and t for a bill of £5 with

i Q-Mobile **ii** Pear.

b Draw a graph of your equations.

Chloe uses Q-Mobile and Daisy uses Pear.

c One week they use the same number of call minutes as each other and send the same number of texts. They both use exactly £5 of credit.

How many call minutes and texts do they each use?

d Another week both girls made 4 minutes of calls and sent 40 texts.

i Mark a point on your graph to show this.

Both girls have £5 credit.

ii Who has to buy more credit? How much more credit does she need?

iii Who has credit left over? How much?

Do I know it now?

① **a** Use a graph to solve the simultaneous equations $y = 2x - 5$, $y = 3x - 7$.
b Check your solution by substituting the x- and y-values in both equations.

② **a** Use a graph to solve the simultaneous equations $y = 3x + 4$, $y = x + 2$.
b Check your answer algebraically by using the method of substitution.
c Check your answer again, this time substituting the x- and y-values in both equations.

Can I apply it now?

① Avonford College compares the cost of two coach companies for a school trip.

Speedy Coaches	**Get-Aways!**
£150 per day, £2 per mile	£100 per day, £4 per mile

a Write down an equation for the cost, £C, of hiring a coach for one day and for m miles for

i Speedy Coaches **ii** Get-Aways!

b On the same pair of axes, draw two lines to illustrate the daily cost of hiring a coach from each company for values of m from 0 to 100.

c **i** Read from your graph the number of miles for which both firms charge the same amount.
ii How much do they each charge for this number of miles?
iii Check your answer to **c i** by solving your equations algebraically using the substitution method.

d Avonford College is organising a coach for an 80-mile round trip.
i Which company would you recommend?
ii How much does Avonford College save by choosing this company?

CHAPTER 12

NEXT STEPS – ALGEBRA

Working with quadratics

12.1 Factorising quadratics

What you need to know

A quadratic expression can sometimes be written as the product of two linear expressions, for example

$$(x + 2)(x - 4)$$

Expanding the brackets

$$\begin{aligned}(x + 2)(x - 4) &= x^2 + 2x - 4x - 8 \\ &= x^2 - 2x - 8\end{aligned}$$

Factorising means writing a number or expression as a product of two factors. It is the reverse of expanding the brackets. So the expression $x^2 - 2x - 8$ factorises to $(x + 2)(x - 4)$.

$x + 2$ and $x - 4$ are **factors** of the original expression.

In general,

$$\begin{aligned}(x + a)(x + b) &= x^2 + ax + bx + ab \\ &= x^2 + (a + b)x + ab\end{aligned}$$

Multiply a and b to give the constant term.

Add a and b to give the coefficient of the x term.

So to factorise $x^2 + 5x + 6$, first find numbers that multiply to give 6.

Possibilities are:

1×6, 2×3, -1×-6 and -2×-3

Of these, find the two numbers that add up to give 5:

$1 + 6 = 7$ ✗

$2 + 3 = 5$ ✓

$-1 + -6 = -7$ ✗

$-2 + -3 = -5$ ✗

So $x^2 + 5x + 6$ factorises to $(x + 2)(x + 3)$.

Check by expanding the brackets to get back to the original expression.

Did you know?

Simplifying the formulae in a spreadsheet using techniques such as factorising and cancelling can avoid a situation where you divide by zero – spreadsheets don't like that!

47,896.49	7130	3565	2139
54,815.62		4896	2448
5,253.16	#DIV/0!	#DIV/0!	#DIV/0!
8,397.00		750	375
101,519.73	15113	7556	4534
97,808.26		8736	4368
10,042.81	2990	1495	897
16,374.15		1463	731

Special case

$(x + a)(x - a) = x^2 + ax - ax - a^2$

$= x^2 - a^2$

$x^2 - a^2$ is known as the **difference of two squares**.

Look out for this when you are asked to factorise.

For example, $x^2 - 100$ is the difference of two squares ($100 = 10^2$).

So $x^2 - 100 = (x + 10)(x - 10)$.

Check:

	x	-10
x	x^2	$-10x$
$+10$	$10x$	-100

The terms in x cancel out.

How to do it

Factorising quadratics

a Factorise $x^2 - 7x + 10$.

b Factorise $x^2 - x - 12$.

Solution

You need to reverse the process of expanding two brackets.

The first term to consider is the **constant term** or number 'on its own'.

a $x^2 - 7x + 10 = (x + \square)(x + \square)$

You need two numbers that multiply to give $+10$…

$+2$ and $+5$, -2 and -5, $+10$ and $+1$, -10 and -1

…and add up to give -7.

$(+2) + (+5) = +7$, right number, wrong sign

$(-2) + (-5) = -7$ ✓

So the two numbers are -2 and -5.

Therefore, $x^2 - 7x + 10 = (x - 2)(x - 5)$

	x	-2
x	x^2	$-2x$
-5	$-5x$	$+10$

b $x^2 - x - 12 = (x + \square)(x + \square)$

$+12$ and -1, -12 and $+1$, $+6$ and -2, -6 and $+2$, $+4$ and -3, -4 and $+3$

You need two numbers that multiply to give -12…

…and add to give -1.

$(+12) + (-1) = (+11)$ ✗ and $(-12) + (+1) = (-11)$ ✗ $(+6) + (-2) = (+4)$ ✗ and $(-6) + (+2) = (-4)$ ✗

$(+4) + (-3) = 1$, right number, wrong sign

$(-4) + (+3) = -1$ ✓

So the two numbers are -4 and $+3$.

Therefore, $x^2 - x - 12 = (x - 4)(x + 3)$.

	x	-4
x	x^2	$-4x$
$+3$	$3x$	-12

Difference of two squares

a Factorise $x^2 - 36$.

b Factorise $x^2 - 1$.

Solution

It is worth checking for the difference of two squares format when a quadratic expression has only two terms.

a The first square is x^2 so the brackets have x at the start:

$(x + \square)(x - \square)$

The second square is 36, so 6 fills in the other places.

$x^2 - 36 = (x + 6)(x - 6)$

	x	$+6$
x	x^2	$6x$
-6	$-6x$	-36

The terms in x cancel out.

b The first square is x^2 so the brackets have x at the start:

$(x + \square)(x - \square)$

The second square is 1, so 1 fills in the other places.

$x^2 - 1 = (x + 1)(x - 1)$

	x	$+1$
x	x^2	x
-1	$-x$	-1

The terms in x cancel out.

Learning exercise

1. Expand and simplify where possible.

   a $x(x + 3)$
   b $x(x - 1)$
   c $(x - 5)(x + 5)$
   d $(x - 2)(x + 3)$
   e $(x + 5)(x - 8)$
   f $(x - 4)(x - 1)$
   g $(x - 2)(x + 2)$

2. Find pairs of numbers which

   a add up to give 6 and multiply to give 8
   b add up to give 2 and multiply to give -8
   c add up to give -2 and multiply to give -8
   d add up to give -6 and multiply to give 8
   e add up to give 9 and multiply to give 8
   f add up to give 7 and multiply to give -8
   g add up to give -7 and multiply to give -8
   h add up to give -9 and multiply to give 8.

3. Complete these simplifications.

   a $2(a + 5) + 7(a + 5) = \square(a + 5)$
   b $12(b - 6) - 3(b - 6) = \square(b - 6)$
   c $8(c + 2) + (c + 2) = \square(c + 2)$
   d $-2(d - 7) + (d - 7) = \square(d - 7)$

④ Complete these simplifications.

a $x(x + 5) + 7(x + 5) = (\square + \square)(x + 5)$

b $x(x - 6) - 3(x - 6) = (\square - \square)(x - 6)$

c $x(x + 2) + (x + 2) = (\square + \square)(x + 2)$

d $-x(x - 7) + (x - 7) = (\square x + \square)(x - 7)$

⑤ Factorise each expression.

a $x^2 + 8x + 15$ **b** $x^2 - 9x + 20$ **c** $x^2 + 8x + 16$

d $x^2 + 4x - 21$ **e** $x^2 - x - 6$ **f** $x^2 + 17x + 16$

g $x^2 - 15x - 16$ **h** $x^2 - 8x - 20$

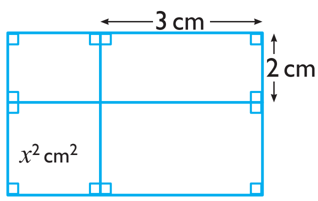
⑥ Factorise each expression.

a $x^2 + 8x$ **b** $x^2 - 100$ **c** $x^2 - 16x$

d $x^2 - 16$ **e** $x^2 + 7x + 6$ **f** $x^2 - 1$

g $x^2 - 144$ **h** $x^2 - 25$

⑦ This rectangle is divided into four regions. One of them is a square of area x^2 cm^2. The others are rectangles. The diagram shows that one of the rectangles is 3 cm by 2 cm.



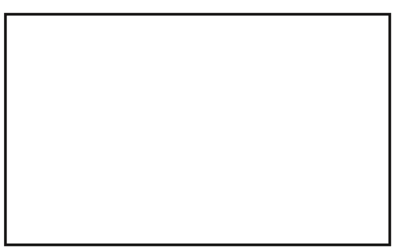
a Copy the diagram and label the areas of the four regions.

b Work out the length and width of the whole rectangle.

c Use the diagram to factorise $x^2 + 5x + 6$.

d Use algebra to check your answer to part **c**.

⑧ This rectangle has an area of $x^2 + 6x + 8$ cm^2.



a Make a copy and divide the rectangle into four regions with areas x^2 cm^2, $4x$ cm^2, $2x$ cm^2 and 8 cm^2.

b Show that the perimeter of the rectangle is $(4x + 12)$ cm and calculate its area.

c Use your diagram to factorise $x^2 + 6x + 8$.

d Use algebra to check your answer to part **c**.

⑨ **a** Expand these quadratic expressions.

i $(x + 7)(x + 3)$

ii $(x + 7)(x + 2)$

b Simplify $(x + 7)(x + 3) - (x + 7)(x + 2)$

c How can you work out the answer to part **b** without expanding the brackets?

Do I know it now?

① Factorise these quadratics.

a $x^2 - 6x + 8$ **b** $x^2 + x - 12$ **c** $x^2 - x - 12$

d $x^2 + 3x - 10$ **e** $x^2 - 3x - 10$ **f** $x^2 - 8x + 16$

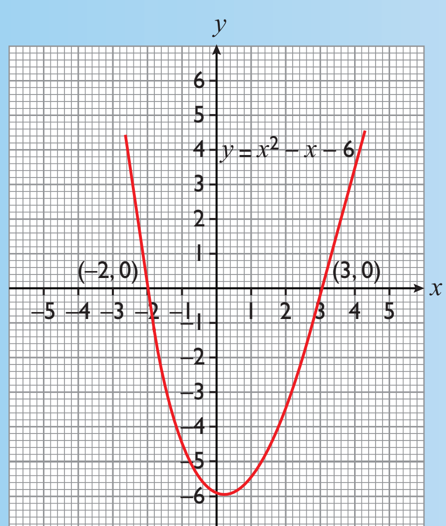
g $x^2 - 49$ **h** $x^2 - 9$

② Show that $(x + 7)(x + 1) - (x + 2)(x + 6) + (x + 2)(x - 2) = (x + 3)(x - 3)$.

12.2 Solving equations by factorising

What you need to know

Here is a **quadratic graph**.

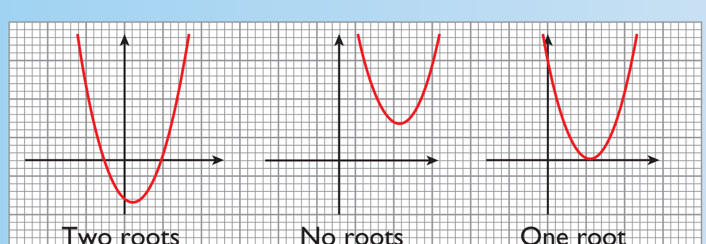


The **parabola** crosses the x axis at $(-2, 0)$ and $(3, 0)$. -2 and 3 are known as the **roots** of the equation $y = x^2 - x - 6$.

They are also the solution to the equation $x^2 - x - 6 = 0$.

This equation therefore has two solutions or roots, $x = -2$ and $x = 3$.

A quadratic function can have 0, 1 or 2 roots, depending on where its curve crosses the x axis.



Some quadratic equations can be factorised and this gives an algebraic method for finding the roots.

The algebraic method uses the fact that if two numbers multiply to give zero, then at least one of those numbers must be zero.

$x^2 + (a + b)x + ab = (x + a)(x + b)$ ← Factorising.

$(x + a)(x + b) = 0$ ← Roots are where its curve crosses the x axis, i.e. $y = 0$.

Either $x + a = 0$

$x = -a$ ← Rearranging.

or $x + b = 0$

$x = -b$ ← Rearranging.

For our example, we can find the roots of $y = x^2 - x - 6$ by factorising.

$x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$ ← Factorising, $2 - 3 = -1$, $2 \times -3 = -6$.

Either $x + 2 = 0$ so $x = -2$

or $x - 3 = 0$ so $x = 3$.

You can check your answer by substituting back into the equation.

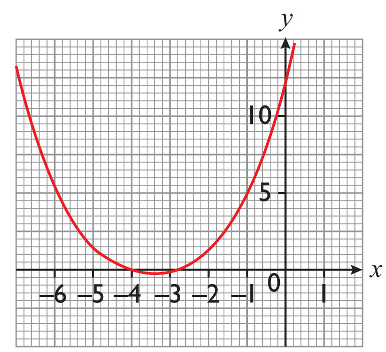
How to do it

Solving equations

The graph shows the function $y = x^2 + 7x + 12$.

a Solve $x^2 + 7x + 12 = 0$ algebraically.

b Solve $x^2 - 3x = 0$.



Solution

a $x^2 + 7x + 12 = 0$

Factorise first:

$(x + 3)(x + 4) = 0$ ← Two numbers that multiply to give +12 and add to give +7 are +3 and +4.

Either $x + 3 = 0$

$x = -3$ ← Check: $(-3)^2 + 7 \times (-3) + 12 = 9 - 21 + 12 = 0$ ✓

or $x + 4 = 0$

$x = -4$ ← Check: $(-4)^2 + 7 \times (-4) + 12 = 16 - 28 + 12 = 0$ ✓

b $x^2 - 3x = 0$

Factorise first:

$x(x - 3) = 0$ ← There is an 'x' in both terms.

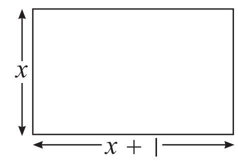
Either $x = 0$ ← Check: $0^2 - 3 \times 0 = 0$ ✓

or $x - 3 = 0$

$x = 3$ ← Check: $3^2 - 3 \times 3 = 9 - 9 = 0$ ✓

Solving geometrical problems

A rectangle has width x metres and length one metre more than its width. The area of the rectangle is $20\,m^2$.



a Form a quadratic equation for x.

b Solve the equation and find the length and width of the rectangle.

Solution

a Area of rectangle $= x(x + 1)$

So, $x(x + 1) = 20$ ← The area is $20\,m^2$.

$x^2 + x = 20$ ← Expanding.

$x^2 + x - 20 = 0$ ← You need to rearrange a quadratic equation into the form $ax^2 + bx + c = 0$ before you can solve it.

b $x^2 + x - 20 = 0$ ← Two numbers that multiply to give -20 and add to give $+1$ are $+5$ and -4.

$(x + 5)(x - 4) = 0$

Either $x + 5 = 0$

$x = -5$ ← Not a valid solution to this problem.

or $x - 4 = 0$

$x = 4$

The width of the rectangle is 4 metres.

The length of the rectangle is 5 metres. ← The length is 1 metre more than the width.

Check: $4 \times 5 = 20$ ✓

Learning exercise

① Solve these equations.

a $(x + 4)(x + 1) = 0$
b $(x - 3)(x + 7) = 0$
c $(x - 1)(x - 1) = 0$
d $(x + 2)(x + 1) = 0$

② Solve these equations.

a $x^2 - 7x + 12 = 0$
b $x^2 - x - 2 = 0$
c $x^2 + 2x - 15 = 0$
d $x^2 + 6x + 5 = 0$
e $x^2 - 4 = 0$
f $x^2 - 7x = 0$
g $x^2 + x - 12 = 0$
h $x^2 - 9 = 0$

③ Rearrange and solve these equations.

a $x^2 + 4x = -3$
b $x^2 + 2x = 8$
c $x^2 = -x$
d $x^2 = 3x + 4$
e $x^2 = 49$

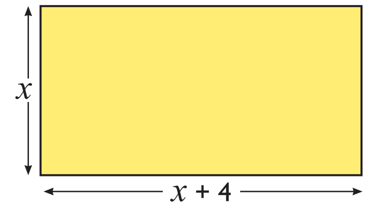
④ Solve these equations.

a $x^2 - 64 = 0$
b $x^2 = 9x + 36$
c $(x + 8)(x - 2) = 0$
d $x^2 + x - 2 = 0$
e $x^2 - 9x = 0$

⑤ These are the solutions of quadratic equations. Write down each quadratic equation in the form $x^2 + bx + c = 0$.

a 3 or 5 **b** -3 or -5 **c** 6 or -8 **d** 1 or -1 **e** 9 or -10 **f** 0 or -6

⑥ A rectangle measures x cm by $(x + 4)$ cm.



The area of the rectangle is 45 cm^2.

a Write the area of the rectangle in terms of x.

b Form a quadratic equation in terms of x and rearrange it to the form $x^2 + bx + c = 0$.

c Solve the quadratic equation.

d What are the length and width of the rectangle?

⑦ A rectangle measures x cm by $(x - 2)$ cm and has area 48 cm^2.

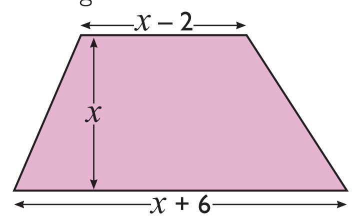
a Form a quadratic equation in terms of x and solve it.

b What are the dimensions of the rectangle?

⑧ A triangle has height $(x + 6)$ cm and base x cm. The area of the triangle is 8 cm^2.

Form and solve a quadratic equation to work out the dimensions of the triangle.

⑨ The diagram shows the dimensions of a trapezium.

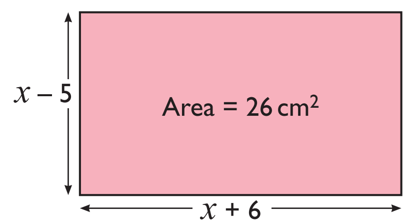


The area of the trapezium is 35 cm^2.

Form and solve a quadratic equation to work out the dimensions of the trapezium.

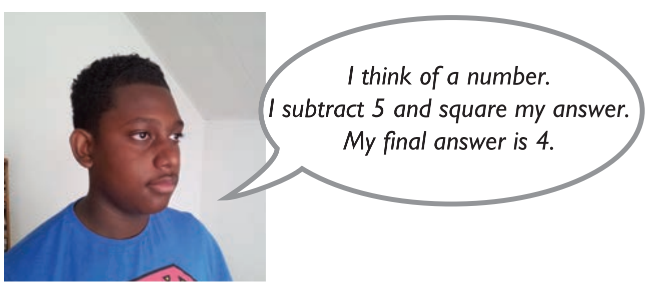
Problem solving exercise

① A rectangle has length $(x + 6)$ cm and width $(x - 5)$ cm.



Work out the length and the width of the rectangle.

② What numbers could Andy be thinking about?



Do I know it now?

① Solve these equations.

a $x^2 + 5x + 6 = 0$	**b** $x^2 - 6x + 8 = 0$	**c** $x^2 - x - 2 = 0$
d $x^2 + 3x - 28 = 0$	**e** $x^2 + 5x = -4$	**f** $x^2 + 6 = 10$
g $x(x - 2) = 15$	**h** $x^2 - 10 = 0$	

Can I apply it now?

① A rectangle has width 3 cm less than its length.

The area of the rectangle is 54 cm^2.

Form and solve a quadratic equation to work out the dimensions of the rectangle.

CHAPTER 13

NEXT STEPS – GEOMETRY AND MEASURES

Units and scales

13.3 Working with compound units

What you need to know

Did you know?

Water has a density of $1\,g/cm^3$ and so all other materials are compared with water. If their density is greater than water, they sink; if they are less dense than water, they float. A boat, or any other item, will float if the average density of the material, including the air contained within it, is less than $1\,g/cm^3$. This means that boats can be made of heavier material or carry heavier loads than you might expect.

The table shows some common compound measures.

Measure	Description	Common units
Speed	Distance travelled in 1 unit of time	m/s, km/h
Acceleration	Change in speed over 1 unit of time	m/s^2, km/h^2
Density	Mass of 1 unit of volume	g/cm^3, kg/m^3
Unit price	The price of an item for 1 unit of weight, area or volume	pence/gram, £/kg, pence/litre, £/gallon, £/m^2
Population density	Number of people in 1 unit of area	people/km^2
Pressure	Force exerted on a square metre	Newtons/m^2

The units tell you the calculation to perform to work out the compound measure.

For example, speed in m/s is calculated as:

metres (distance) ÷ seconds (time).

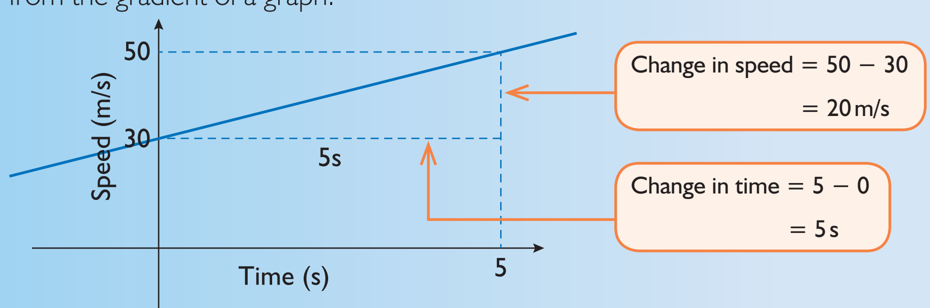
So, to convert from one compound unit to another, e.g. convert $6.7\,g/cm^3$ into kg/m^3:

$6.7\,g = 0.0067\,kg$ ← 1 kg = 1000 g

$1\,cm^3 = 0.000\,001\,m^3$ ← $1\,cm^3$ is 0.01 m by 0.01 m by 0.01 m.

$$\text{Density} = \frac{6.7\,g}{1\,cm^3} = \frac{0.0067\,kg}{0.000\,001\,m^3} = 6700\,kg/m^3$$

Speed and acceleration are also known as **rates of change**. They can often be found from the gradient of a graph.



Acceleration is the gradient of a graph of speed against time.

$$\text{Acceleration} = \left(\frac{20}{5}\right) = 4\,m/s^2$$

How to do it

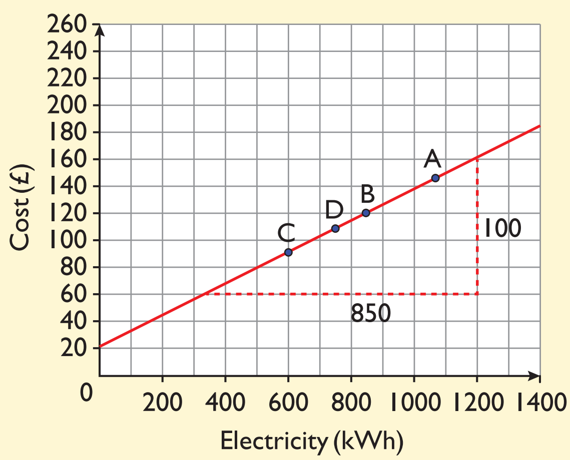
Electricity bills

Totals from SuLin's last four electricity bills are given below.

A 1070 kWh   Total cost £145.00
B 850 kWh   Total cost £119.30
C 600 kWh   Total cost £90.09
D 750 kWh   Total cost £107.61

SuLin wants to find out how much she pays per kWh.
Draw a graph and use it to find this information.

Solution



$$\text{Cost per kWh} = \frac{100\,(£)}{850\ (\text{kWh})}$$
$$= £0.12 \text{ per kWh}$$
$$= 12\text{p per kWh}$$

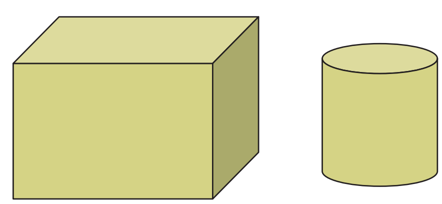
Problems involving density

A brass block has a mass of 187 g and a volume of 22 cm^3.

a Calculate the density of brass in g/cm^3.

A brass cylinder has a mass of 53 g.

b Calculate the volume of the cylinder.



Solution

a Density = mass ÷ volume
= 187 ÷ 22
= 8.5 g/cm^3

b Density = mass ÷ volume
$8.5 = 53 \div v$
$8.5v = 53$ Multiply both sides by v.
$v = 53 \div 8.5 = 6.2$ Divide both sides by 8.5.
So the volume is 6.2 cm^3.

Learning exercise

① Which is better value? Show your method clearly.

a 1.5 litres of lemonade for 65p or 1 litre for 44p?

b 15 pencils for 99p or 12 for 75p?

c 3 kg of grass seed for £4.20 or 85 g for 96p?

d 650 g of dog food for £2.15, 1.5 kg for £4.65 or 5 kg for £18?

② Lucy swims 500 m in 2 minutes and 20 seconds.

Calculate her speed in

a metres per second **b** kilometres per hour.

③ A block of metal has a mass of 475 g and a volume of 225 cm^3.

Calculate the density in

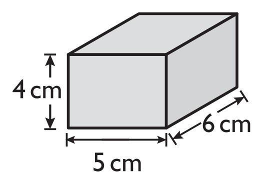
a g/cm^3 **b** kg/m^3.

④ A gymnast of weight 720N does a handstand. The area of his hands is 300 cm^2. What pressure do his hands exert on the floor? Give your answer in N/m^2

⑤ Dave runs n metres in t seconds. Write his speed in

a m/s **b** km/h.

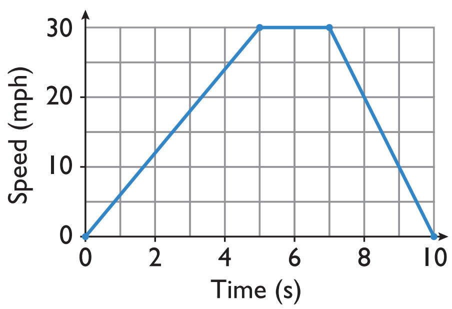
⑥ The mass of this block of silver is 1260 g.



a Calculate the density of silver in g/cm^3.

b Find the mass of a 20 centimetre cube of silver in kilograms.

⑦ This graph shows the way that the speed of one car changes in a certain 10-second period.



a What is happening to the car between 5 and 7 seconds into its journey?

b What is the rate of change of speed during the first 5 seconds?

c What term describes the rate of change of speed?

⑧ Jean is driving from the south of France to Paris.

She sees a road sign.

She knows that it takes her 1 hour to travel 60 miles.

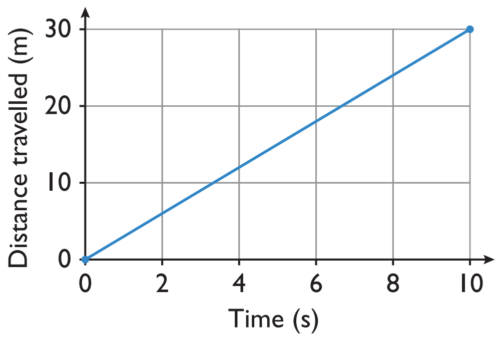
The time now is 10 p.m.

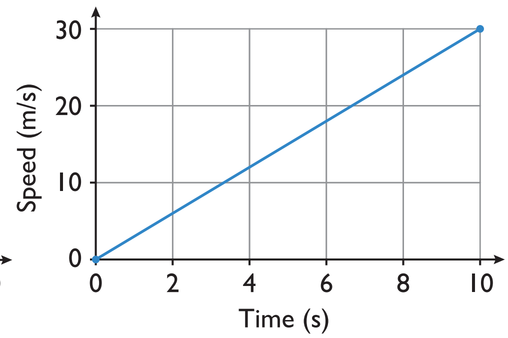
Jean wants to get to Paris by 2.30 a.m.

Will she be able to get to Paris by 2.30 a.m.?

⑨ Salima knows that the fuel tank on her car has a capacity of 9 gallons.
When she was in France she filled up the tank from half full at a cost of €30.
£1 = €1.25
Work out the cost in pounds of a litre of fuel in France.

⑩ Andy says that these two graphs are for the same car at the same time.
Is Andy correct?
Explain how you know.





⑪ The pressure exerted by an elephant's foot is f N/m^2 (Newtons per square metre).
Write this in Newtons per square centimetre.

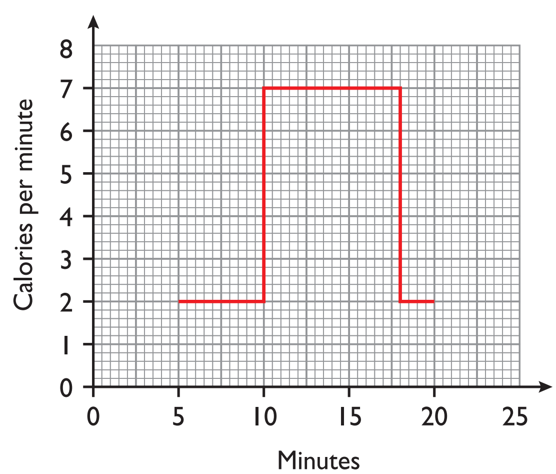
⑫ A concrete block has a volume of v cm^3 and a mass of m g.
Write the density in kg/m^3.

⑬ Show that V metres per second is about the same speed as $2.25V$ miles per hour.

Problem solving exercise

① Bob is following a fitness programme.
He wears a device that monitors his calorie burn.

The graph shows some information about his calorie burn during a period of 15 minutes.



a Work out the number of calories Bob burned over the first 5 minutes of the 15-minute period.

b Work out the average rate at which he burned calories over the 15 minutes.

Bob found that his average rate of calorie burn was 2.5 calories per minute when he was awake and 1.5 calories per minute when he was asleep.

c Work out the total number of calories Bob burned during a 24-hour period when he spent 8 hours asleep.

2 The diagram shows two bottles of the same make of shampoo.

Which bottle gives better value for money?

Show your working.

3 The density of wood is 0.9 g/cm^3.

a Work out the density of the wood in kg/m^3.

x litres of oil have a mass of y grams.

b Work out an expression for the density of the oil in kg/m^3.

4 Jim uses a water meter in his house.

The cost of the water he uses during one year is the sum of the meter charge + the charge for the water used.

The meter charge is £41.

The charge for the water used is £2.10 per 1000 litres.

On average Jim uses 170 litres of water per day.

Work out the cost of the water Jim uses in one year.

5 The BMI of a person is calculated using the rule

BMI = mass in kg ÷ height in m^2

The ideal range for BMIs is between 18 and 25.

Bill weighs 154 pounds and has a height of 183 cm.

Is Bill's BMI in the ideal range? Give a reason for your answer.

Do I know it now?

1 Peter is driving in France.

At 10 a.m. he sees a road sign which says Calais 308 km.

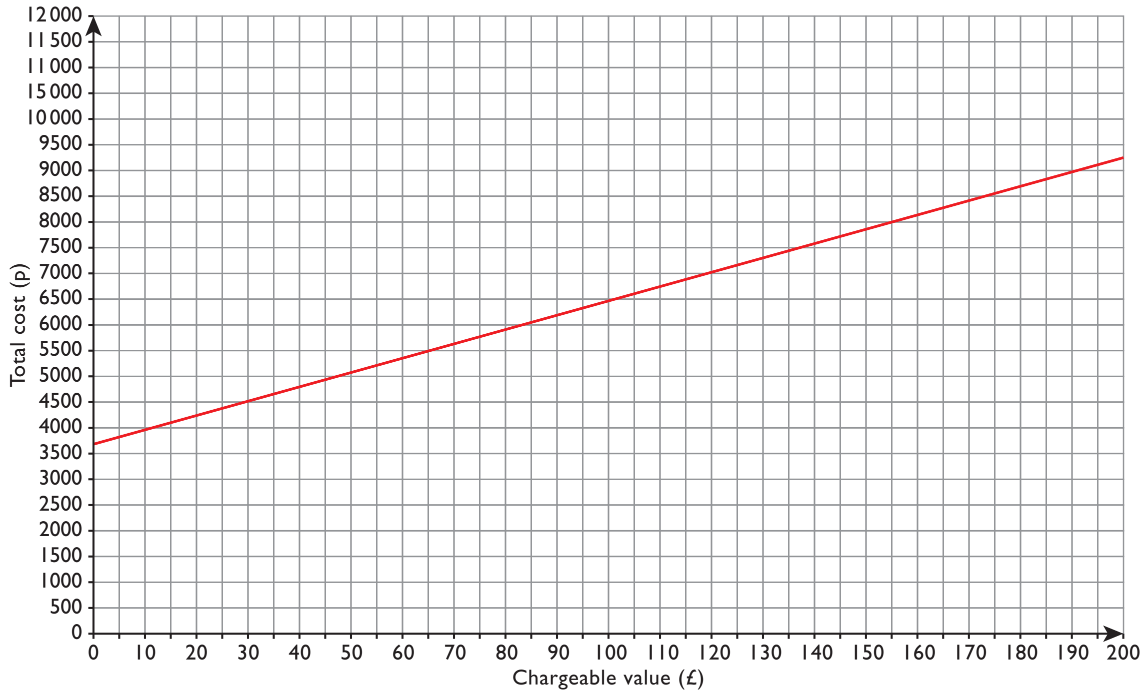
Peter knows that he can drive at a maximum speed of 55 miles per hour.

Work out the earliest time Peter can get to Calais.

2 A 275 cm^3 block of expanded polystyrene has a mass of 15 g.

What is the density of expanded polystyrene in kg/m^3?

③ Peter's water is unmetered. His water company provides the graph below so residents can work out how much they will be charged. The chargeable value of Peter's house is £140.



a How much will Peter be charged? Give your answer in pounds.

b Estimate how much the water company charges per pound of chargeable value.

Can I apply it now?

① A company sells gold bars in the shape of cuboids.

The measurements of a bar are 80 mm by 40 mm by 16 mm.

The density of gold is 19.32 g/cm^3.

Tom thinks that the mass of the block is over 1 kilogram.
Is he correct? Explain your answer.

CHAPTER 14

NEXT STEPS – GEOMETRY AND MEASURES

Properties of shapes

14.3 Congruent triangles and proof

What you need to know

Did you know?

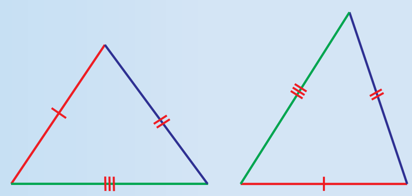
Builders use triangular supports to give buildings strength because triangles are rigid shapes.

A **proof** is a water-tight argument that cannot be disputed. A mathematical proof starts from a position that is known to be true. Each step is justified. The explanation is often written in brackets. The proofs in this unit involve angle facts and congruent triangles.

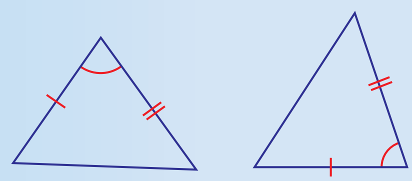
Congruent shapes are exactly the same shape and size.

There are four sets of conditions that prove two triangles are congruent.

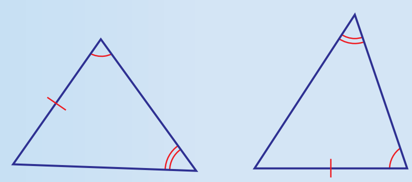
① **SSS**. The three sides of one triangle are equal to the three sides of the other.



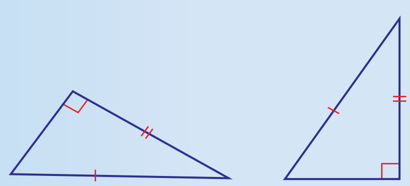
② **SAS**. Two sides of one triangle are equal to two sides of the other, and the angles between those sides are equal.



③ **ASA**. Two angles of one triangle are equal to two angles of the other, and a pair of corresponding sides are equal.



④ **RHS**. Two right-angled triangles have equal hypotenuses and another pair of equal sides.

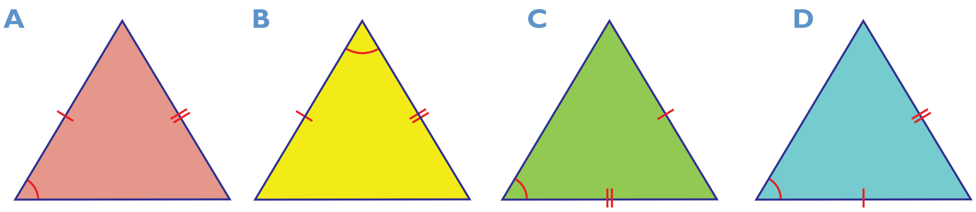


Note that three angles being equal in a pair of triangles (AAA) is not proof of congruence. One triangle may be bigger than the other. It does, however, prove they are similar.

How to do it

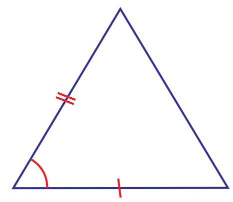
➤ Recognising congruence

These four triangles have two sides and one angle equal.



a Are any pairs of these triangles congruent?

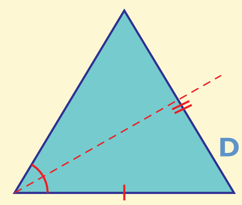
b Which triangle is congruent to this one?



Solution

a A and D are congruent. If you reflect D in the line bisecting the marked angle, you get A (SAS).

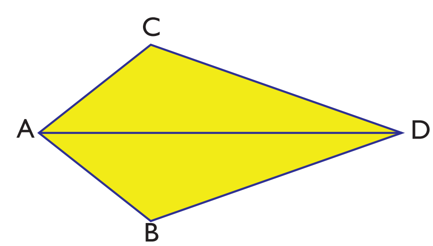
b The yellow triangle (B) is congruent to the white one (SAS).



➤ Using congruent triangles

In this kite, AB = AC and BD = CD.

Prove that angles ABD and ACD are equal.



Solution

In triangles ABD and ACD:

AB = AC   Given

BD = CD   Given

AD is common.

Therefore triangles ABD and ACD are congruent.   SSS

So, ∠ABD = ∠ACD   Corresponding angles of congruent triangles.

Learning exercise

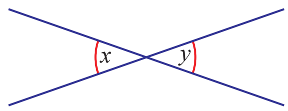
① Say why each of the following pairs of angles are equal.

In each case choose a reason from this list.

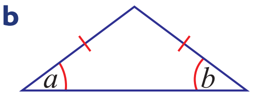
Alternate angles | Corresponding angles | Vertically opposite angles

Base angles in an isosceles triangle | Opposite angles of a parallelogram

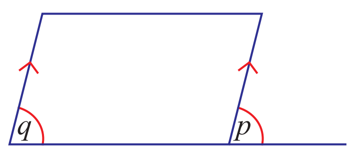
a



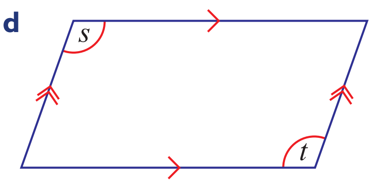
b



c

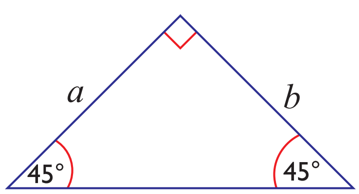


d

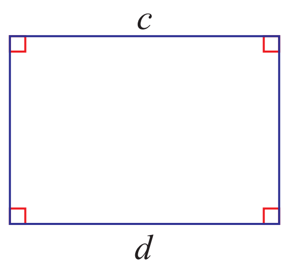


② For each part, say why the pairs of lettered lines are equal.

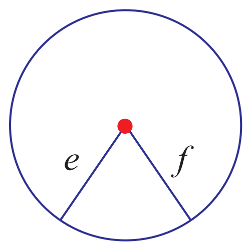
a



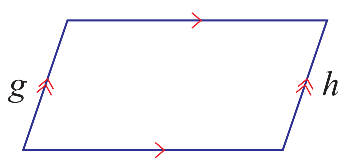
b



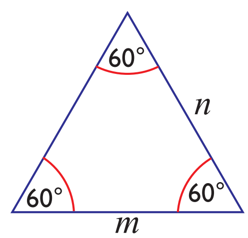
c



d



e

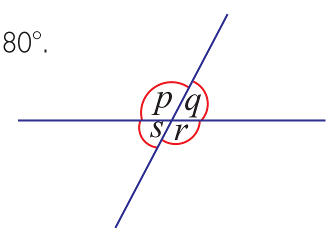


③ **a** State two pairs of equal angles in this diagram.

b State four different pairs of angles that add up to 180°.

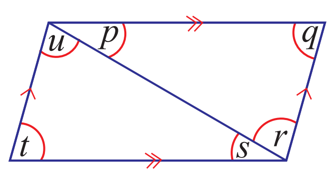
c Copy and complete this statement.

$p + q + r + s =$ ___ (angles round a _______)



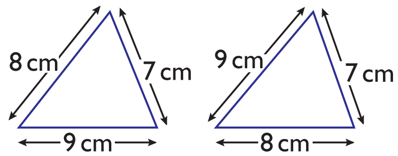
④ **a** State, giving reasons, three pairs of equal angles in this diagram.

b State, giving reasons, four different sets of three angles from the diagram that add up to 180°.

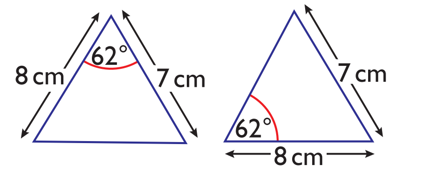


⑤ Say whether the triangles in each pair are congruent. If they are, give a reason (SSS, SAS, ASA, RHS).

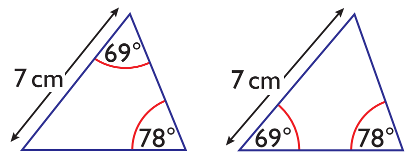
a



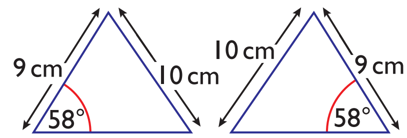
b



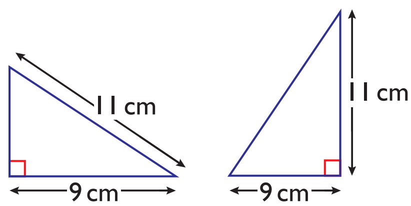
c



d



e



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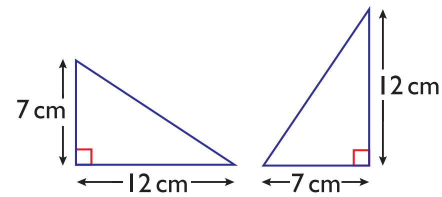

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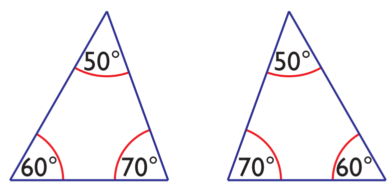

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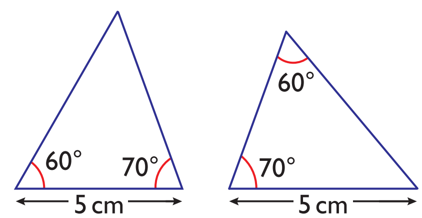
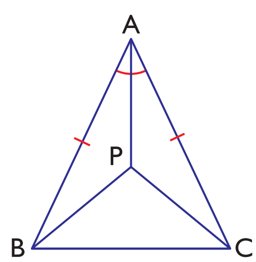


⑥ ABC is an isosceles triangle with AB = AC.

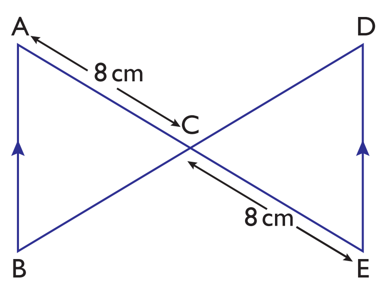
AP bisects ∠BAC.

Prove that PB = PC.



⑦ In the diagram, AB and DE are parallel.

AC and CE are both 8 cm long.



a Copy and complete this proof that triangles ABC and CDE are congruent.

In triangles ABC and CDE:

AC = ___ (both given as 8 cm)

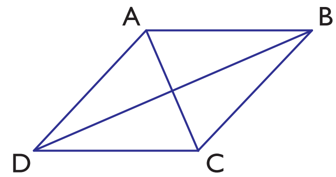
∠ACB = ∠ECD (___________)

∠BAC = _____ (alternate angles)

So triangles ABC and CDE are congruent (____).

b What does this tell you about lines AB and DE?

⑧ The diagram shows a rhombus, ABCD.



a Copy and complete this proof that triangles DAB and DCB are congruent.

In triangles DAB and DCB:

AB = CB (a rhombus has equal sides)

AD = CD (___________)

BD is common to both triangles.

So triangles DAB and DCB are congruent (_____).

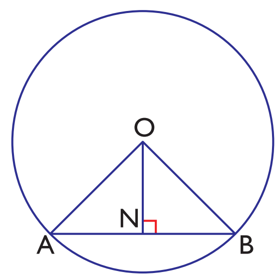
b What does this tell you about ∠ADB and ∠CDB?

⑨ AB is a chord of a circle, centre O.

ON is perpendicular to AB.

a Prove that triangles OAN and OBN are congruent.

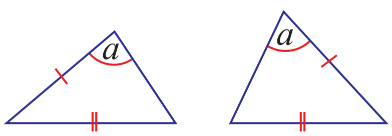
b What does this tell you about point N?



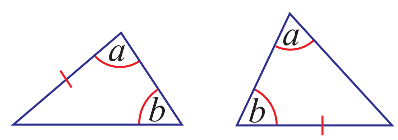
⑩ Peter has given reasons why the triangles below are congruent.

Explain his errors.

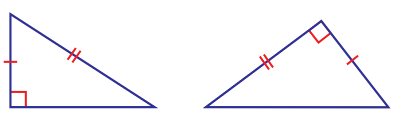
a SAS



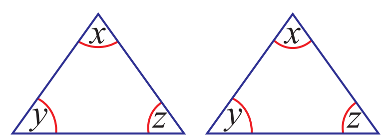
b ASA



c RHS

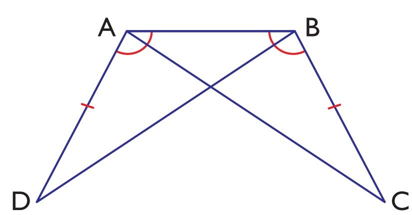


d AAA



⑪ In this diagram, angles DAB and CBA are equal and AD = BC.

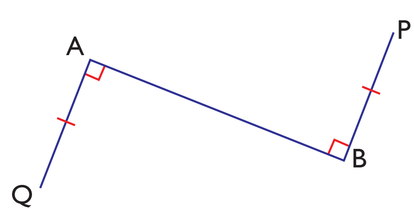
Prove that BD = AC.



⑫ In this diagram, AQ and BP are perpendicular to AB.

AQ = BP

Prove that AB bisects PQ.

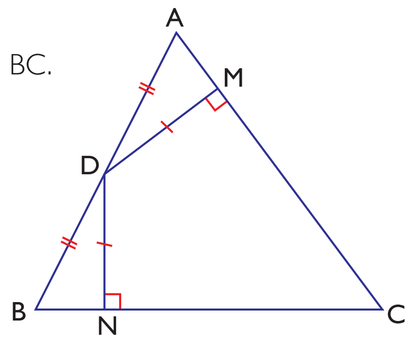


⑬ In this diagram, D is the mid-point of the line AB.

DM and DN are perpendiculars from D to AC and BC.

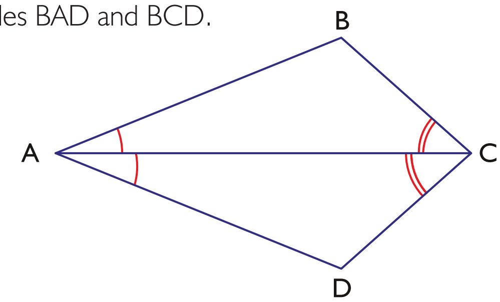
DM = DN

Prove that triangle ABC is isosceles.



⑭ ABCD is a quadrilateral. The diagonal AC bisects the angles BAD and BCD.

Prove that the quadrilateral is a kite.



Problem solving exercise

① ABCDE is a regular pentagon.

a What does this tell you about its sides?

b What does this tell you about its interior angles?

c Use congruent triangles to prove that BE = BD.

d Which other lines are diagonals of this pentagon?

Explain how you know that all the diagonals are equal in length.

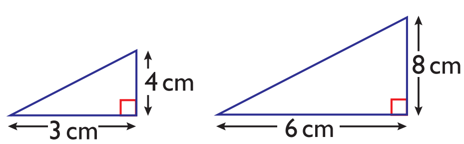
Do I know it now?

① Say whether the following pairs of triangles are congruent or similar.

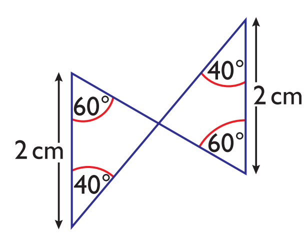
In each case, choose a reason from this list.

SSS   SAS   ASA   RHS

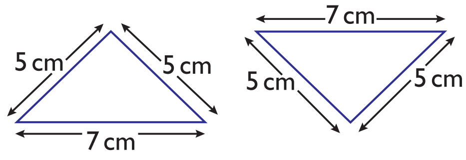
a



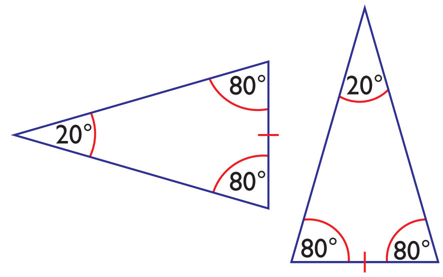
b



c



d



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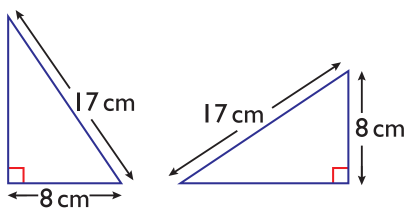

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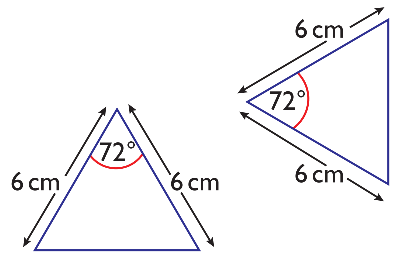
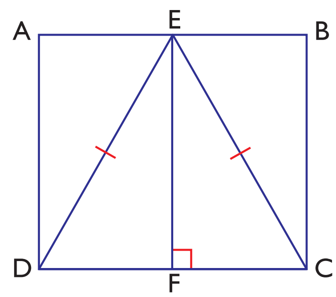


② CDE is an isosceles triangle with CE = DE.
ABCD is a rectangle, and EF is perpendicular to CD.

a Prove that triangles ADE and BCE are congruent.

b What does this tell you about point E?



Can I apply it now?

① ABCD is a parallelogram. Therefore its opposite sides are equal and parallel.
Prove that the diagonals AC and BD bisect each other.

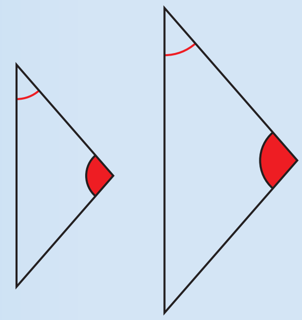
14.4 Proof using similar and congruent triangles

What you need to know

You have met several standard geometry results. These can be proved using angle facts, congruent triangles and similar triangles.

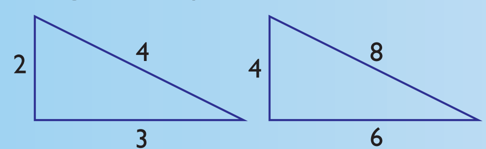
Similar triangles are the same shape but not necessarily the same size.

To prove two triangles are similar, you need to show **two pairs of angles are equal (AA)**. Since the angles of each triangle add up to 180°, the third pair must also be equal.



or

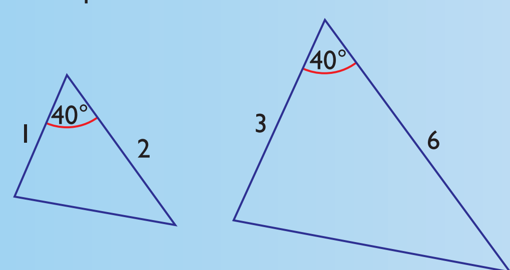
The ratios of the lengths of corresponding sides of similar triangles are equal.



or

A combination of these gives one more option.

The ratios of two corresponding sides are the same and the angles between these sides are equal.

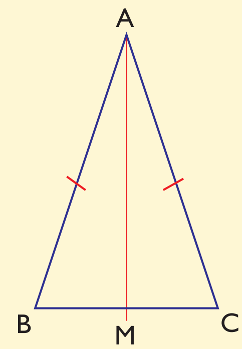


How to do it

Using congruent triangles to prove a standard result

ABC is an isosceles triangle with AB = AC.
Prove that ∠ABC = ∠ACB.

Solution



Mark the mid-point of BC as M.

Join AM.

In the triangles ABM and ACM:

AB = AC ← Given

BM = CM ← M is mid-point of BC.

AM is common.

Therefore triangles ABM and ACM are congruent. ← SSS

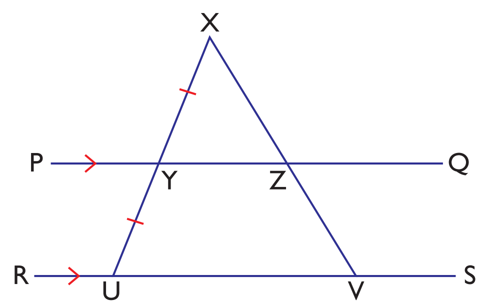
So ∠ABM = ∠ACM ← Corresponding angles of congruent triangles

These are the required angles.

∠ABC = ∠ACB

Proof using similar triangles

In the diagram, PQ and RS are parallel lines.
XYU and XZV are straight lines.
XY = YU
Prove that UV = 2YZ.



Solution

In the triangles XYZ and XUV:

∠XYZ = ∠XUV (Corresponding angles) This is because lines PQ and RS are parallel.

∠YXZ is common.

Therefore triangles XYZ and XUV are similar (AA).

So, the sides are in ratio.

$\frac{XU}{XY} = \frac{UV}{YZ} = \frac{VX}{ZX}$

Since XY = YU (given), XU = 2XY and $\frac{XU}{XY} = 2$.

Therefore $\frac{UV}{YZ} = 2$ and UV = 2YZ.

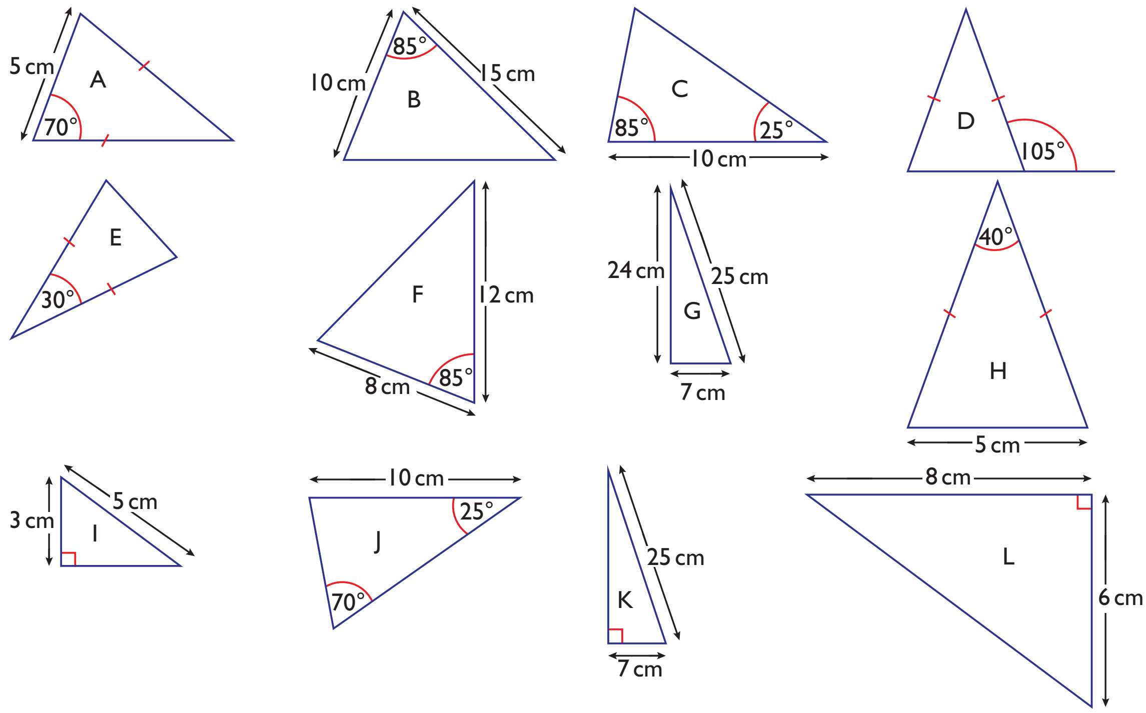
Learning exercise

① Here are 12 triangles. They are 6 pairs.

The pairs are either similar or congruent.

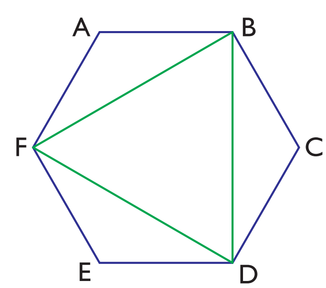
Identify the pairs.

For each pair, justify why you are linking them.



② ABCDEF is a regular hexagon.

Use congruent triangles to prove that BDF is an equilateral triangle.



③ ABCD is a quadrilateral.

AB = CD and AD = BC

The angles at A, B, C and D are all 90°.

a What sort of quadrilateral is ABCD?

b Prove triangles ADB and BCA are congruent.

c Hence show that AC = BD.

d What general result have you proved?

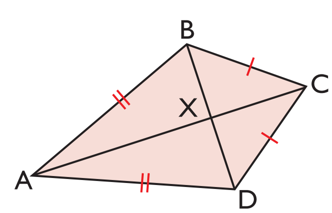
④ ABCD is a kite. The diagonals meet at X.

a Prove triangles ABC and ADC are congruent.

b Hence prove triangles ABX and ADX are congruent.

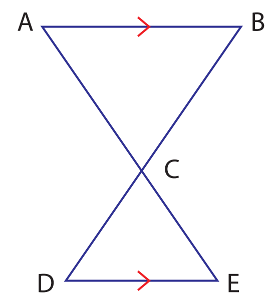
c Show that X is the mid-point of BD.

d State this as a general result that is true for all kites.



5 In the diagram, the lines AB and DE are parallel.

AC = 2CE



a Copy and complete this proof that triangles ABC and DEC are similar.

In triangles ABC and DEC

$\angle ACB = \angle$____ (vertically opposite)

$\angle ABC = \angle$____ (alternate)

Therefore the triangles are similar (_____).

b Write down the value of $\frac{AC}{EC}$.

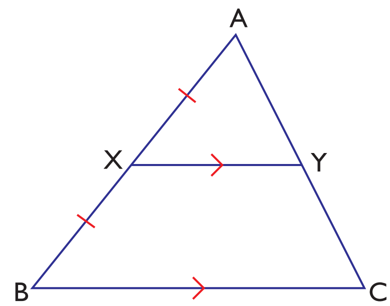
c What fraction of the way along AE is the point C?

6 ABC is a triangle.

X is the mid-point of AB.

Y is a point on AC such that XY is parallel to BC.

Prove that Y is the mid-point of AC and $XY = \frac{1}{2}BC$.



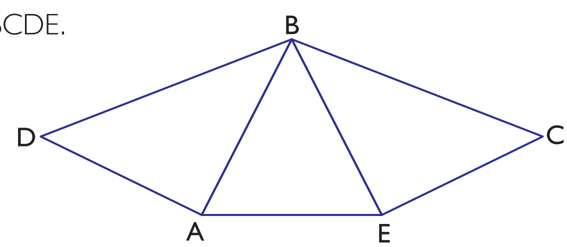
7 The diagram shows the pentagon ABCDE.

ABE is an equilateral triangle.

$\angle DAB = 90°$

$\angle CEB = 90°$

DB = BC.



a Prove that triangles DAB and CEB are congruent.

b Prove that $\angle DBE = \angle CBA$.

c Explain whether the result in part **a** is still true if triangle ABE is isosceles.

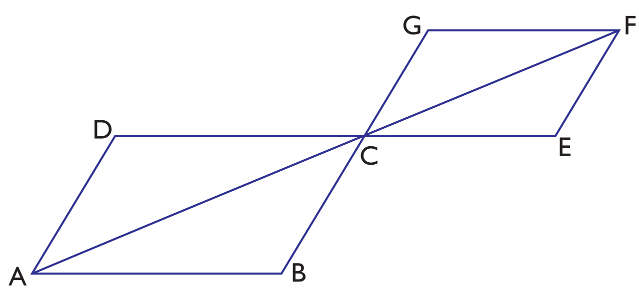
8 ABCD and CEFG are parallelograms.

BCG and DCE are straight lines.

a Prove that triangle CFG is similar to triangle CBA.

Write down reasons at each stage of your working.

b Given that DC : CE = 3 : 2, find the value of AF : AC.



9 In this diagram, ABCD is a parallelogram.

The diagonals AC and BD bisect each other at O.

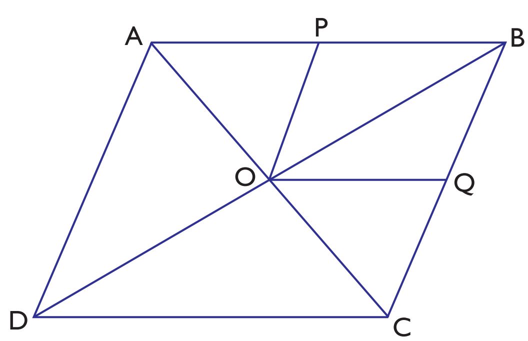
OP is parallel to DA and OQ is parallel to DC.

a Prove that triangles OPB and DAB are similar.

b Write down another pair of similar triangles.

c Prove that P is the mid-point of AB.

d Prove that $PQ = \frac{1}{2}AC$.



⑩ PQR is a right-angled triangle. QRS is an isosceles triangle with SQ = SR.

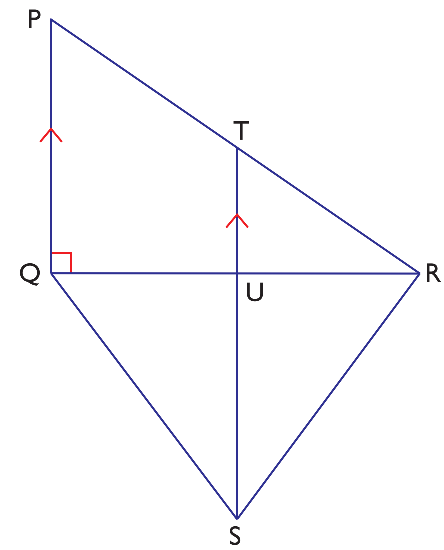
ST is parallel to QP.

a Prove that triangles SQU and SRU are congruent.

b Write down the ratio QR : UR.

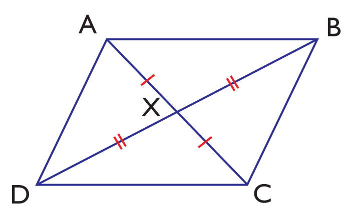
c Prove that triangles PQR and TUR are similar.

d Show that PQ = 2TU.



Do I know it now?

① In the diagram, AC and BD are two straight lines that bisect each other at X.



a Prove that AB = CD and ∠BAX = ∠DCX.

b What does this tell you about the quadrilateral ABCD? Give reasons for your answer.

CHAPTER 15

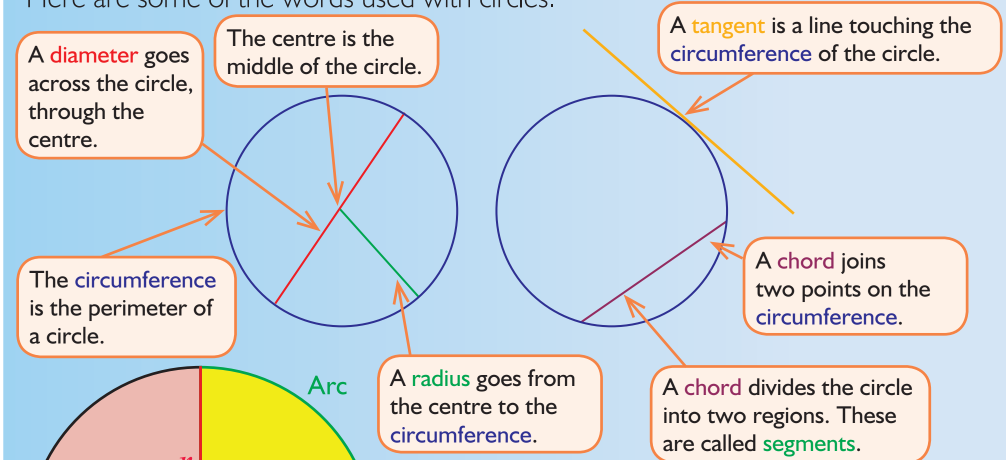
NEXT STEPS – GEOMETRY AND MEASURES

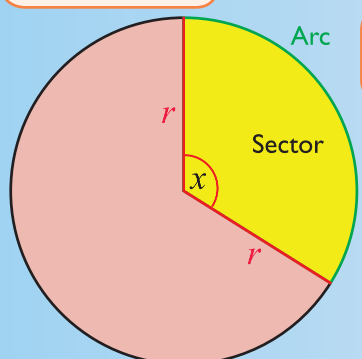
Measuring shapes

15.3 Arcs and sectors

What you need to know

Here are some of the words used with circles.





The yellow region of this circle is a **sector**.

The sector is bounded by two radii and an arc.

The sector makes an angle $x°$ at the centre of the circle.

The arc length, $l = \frac{x}{360} \times \pi d$

The area of the sector, $A = \frac{x}{360} \times \pi r^2$

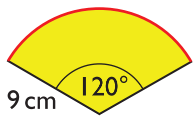
Did you know?

Many garden designs involve arcs or sectors. It is important to calculate the length of the border or the area to be covered accurately so that the right number of plants or edging sections can be bought.

How to do it

Arcs

A sector has a radius of 9 cm and an angle at the centre of 120°.



The arc is marked in red.

a Calculate the length of the arc.

b Find the perimeter of the sector.

c Another arc has the same radius and a length of 9.4 cm.

Calculate the angle at the centre.

Solution

a Length of arc $= \dfrac{x}{360} \times \pi d$

$= \dfrac{120}{360} \times \pi \times 18$ ← The radius is 9 cm so $d = 18$ cm.

$= 6\pi$ ← You will often give your answer like this, as a multiple of π.

$= 18.8$ cm ← To 1 decimal place.

b The perimeter of the sector is made up of two radii and the arc.

Perimeter $= 9 + 9 + 6\pi$

$= (18 + 6\pi)$ cm ← Leaving the answer in terms of π.

$= 36.8$ cm ← To 1 decimal place.

c Arc length $= \dfrac{x}{360} \times \pi d$

$9.4 = \dfrac{x}{360} \times \pi \times 18$ ← Replace arc length with 9.4 and d with 18.

$9.4 \times 360 = x \times \pi \times 18$ ← Multiply both sides by 360.

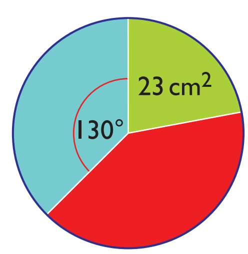
$\dfrac{3384}{18\pi} = x$ ← Divide both sides by 18π.

$x = 59.8$

So the angle at the centre is 59.8°.

Area of a sector

A circle of radius 6 cm is divided into three sectors as shown.



a The blue sector has an angle of 130°.

Calculate the area of the blue sector.

b The green sector has an area of 23 cm².

Calculate the angle of the green sector.

c Show that the area of the red sector is $23(\pi - 1)$ cm².

Solution

a Area of blue sector

$$A = \frac{x}{360} \times \pi r^2$$

$$= \frac{130}{360} \times \pi \times 6^2$$ Replace x with 130 and r with 6.

$$= \frac{130 \times 36}{360} \times \pi$$ Simplifying.

$$= 13\pi$$ This is the answer in terms of π.

$$= 40.8\,\text{cm}$$ To 1 d.p.

b Area of green sector

$$A = \frac{x}{360} \times \pi r^2$$

$$23 = \frac{x}{360} \times \pi \times 6^2$$ Replace A with 23 and r with 6.

$$23 \times 360 = x \times \pi \times 36$$ Multiply both sides by 360.

$$\frac{8280}{36 \times \pi} = x$$ Divide both sides by 36π.

$$x = 73.2$$

The angle of the green sector is 73.2°.

c Area of red sector = area of whole circle − area of blue sector − area of green sector

$$= \pi \times 6^2 - 13\pi - 23$$ Area of blue sector is 13π from part **a**.

$$= 36\pi - 13\pi - 23$$

$$= 23\pi - 23$$

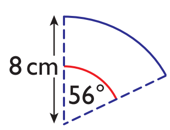
$$= 23(\pi - 1)\,\text{cm}^2$$ Factorising.

Learning exercise

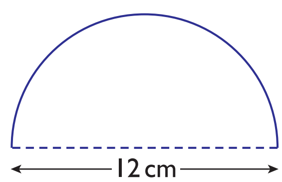
① Calculate the lengths of these arcs.

Give your answers correct to 1 decimal place.

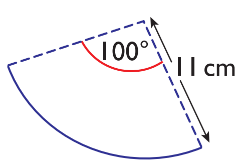
a



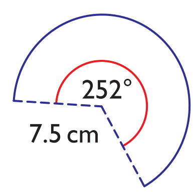
b



c



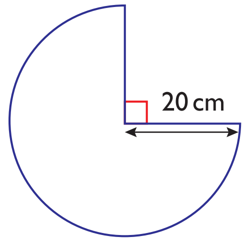
d



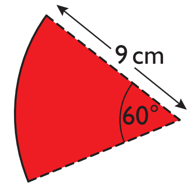
② Calculate the areas of these sectors.

Give your answers correct to 1 decimal place.

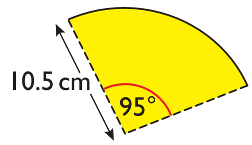
a



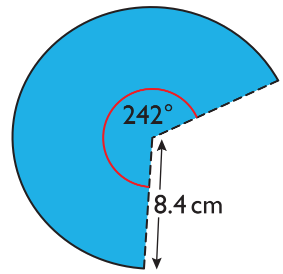
b



c



d



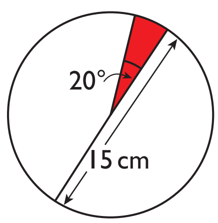
③ For each sector that is shaded red, find the

i arc length

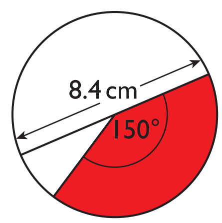
ii area.

Give your answers correct to 1 decimal place.

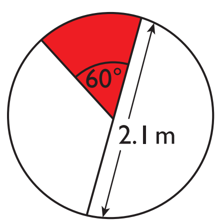
a



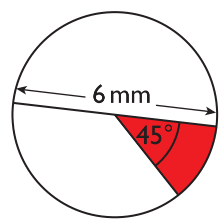
b



c



d



④ **a** An arc has a length of 35 cm and a radius of 8 cm.

Calculate the reflex angle at the centre.

Give your answer to the nearest degree.

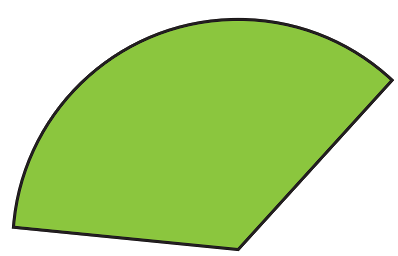
b An arc has a length of 4π cm and a radius of 12 cm.

Calculate the acute angle at the centre.

Give your answer to the nearest degree.

c In the diagram, the sector has a radius of 7 cm and an area of 30 cm^2.

Calculate the acute angle at the centre.



⑤ The shape of a flower bed is a sector of radius 2.4 m.

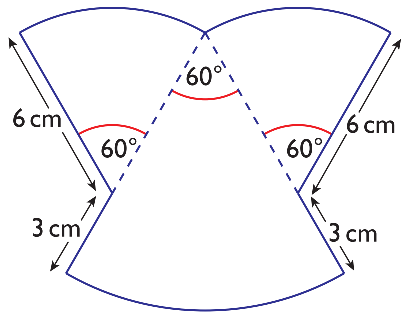
The angle at the centre is 115°. Calculate, to 1 decimal place,

a the perimeter of the flower bed

b the area of the flower bed.

⑥ This shape is made from sectors of circles.

Show that the perimeter of this shape is $(7\pi + 18)$ cm.

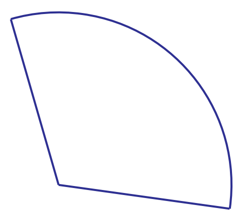


⑦ A sector has an angle of 64° and an area of $10\pi\,\text{cm}^2$.

Calculate the radius of the sector.

⑧ This sector has arc length of 3.42π cm and angle of 114°.

Calculate the area of the sector, giving your answer in terms of π.

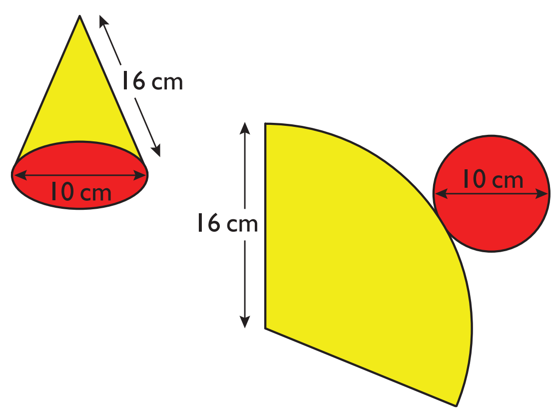


⑨ Here is a cone.

The net of the cone consists of a sector and a circle.

The circumference of the red circle is equal to the length of the yellow arc.

Calculate the angle at the centre of the sector.



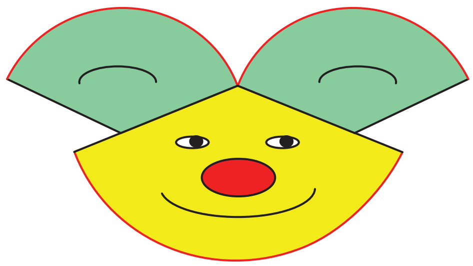
⑩ This creature is made from three sectors.

Each sector has an angle at the centre of 130°.

The two green sectors each have a radius of 7.5 cm.

The yellow sector has a radius of 9.7 cm.

Calculate the total area of the creature, giving your answers in cm^2, correct to 1 decimal place.



⑪ AB is a diameter of a circle.

O is the centre of the circle.

CD is an arc, equal in length to the diameter AB.

Calculate the size of ∠AOB.

Problem solving exercise

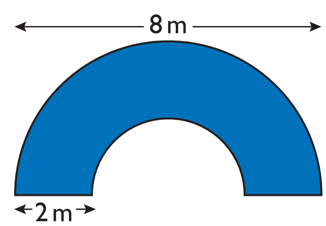
① Here is a design for a company logo.

The logo is the region between two semicircles.

The semicircles have the same centre.

The width of the logo is 2 m.

The diameter of the larger semicircle is 8 m.



The company wants to put wire all along the edges of the logo.

a Calculate the length of wire required.

The company wants to paint the logo.

b Calculate the area of the logo.

② The diagram represents the plan for a scented garden.

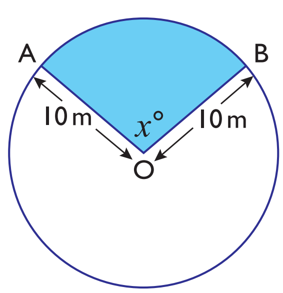
The scented garden will be planted in the sector AOB.

The sector has a radius of 10 m.

The scented garden has to be fenced all round.

The gardener uses 40 m of fencing.

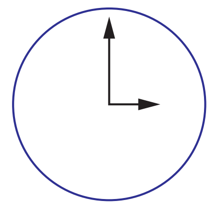
Work out the value of x.



Do I know it now?

① The length of the minute hand on a clock is 4.2 cm.

Find the distance moved by the tip of the minute hand between



a 3 p.m. and 3:15 p.m.

b 3:15 p.m. and 3:45 p.m.

c 3:45 p.m. and 4:30 p.m.

d 4:30 p.m. and 4:40 p.m.

Give your answers to the nearest millimetre.

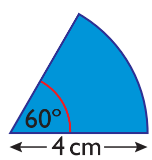
② Calculate

a the perimeter **b** the area

of each of the shaded regions.

Give your answers correct to 2 decimal places.

i



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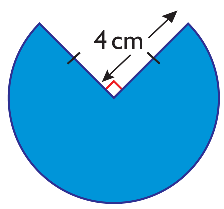

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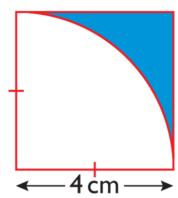

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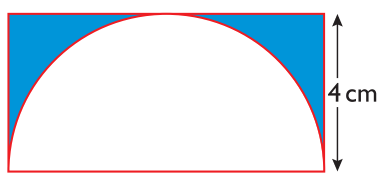


③ A throwing area on a sports field is a 30° sector of a circle with radius 80 m. Calculate the perimeter and area of the throwing area in terms of π.

Can I apply it now?

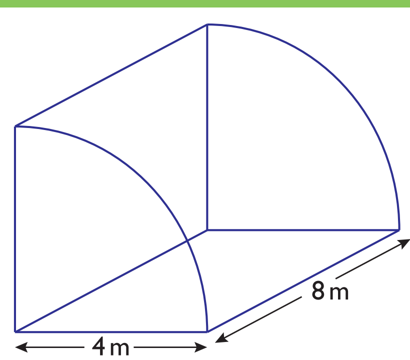
① The diagram represents a framework for a greenhouse.

The cross-section of the framework is a quarter-circle of radius 4 metres.

The length of the greenhouse is 8 metres.

a Work out the total amount of the framework required.

b Investigate whether the volume enclosed will exceed 100 m^3.



CHAPTER 17

NEXT STEPS – GEOMETRY AND MEASURES

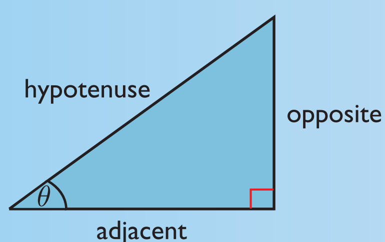
Transformations

17.2 Trigonometry

What you need to know

Each side in a **right-angled triangle** has a name.

- The longest side is called the **hypotenuse** (H).
- The side opposite the marked angle is called the **opposite** (O).
- The remaining side, next to the marked angle, is called the **adjacent** (A).



The ratio of the lengths of the sides are given special names.

cosine (cos) $\theta = \dfrac{\text{adjacent}}{\text{hypotenuse}}$

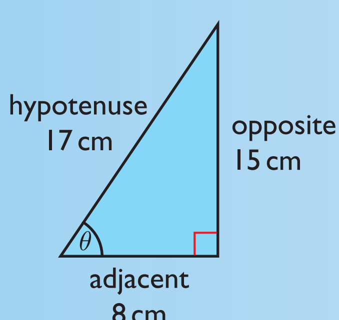
tangent (tan) $\theta = \dfrac{\text{opposite}}{\text{adjacent}}$

You will find keys for sin, cos and tan on your calculator.

sine (sin) $\theta = \dfrac{\text{opposite}}{\text{hypotenuse}}$

These ratios are constant for similar right-angled triangles.

You can use this information to find angles and lengths in triangles.



For this triangle

cos $\theta = \dfrac{8}{17}$

sin $\theta = \dfrac{15}{17}$

tan $\theta = \dfrac{15}{8}$

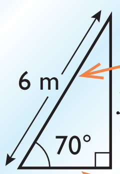
Note that the ratios sin θ, cos θ and tan θ are the same for all triangles that are similar to this one.

How to do it

Using trigonometry to find a length

John is a window cleaner. His ladder is 6 m long.
The angle between the ladder and the ground is 70°.
Work out the height that his ladder reaches up the wall.

Solution



The ladder is the hypotenuse. It is 6 m long.

The height the ladder reaches up the wall is the opposite side to the angle of 70°.

The distance from the bottom of the ladder to the wall is the adjacent side.

$\sin\theta = \frac{O}{H}$

$\sin 70° = \frac{y}{6}$

so $y = 6 \times \sin 70°$ — Multiply by 6.

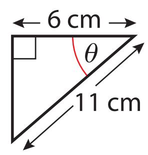
$y = 6 \times 0.94$ — $\sin 70° = 0.94$ from a calculator.

$y = 5.64$ m

Using trigonometry to find an angle

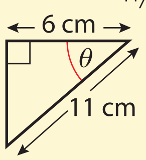
a Look at the diagram. Which ratio would you use to find the value of θ?

b Find the value of θ.



Solution

a Use $\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$



The adjacent side is 6 cm. The hypotenuse is 11 cm.

b $\cos\theta = \frac{6}{11}$

$\theta = \cos^{-1}\left(\frac{6}{11}\right)$

$\theta = 56.9°$

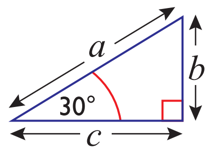
You can write $\theta = \cos^{-1}\left(\frac{6}{11}\right)$ or $\theta = \arccos\left(\frac{6}{11}\right)$.
Find this using your calculator.

Learning exercise

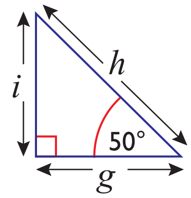
① For each triangle, write down which side is

i the hypotenuse
ii opposite
iii adjacent.

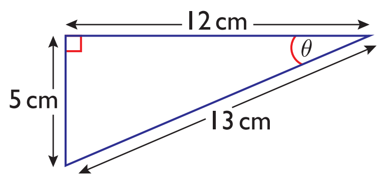
a



b



② Which of these are true in each case?



a $\tan\theta = \frac{5}{12}$ $\quad \tan\theta = \frac{13}{12}$ $\quad \tan\theta = \frac{12}{5}$

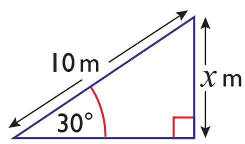
b $\cos\theta = \frac{5}{12}$ $\quad \cos\theta = \frac{5}{13}$ $\quad \cos\theta = \frac{12}{13}$

c $\sin\theta = \frac{5}{12}$ $\quad \sin\theta = \frac{12}{5}$ $\quad \sin\theta = \frac{5}{13}$

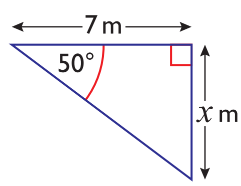
③ For each triangle:

i Make a sketch.
ii Label the sides H, O and A.
iii Decide whether to use sin, cos or tan.
iv Work out the value of x correct to 1 decimal place.

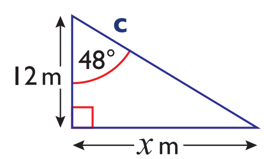
a



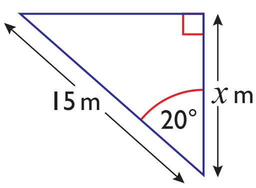
b



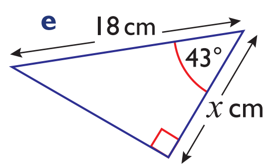
c



d



e



④ Use your calculator to find the value of θ to the nearest degree.

a $\sin\theta = 0.5$ **b** $\tan\theta = 1$ **c** $\cos\theta = 0.5$

d $\tan\theta = 0.839$ **e** $\sin\theta = 0.951$ **f** $\cos\theta = 0.139$

g $\sin\theta = \frac{4}{5}$ **h** $\cos\theta = \frac{2}{3}$ **i** $\tan\theta = \frac{9}{5}$

5. For each triangle:

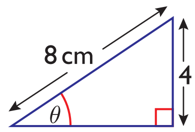
i Make a sketch.

ii Label the sides H, O and A.

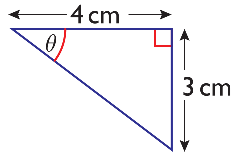
iii Decide whether to use sin, cos or tan.

iv Work out the value of θ correct to 1 decimal place.

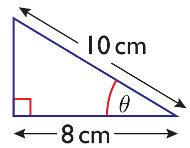
a



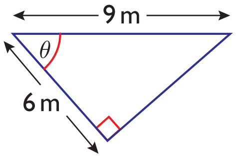
b



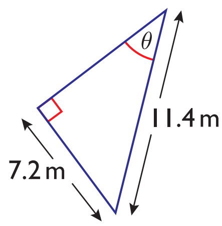
c



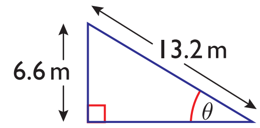
d



e



f



v Which two triangles are similar?

6. For each triangle:

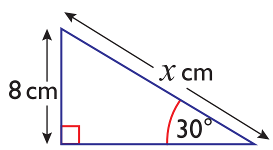
i Make a sketch.

ii Label the sides H, O and A.

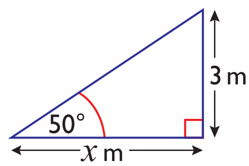
iii Decide whether to use sin, cos or tan.

iv Work out the value of x correct to 1 decimal place.

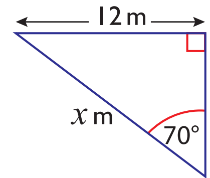
a



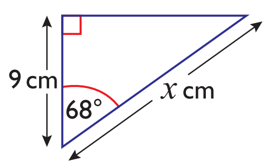
b



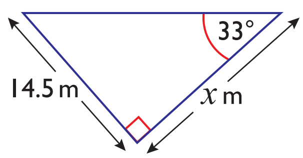
c



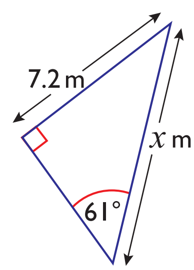
d



e



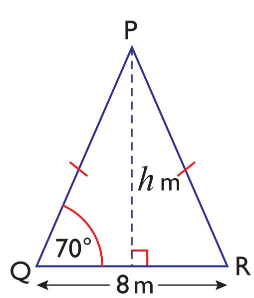
f



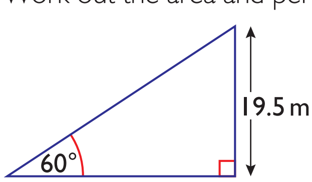
7. **a** Work out the perpendicular height, h m, of this triangle.

b Calculate the area of the triangle.

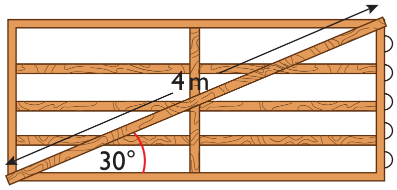
c Calculate the perimeter of the triangle.



8. Work out the area and perimeter of this triangle.



9 Joshua has made a 5-barred gate. The diagonal is 4 m long and makes an angle of 30° with the horizontal. What length of wood did he need?

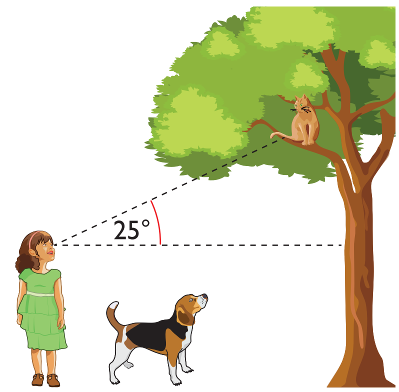


10 Alice has lost her cat Truffles. After some searching, she spots Truffles up a tree.

When Alice is 30 m from the tree, she sees Truffles at an angle of elevation of 25°.

Alice's eye is 1.5 m above the ground.

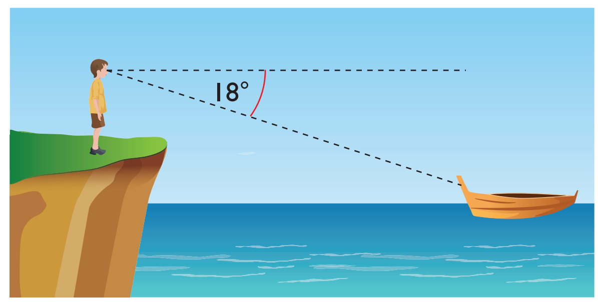
How high up the tree is Truffles?



11 Robin is standing on top of a vertical cliff looking at a small boat.

He sees it at an angle of depression of 18°. The top of the cliff is 50 m above the sea and Robin's eye height is 1.5 m.

How far from the base of the cliff is the boat?



12 Look at triangle ABC.

You are given the lengths of AB and AC.

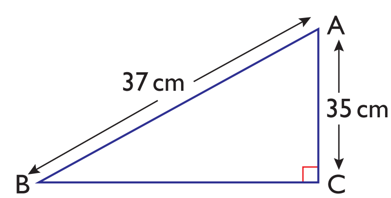
There are two different ways to find the length of BC.

a **i** Use trigonometry to work out ∠ABC.

ii Use trigonometry again to calculate the length of BC.

b Use Pythagoras' theorem to find the length of BC.

c Do the two methods give the same answer?



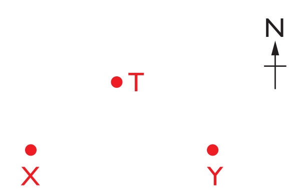
13 Point X is 1800 m due west of point Y.

T is the base of a tower.

The bearing of T from X is 040° and the bearing of T from Y is 320°.

Floella walks from X to T and then from T to Y.

Work out how far she walks.



Problem solving exercise

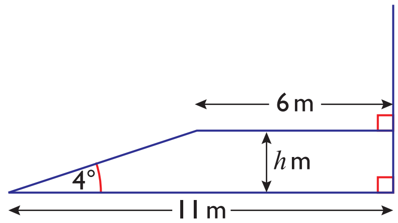
① The diagram shows a ramp for wheelchair access to a building.

The slope of the ramp is 4°.

The top of the ramp is 6 m from the building.

The bottom of the ramp is 11 m from the building.

Find the value of h.



② A supermarket has a moving walkway between the ground floor and the first floor.

The top of the walkway is 3.5 m above the ground floor.

The walkway makes an angle of 10° with the horizontal.

a Work out the horizontal distance covered by the walkway.

b Work out the length of the walkway.

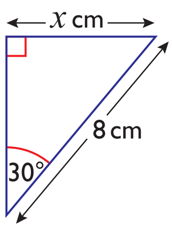
Do I know it now?

① For each triangle:

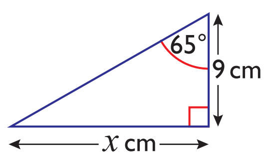
i Write down which side is the hypotenuse, which side is the opposite and which side is the adjacent.

ii Choose the correct ratio and work out the value of x.

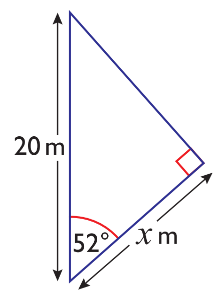
a



b



c



② For each triangle:

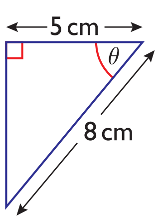
i Make a sketch.

ii Label the sides H, O and A.

iii Decide whether to use sin, cos or tan.

iv Work out the value of θ correct to 1 decimal place.

a



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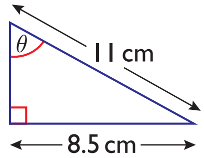

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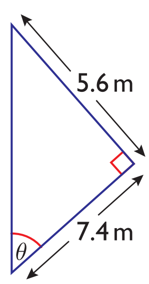
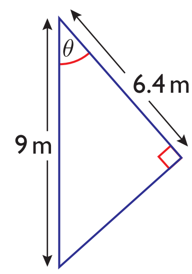
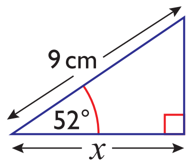


d

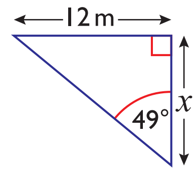


③ For each triangle, work out the value of x correct to 1 decimal place.

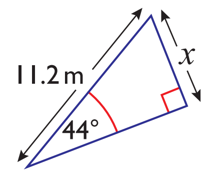
a



b

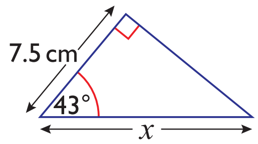


c

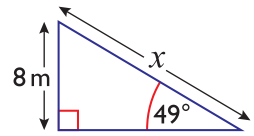


④ For each triangle, work out the value of x correct to 1 decimal place.

a



b



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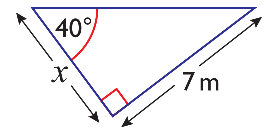
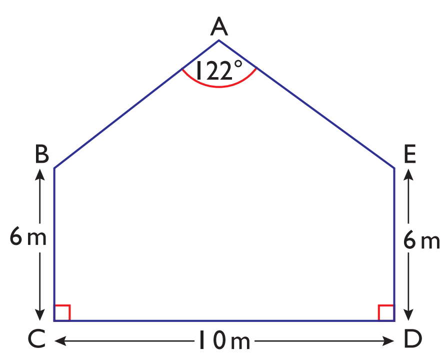


Can I apply it now?

① The diagram shows the end view of a house, ABCDE.

The dimensions are as shown. There is a line of symmetry through A.

Find the height of point A.



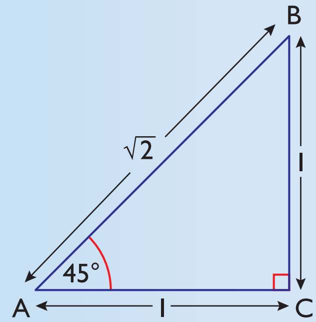
17.3 Trigonometry for special angles

What you need to know

Using exact values of some sines, cosines and tangents avoids rounding errors.

$\sin 45° = \cos 45° = \frac{1}{\sqrt{2}}$

$\tan 45° = 1$



$\sin 30° = \cos 60° = \frac{1}{2}$

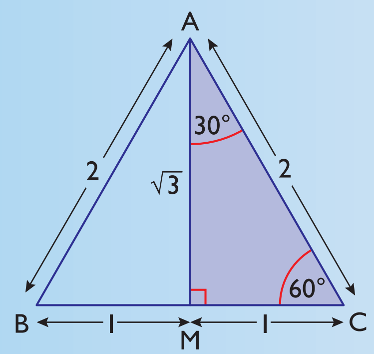
$\sin 60° = \cos 30° = \frac{\sqrt{3}}{2}$

$\tan 30° = \frac{1}{\sqrt{3}}$

$\tan 60° = \sqrt{3}$

$\sin 90° = 1$

$\cos 90° = 0$



Remember when working with surds:

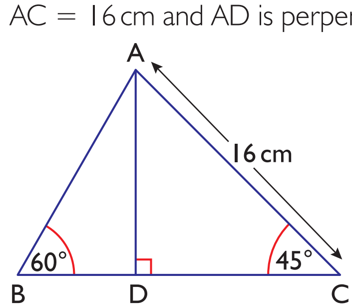
$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2} \times \sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{3}$$

How to do it

Using exact values

In the diagram, triangle ABC has angle B = 60° and angle C = 45°.
AC = 16 cm and AD is perpendicular to BC.



a Calculate the exact length of AD.

b Calculate the exact area of the triangle ABC.

Solution

a $\frac{AD}{16} = \sin 45°$

$\frac{AD}{16} = \frac{1}{\sqrt{2}}$ ← $\sin 45° = \frac{1}{\sqrt{2}}$

$AD = \frac{16}{\sqrt{2}}$ ← Multiplying by 16

$AD = \frac{16\sqrt{2}}{2}$ ← $\left(\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}\right)$

$AD = 8\sqrt{2}$ cm

b Area of triangle ABC = area of triangle ABD + area of triangle ACD

Triangle ABD

Area $= \frac{1}{2} \times BD \times AD$

To find BD, use $\frac{BD}{AD} = \tan 30°$

$\frac{BD}{8\sqrt{2}} = \frac{1}{\sqrt{3}}$

$BD = 8 \times \sqrt{2} \times \frac{\sqrt{3}}{3}$

Area of triangle ABD

$= \frac{1}{2} \times 8 \times \sqrt{2} \times \frac{\sqrt{3}}{3} \times 8 \times \sqrt{2}$

$= 64\frac{\sqrt{3}}{3}$

Triangle ACD

Area $= \frac{1}{2} \times DC \times AD$

To find DC, use $\frac{DC}{AD} = \tan 45°$

$\frac{DC}{8\sqrt{2}} = 1$

$DC = 8\sqrt{2}$

Area of triangle ACD

$= \frac{1}{2} \times 8 \times \sqrt{2} \times 8 \times \sqrt{2}$

$= 64$

Total area of triangle ABC $= 64\frac{\sqrt{3}}{3} + 64$

$= 64\left(\frac{\sqrt{3}}{3} + 1\right)$ cm^2

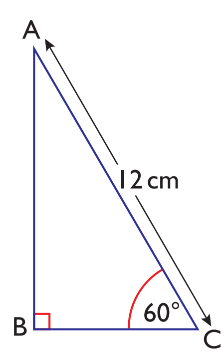
Learning exercise

Give all answers exactly.

① Calculate the length of

a BC

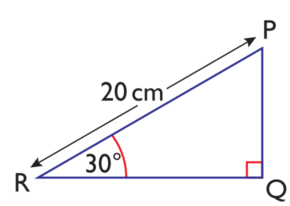
b AB.



② Calculate the length of

a PQ

b RQ.

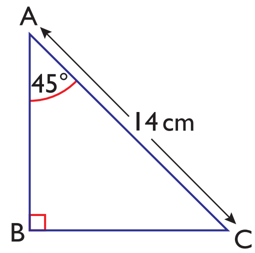


③ Calculate

a the length of BC

b the length of AB

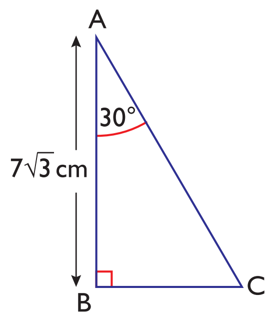
c the area of triangle ABC.



④ Calculate the length of

a BC

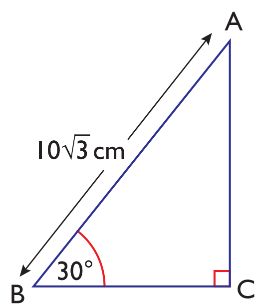
b AC.



⑤ Calculate the length of

a AC

b BC.



⑥ The length of the diagonal of a square is 26 cm.

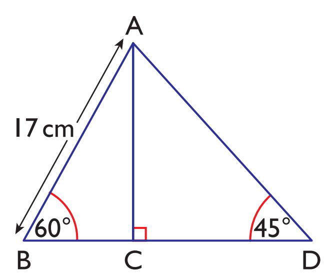
Calculate the length of each side.

⑦ An equilateral triangle has a base of 16 cm.

Calculate the area of the triangle.

⑧ In the diagram, ABC and ACD are right-angled triangles.

Calculate the length of AD.



Do I know it now?

① Calculate

a the lengths of AB and BC

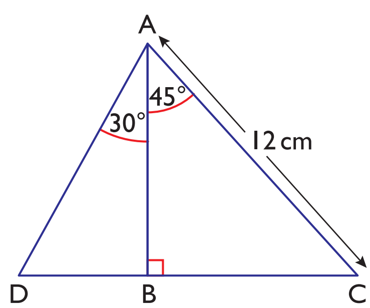
b the area of triangle ABC

c the length of BD

d the area of triangle ABD

e the area of triangle ADC.

Give all answers exactly.



17.4 Finding centres of rotation

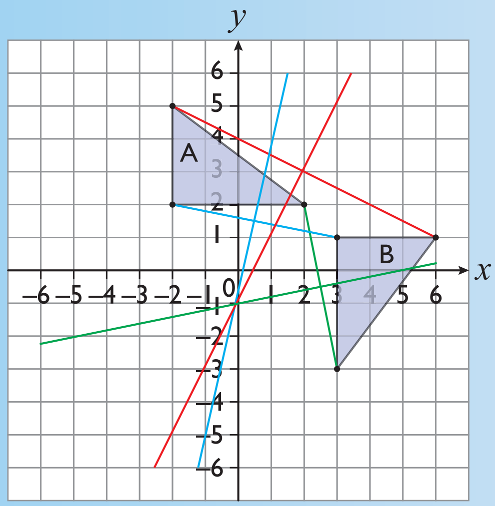
What you need to know

Did you know?

Wallpaper designers use transformations to create a repeating design.

The **centre of rotation** is the point that does not move as a result of the rotation.

During a rotation, an object revolves around the centre of rotation to form the **image**.



To find the centre of rotation that maps triangle A onto triangle B:

- join the corresponding points on the two shapes
- draw the perpendicular bisectors of the joining lines
- the centre of rotation is where the perpendicular bisectors meet.

In this example, the rotation that maps triangle A onto triangle B is a 90° clockwise rotation about (0, −1).

Note that you need the centre, angle and direction to fully describe a rotation. It is not necessary to specify clockwise or anticlockwise for rotations of 180°.

Sometimes, if the shapes are drawn on a co-ordinate grid, you can find the centre of rotation 'by eye'.

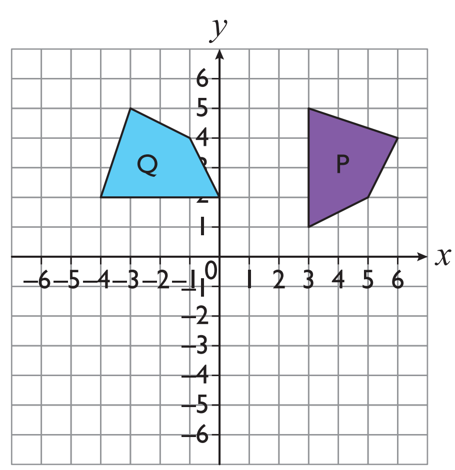
How to do it

Finding the centre of rotation

Quadrilateral Q is the image of quadrilateral P under a rotation.

a Find the centre of rotation.

b Describe fully the transformation that maps P onto Q.

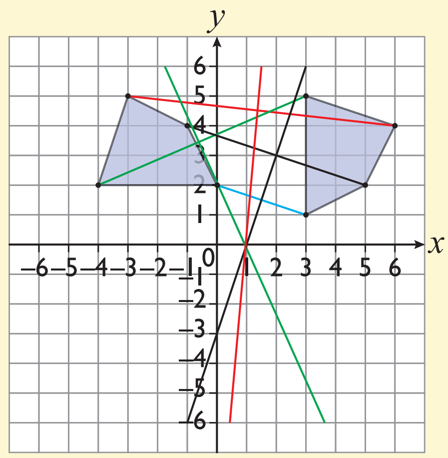


Solution

a Join each vertex of Q to the corresponding vertex of P.
Draw the perpendicular bisector of each of these lines.
The bisectors meet at the centre of rotation.
So the centre of rotation is (1, 0).

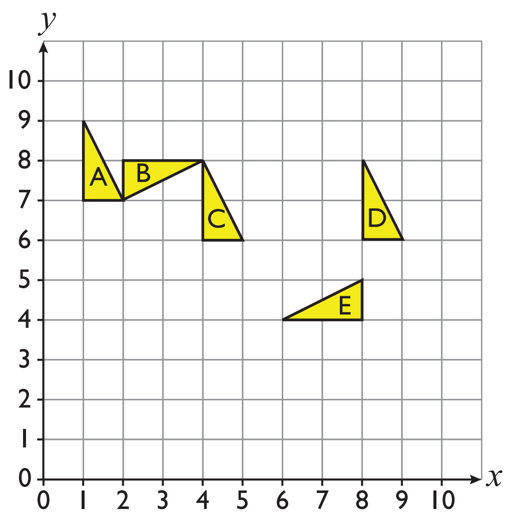
b The transformation is a 90° anticlockwise rotation about (1, 0).

Remember to state angle, direction and centre.



Learning exercise

① Copy and complete the diagram and statements.



a When triangle A is rotated ☐° clockwise about (☐, ☐), it moves to position B.

b When triangle B is rotated 90° ________ about (4, 8), it moves to position ☐.

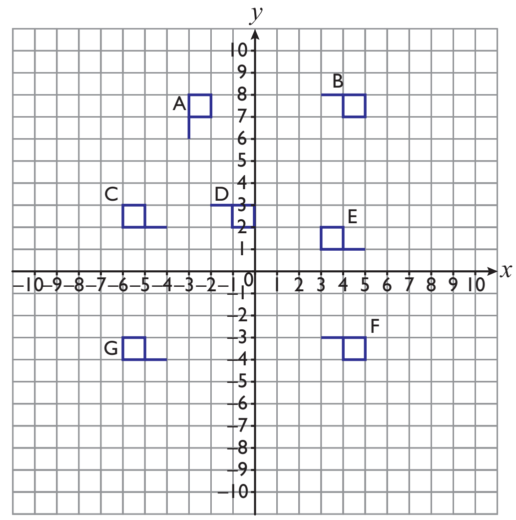
c When triangle C is rotated 90° ________ about (☐, ☐), it moves to position E.

d When triangle ☐ is rotated 90° clockwise about (9, 5), it moves to position ☐.

② Copy the diagram.

Find the centre and angle of rotation for each of these.

a A → B **b** B → C
c C → D **d** D → E
e E → F **f** F → G
g G → A

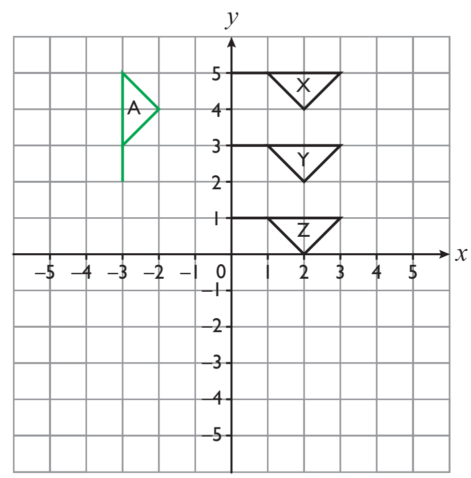


③ Copy the diagram.

a Find the centre of rotation for each transformation.

i A → X **ii** A → Y
iii A → Z

b Say what you notice about the centres of rotation.



④ Here are five transformations.

a The blue triangle is a 180° rotation of the green triangle.

b The orange triangle is a 90° anticlockwise rotation of the green triangle.

c The purple triangle is a 90° anticlockwise rotation of the orange triangle.

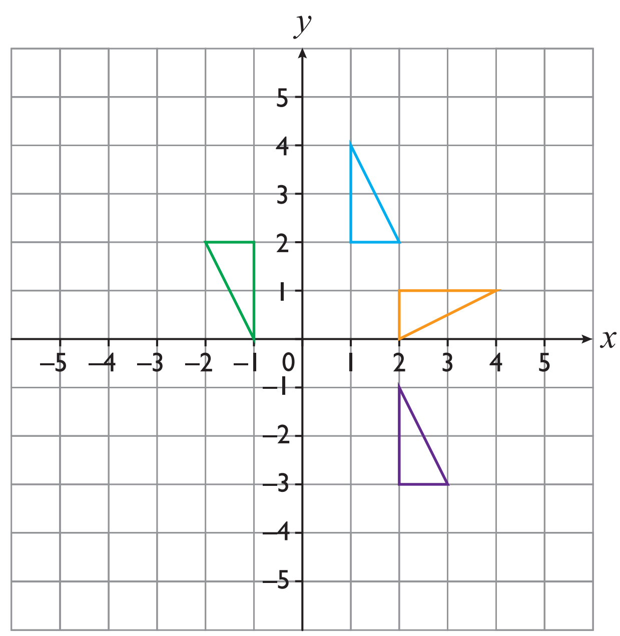
d The purple triangle is a 180° rotation of the green triangle.

e The orange triangle is a 90° clockwise rotation of the blue triangle.

The five centres of rotation are given below.

P(4, −1) Q(0, 2) R(1, 1)
S(0.5, −0.5) T(1, 3)

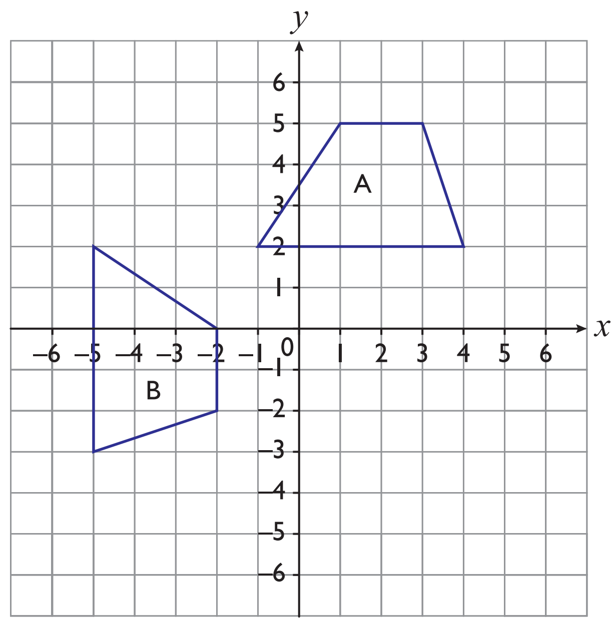
Match each transformation with the correct centre of rotation.



5 Describe fully the rotations that map

a shape A onto shape B.

b shape B onto shape A.

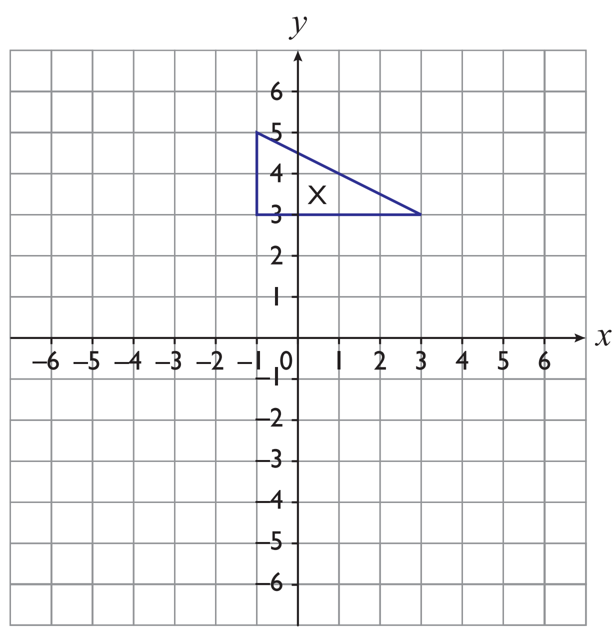


6 Copy the diagram.

a Rotate the triangle X 90° clockwise about (− 1, 6). Label the new triangle A.

b Rotate the triangle X 90° anticlockwise about (5, 4). Label the new triangle B.

c Describe fully the rotation that maps triangle A onto triangle B.



7 The diagram shows four congruent shapes A, B, C and D.

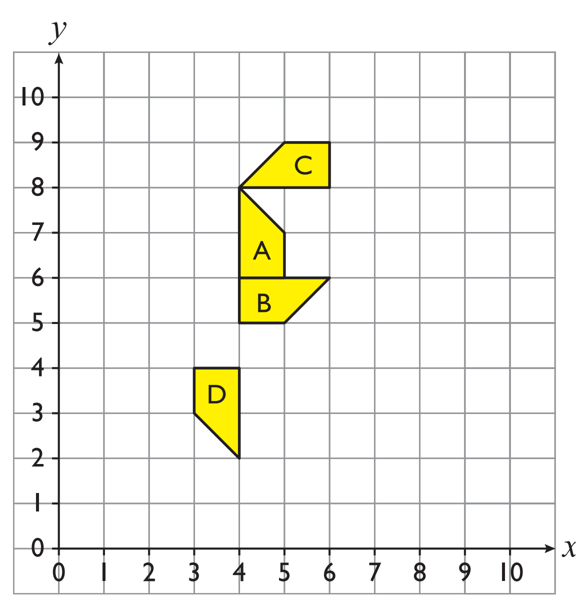
Describe the rotation that maps

a shape A → shape B

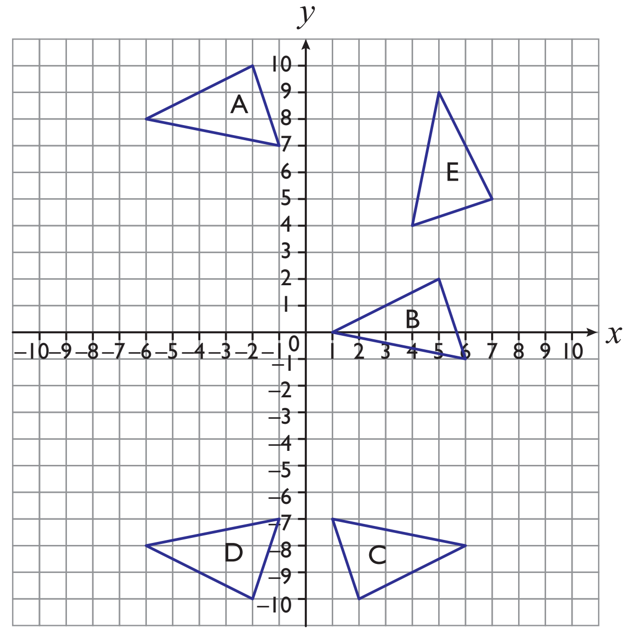
b shape B → shape C

c shape C → shape D

d shape D → shape A.



⑧ This diagram shows five congruent triangles.



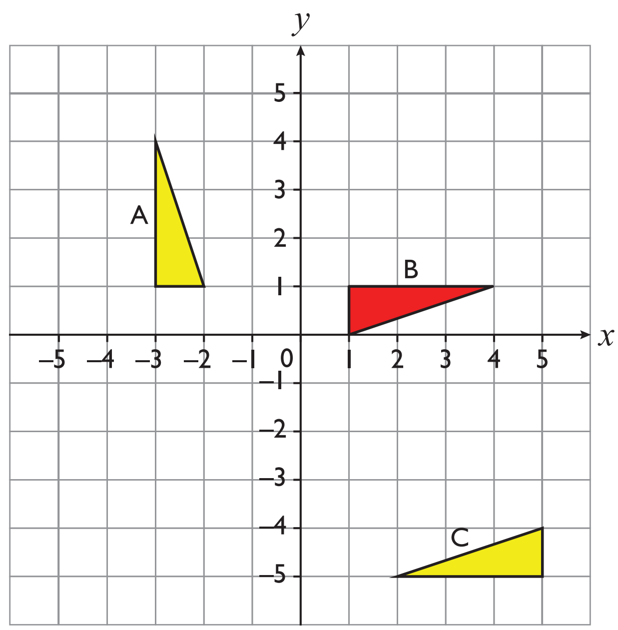
a Describe fully the single transformation that maps

i A → B **ii** B → C
iii C → D **iv** D → A
v A → E **vi** E → C
vii C → B **viii** B → A.

b State what pair of triangles cannot be mapped onto each other by a single transformation.

Do I know it now?

① This diagram shows three congruent triangles A, B and C.



Describe fully the rotations that map

a A → B
b B → A
c C → A
d B → C

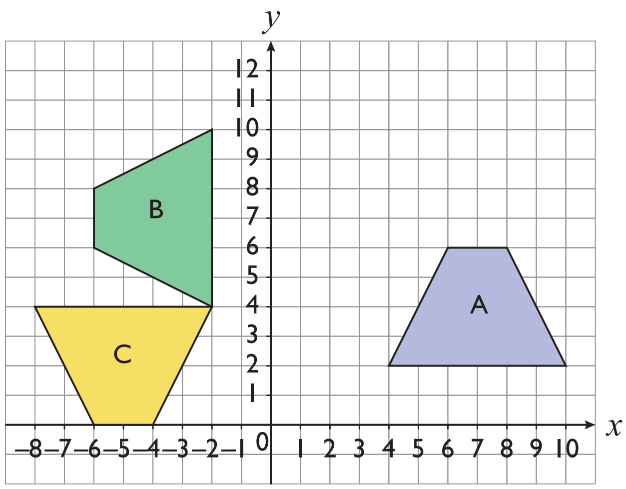
② This diagram shows three congruent trapeziums.

Describe the transformation that maps

a trapezium A to trapezium B

b trapezium B to trapezium C

c trapezium C to trapezium A.



CHAPTER 18

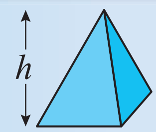
NEXT STEPS – GEOMETRY AND MEASURES

Three-dimensional shapes

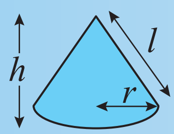
18.3 Surface area and volume of 3-D shapes

What you need to know

The volume of a pyramid $= \frac{1}{3}$ base area $\times$ height



Pyramid



Cone

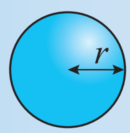
The volume of a cone $= \frac{1}{3}\pi r^2 h$

The surface area of a cone = area of base + area of curved surface

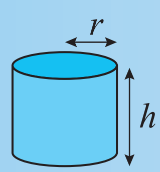
$= \pi r^2 + \pi r l$

The volume of a sphere $= \frac{4}{3}\pi r^3$

The surface area of a sphere $= 4\pi r^2$



Sphere



Cylinder

The volume of a cylinder $= \pi r^2 h$

The surface area of a cylinder $= 2\pi r h + 2\pi r^2$

Did you know?

A dome is a hollow architectural structure in the shape of half a sphere. It is an extremely efficient structure. Domes use less material and are stronger than any other roof shape.

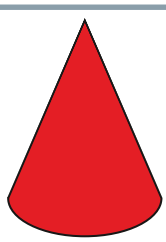
How to do it

Using formulae

A cone has a radius of 7.5 cm and a height of 18 cm.

a Calculate the volume of the cone.

b Calculate the surface area of the cone.



Solution

a Volume $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3}\pi \times 7.5^2 \times 18$ ← $r = 7.5$ and $h = 18$

$= 1060\,\text{cm}^3$ (to the nearest cm^3)

b By Pythagoras' theorem, $l^2 = r^2 + h^2$ ← First find the slant height of the cone.

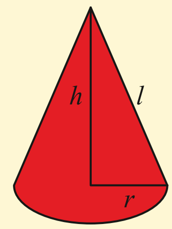
$l^2 = 7.5^2 + 18^2 = 380.25$

$l = \sqrt{380.25} = 19.5\,\text{cm}$

Surface area $= \pi r^2 + \pi r l$ ← $r = 7.5$ and $l = 19.5$

$= \pi \times 7.5^2 + \pi \times 7.5 \times 19.5$

$= 636\,\text{cm}^2$ ← To the nearest cm^2.



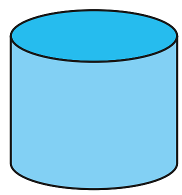
➤ Leaving π in the answer

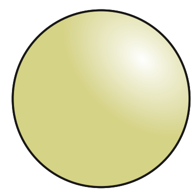
A cylinder has diameter 16 cm and height 16 cm.

A sphere has diameter 16 cm.

Which shape has the greater volume and by how much?

Give your answer in terms of π.





Solution

Volume of cylinder $= \pi r^2 h$ ← Diameter is 16 cm so radius is 8 cm. Height is 16 cm.

$= \pi \times 8^2 \times 16$

$= 1024\pi$

Volume of sphere $= \frac{4}{3}\pi r^3$ ← Diameter is 16 cm so radius is 8 cm.

$= \frac{4}{3}\pi \times 8^3$

$= \frac{2048}{3}\pi$

The cylinder has a larger volume by $1024\pi - \frac{2048}{3}\pi$

$= \frac{3072 - 2048}{3}\pi$ ← $1024\pi = \frac{3072}{3}\pi$

$= \frac{1024}{3}\pi\,\text{cm}^3$

Learning exercise

① A pyramid has a square base with sides of 11 cm.

Its vertical height is 15 cm.

Calculate the volume of the pyramid.

② A cone has a radius of 9 cm, a vertical height of 12 cm and a slant height of 15 cm.

Calculate

a its volume **b** its surface area, including its base.

Give your answers correct to the nearest whole unit.

③ The diameter of a sphere is 25 cm.

Calculate

a its volume **b** its surface area.

Give your answers correct to the nearest whole unit.

④ A square-based pyramid has volume 147 cm^3 and height 9 cm.

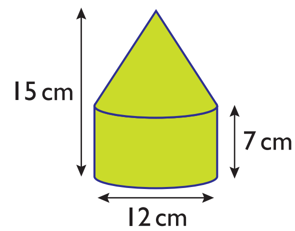
Calculate the length of a side of the base.

⑤ Calculate, in terms of π,

a the volume

b the surface area (including the base)

of this shape.

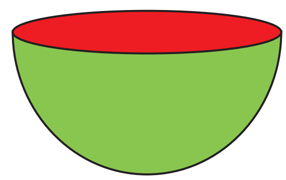


⑥ The radius of this solid wooden hemisphere is 14 cm.

Calculate

a the surface area **b** the volume.

Give your answers to the nearest whole unit.



⑦ A cone has volume 784π cm^3 and height 12 cm.

Calculate the radius of the base.

⑧ A cone has radius 11 cm and slant height 28.6 cm.

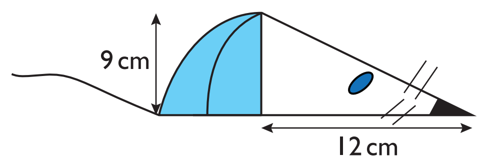
Calculate

a the surface area **b** the volume.

Give your answers in terms of π.

⑨ A cat's toy is made to look like a mouse.

It is made of a quarter sphere with radius 9 cm and half a cone of radius 9 cm and length 12 cm.



The underside is flat.

Calculate

a the volume **b** the surface area

of the toy.

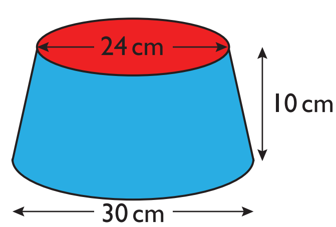
Give your answers in terms of π.

Problem solving exercise

① A truncated cone has a base diameter of 30 cm and a top diameter of 24 cm.

The height of the truncated cone is 10 cm.

Calculate the volume, giving your answer in terms of π.



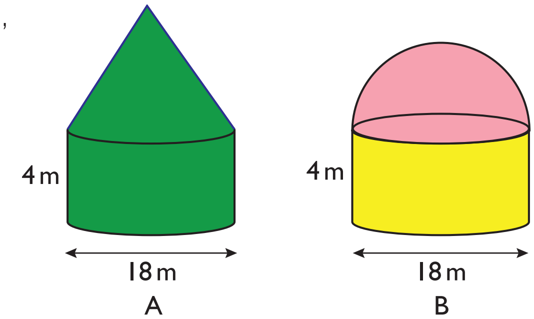
② An architect is considering these two designs, A and B, to cover a new arena.

Each has a cylindrical base of diameter 18 m and height 4 m.

Design A has a conical roof of height 9 m.

The roof of design B is a hemisphere.

Work out how much more volume is enclosed by design B than by design A.

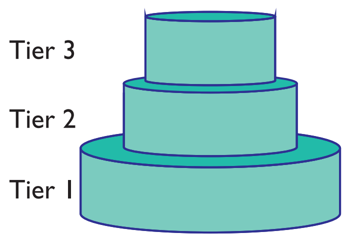


③ Craig makes wedding cakes.

Here is a cake with 3 tiers.

Each tier is 10 cm high.

The diameter of tier 1 is 60 cm.
The diameter of tier 2 is 40 cm.
The diameter of tier 3 is 30 cm.



a Work out the total volume of the cake.

The surfaces of the cake are covered in icing. This includes the complete tops of the tiers but not their underneath.

b Work out the area of icing needed.

Do I know it now?

① A cone has a radius of 15 cm and a height of 20 cm.

Calculate

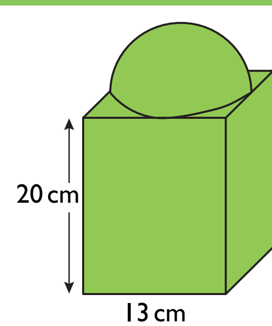
a the volume of the cone **b** the surface area of the cone.

Can I apply it now?

① The diagram shows a square-based cuboid with a hemisphere stuck on the top. The dimensions are as shown.

The hemisphere just meets the edges of the cuboid.

Calculate the surface area of the shape giving your answer correct to the nearest cm^2.



CHAPTER 19

NEXT STEPS – GEOMETRY AND MEASURES

Vectors

19.1 Vectors

What you need to know

Did you know?

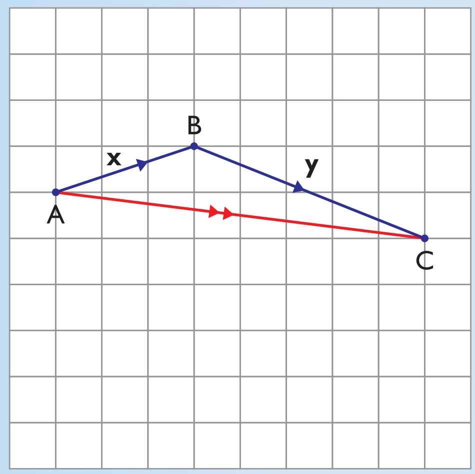
Computer game programmers use vectors to control the movement within games. That may be jumping over a river, shooting at a target or the flow of water.

A **vector** is a quantity that has size (or magnitude) and direction.

A **translation** is a vector.

The vector from A to B can be written in three ways:

$\overrightarrow{AB}$ *or* $\mathbf{x}$ *or* $\begin{pmatrix}3\\1\end{pmatrix}$



Vectors can be added.

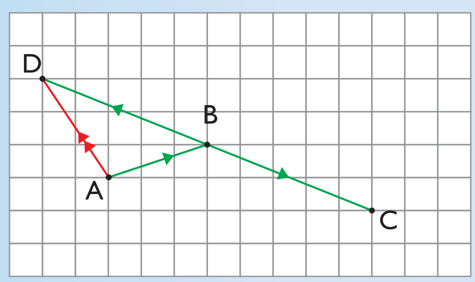
$\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$

$\begin{pmatrix}3\\1\end{pmatrix} + \begin{pmatrix}5\\-2\end{pmatrix} = \begin{pmatrix}8\\-1\end{pmatrix}$

Vectors can be subtracted.

$\overrightarrow{AB} - \overrightarrow{BC} = \overrightarrow{AD}$

$\begin{pmatrix}3\\1\end{pmatrix} - \begin{pmatrix}5\\-2\end{pmatrix} = \begin{pmatrix}-2\\3\end{pmatrix}$



As $\overrightarrow{BD}$ is the same as $-\overrightarrow{BC}$.

Vectors can be multiplied by a scalar.

$2\begin{pmatrix}5\\-2\end{pmatrix} = \begin{pmatrix}10\\-4\end{pmatrix}$

This gives a parallel vector, $\begin{pmatrix}10\\-4\end{pmatrix}$, which is twice as long and parallel to $\begin{pmatrix}5\\-2\end{pmatrix}$.

A scalar is a number with size but no direction.

How to do it

Adding and subtracting vectors

$\overrightarrow{AB} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$, $\overrightarrow{BC} = \begin{pmatrix} 4 \\ -2 \end{pmatrix}$

a Draw vectors representing $\overrightarrow{AB}$ and $\overrightarrow{BC}$ on a grid.

b i Work out the column vector for $\overrightarrow{AB} + \overrightarrow{BC}$.

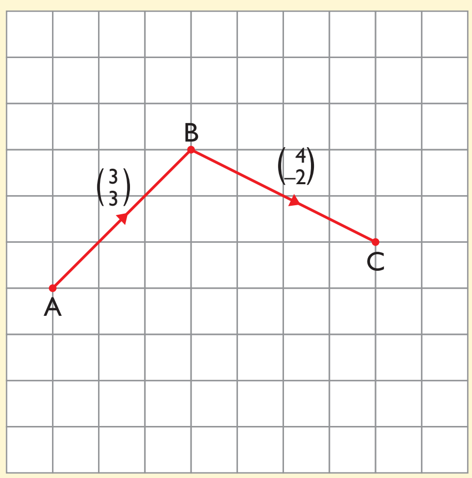
ii Draw the answer on a grid.

c i Work out the column vector for $\overrightarrow{BC} - \overrightarrow{AB}$.

ii Draw the answer on a grid.

Solution

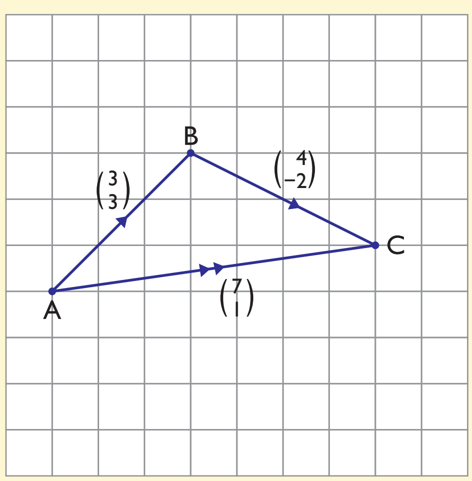
a



b i $\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 3 \\ 3 \end{pmatrix} + \begin{pmatrix} 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 7 \\ 1 \end{pmatrix}$

$3 + 4 = 7$ and $3 + (-2) = 1$.

ii



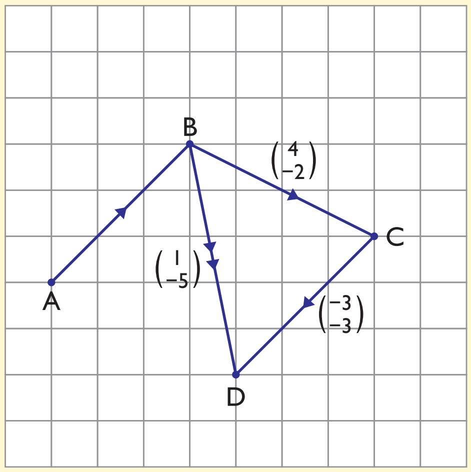
$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{BC}$

You can add vectors by drawing them end-to-end.

c i $\overrightarrow{BC} - \overrightarrow{AB} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} - \begin{pmatrix} 3 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

$4 - 3 = 1$ and $(-2) - 3 = -5$.
This is $\overrightarrow{BD}$ on the diagram.

ii



$\overrightarrow{CD} = -\overrightarrow{AB}$

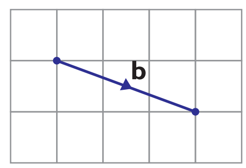
This is the same as BC + (−AB).

➤ Multiplying a vector by a scalar

The diagram shows the vector **b**.

a Write **b** and 3**b** as column vectors.

b Draw the vector 3**b** and describe its relationship to **b**.



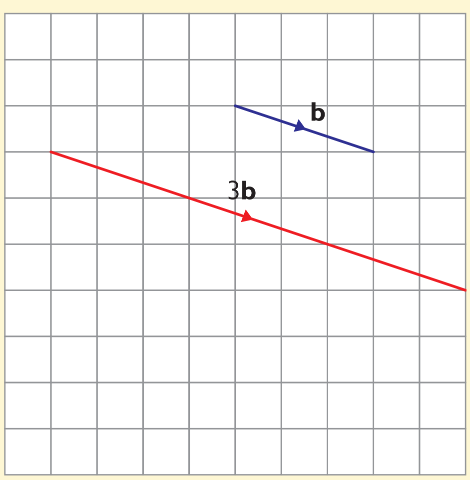
Solution

a $\mathbf{b} = \begin{pmatrix}3\\-1\end{pmatrix}$

$3\mathbf{b} = 3\begin{pmatrix}3\\-1\end{pmatrix} = \begin{pmatrix}9\\-3\end{pmatrix}$

Multiply both numbers in the vector by 3.

b The red vector is 3**b**.



3**b** is three times the length of **b** and is parallel to it.

Notice that **b** and 3**b** can be drawn anywhere on the grid.

Learning exercise

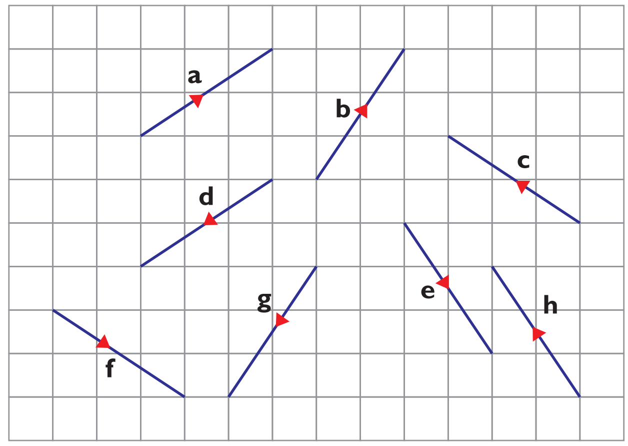
① Match the vectors on the grid with the column vectors.

i $\begin{pmatrix} 3 \\ -2 \end{pmatrix}$ **ii** $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

iii $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ **iv** $\begin{pmatrix} -2 \\ 3 \end{pmatrix}$

v $\begin{pmatrix} -2 \\ -3 \end{pmatrix}$ **vi** $\begin{pmatrix} 3 \\ 2 \end{pmatrix}$

vii $\begin{pmatrix} 2 \\ -3 \end{pmatrix}$ **viii** $\begin{pmatrix} -3 \\ 2 \end{pmatrix}$



② Work out these vector additions. In each case draw a diagram on squared paper to show the vectors.

a $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b $\begin{pmatrix} 5 \\ -1 \end{pmatrix} + \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} 4 \\ 2 \end{pmatrix} + \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

d $\begin{pmatrix} 0 \\ -4 \end{pmatrix} + \begin{pmatrix} -3 \\ -7 \end{pmatrix}$

③ Work out these vector subtractions.

a $\begin{pmatrix} 4 \\ 6 \end{pmatrix} - \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

b $\begin{pmatrix} 3 \\ -1 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \end{pmatrix}$

c $\begin{pmatrix} -2 \\ -3 \end{pmatrix} - \begin{pmatrix} -5 \\ 5 \end{pmatrix}$

d $\begin{pmatrix} 0 \\ -2 \end{pmatrix} - \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

④ Work out these.

a $3\begin{pmatrix} 2 \\ 3 \end{pmatrix}$

b $4\begin{pmatrix} 3 \\ -1 \end{pmatrix}$

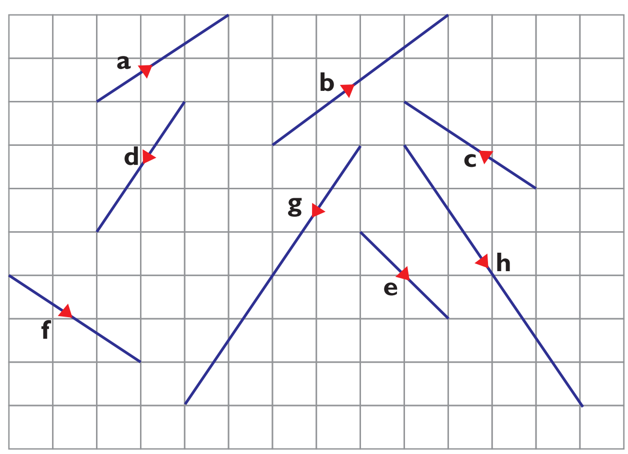
c $4\begin{pmatrix} 2 \\ -3 \end{pmatrix} + 3\begin{pmatrix} -2 \\ 1 \end{pmatrix}$

d $-7\begin{pmatrix} 2 \\ -3 \end{pmatrix} - 3\begin{pmatrix} -2 \\ 4 \end{pmatrix}$

⑤ **a** Which vector is parallel to **d**?

b Find another set of parallel vectors.

c Which vector is perpendicular to **a**?



6 $\mathbf{a} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$, $\mathbf{c} = \begin{pmatrix} 1 \\ 5 \end{pmatrix}$ and $\mathbf{d} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$

a Calculate these as column vectors.

i $2\mathbf{a}$ **ii** $\mathbf{a} + \mathbf{c}$ **iii** $\mathbf{b} - \mathbf{d}$

iv $2\mathbf{b} - \mathbf{a}$ **v** $3\mathbf{b} - 2\mathbf{d}$ **vi** $\mathbf{a} - \mathbf{b} + \mathbf{c} - \mathbf{d}$

b Show that $3\mathbf{a} - \mathbf{b}$ is parallel to $\mathbf{c}$.

7 A shape is translated through $\begin{pmatrix} 1 \\ -5 \end{pmatrix}$.

It is then translated from its new position through $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$.

What translation will return the shape to its original position?

8 Find the value of each letter in these equations.

a $\begin{pmatrix} 2 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$

b $2\begin{pmatrix} 2 \\ -1 \end{pmatrix} - \begin{pmatrix} c \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ d \end{pmatrix}$

c $4\begin{pmatrix} e \\ -3 \end{pmatrix} + 3\begin{pmatrix} -1 \\ f \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$

d $g\begin{pmatrix} 4 \\ -2 \end{pmatrix} - 3\begin{pmatrix} 2 \\ h \end{pmatrix} = \begin{pmatrix} -14 \\ 13 \end{pmatrix}$

9 On this diagram $\overrightarrow{OA}$ and $\overrightarrow{OB}$ represent the vectors $\mathbf{a}$ and $\mathbf{b}$.

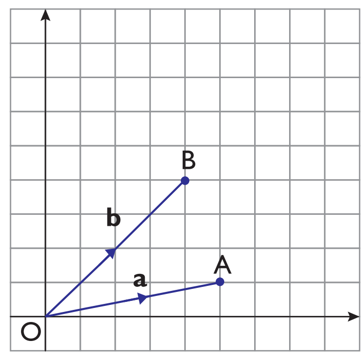
Copy the diagram, then draw these vectors.

i $\overrightarrow{OC}$ where $\mathbf{c} = \mathbf{a} + \mathbf{b}$

ii $\overrightarrow{OD}$ where $\mathbf{d} = 2\mathbf{a} + \mathbf{b}$

iii $\overrightarrow{OE}$ where $\mathbf{e} = -\mathbf{a} - \mathbf{b}$

iv $\overrightarrow{OF}$ where $\mathbf{f} = \mathbf{a} - \mathbf{b}$.



10 $\mathbf{p} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ are vectors representing two translations.

a Work out $\mathbf{p} + \mathbf{q}$ and $\mathbf{p} - \mathbf{q}$.

b Find the combination of the vectors $\mathbf{p}$ and $\mathbf{q}$ that gives each translation.

i $\begin{pmatrix} 6 \\ 0 \end{pmatrix}$ **ii** $\begin{pmatrix} 3 \\ 4 \end{pmatrix}$ **iii** $\begin{pmatrix} -3 \\ -4 \end{pmatrix}$

11 $\mathbf{p}$ and $\mathbf{q}$ are translations.

$\mathbf{p} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$

a Find a combination of $\mathbf{p}$ and $\mathbf{q}$ that gives a translation

i parallel to the y axis

ii parallel to the x axis.

b Find a combination of $\mathbf{p}$ and $\mathbf{q}$ that gives a translation $\begin{pmatrix} 20 \\ 15 \end{pmatrix}$.

12 $\mathbf{p} = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ and $\mathbf{q} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

a Work out $\mathbf{p} + \mathbf{q}$ and $2\mathbf{q} - \mathbf{p}$.

b Deduce values of n and m for which $n\mathbf{p} + m\mathbf{q}$ is equal to the vector $\begin{pmatrix} 3 \\ 6 \end{pmatrix}$.

Do I know it now?

① Work out these vector additions. In each case draw a diagram on squared paper.

a $\begin{pmatrix} 2 \\ 6 \end{pmatrix} + \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ **b** $\begin{pmatrix} -4 \\ -3 \end{pmatrix} + \begin{pmatrix} -5 \\ 2 \end{pmatrix}$

② Work out each of the following.

a $\begin{pmatrix} 1 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **b** $\begin{pmatrix} -3 \\ -3 \end{pmatrix} - \begin{pmatrix} -3 \\ 2 \end{pmatrix}$ **c** $-2\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **d** $3\begin{pmatrix} -3 \\ 1 \end{pmatrix} + \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

CHAPTER 22

NEXT STEPS – STATISTICS AND PROBABILITY

Probability

22.2 Estimating probability

What you need to know

Did you know?

Insurance companies work out your insurance premiums using probability.

A **population** is everything or everyone with some characteristic, for example

- all people over 100 years old
- all red squirrels
- all the students in a school
- all cars of a particular model.

A **sample** is a small set that is chosen from the population to provide information about it.

Samples are often used to estimate population probabilities.

To be useful, a sample must be **representative** of the population. This is often achieved by choosing a **random sample**, but this is not always possible.

In a random sample, every item in the population has the same probability of being selected, so is equally likely.

The larger the sample, the more accurate it is likely to be.

Probability is estimated from what has been observed, using **relative frequency**.

Sometimes you need to carry out an experiment with trials.

Estimated probability (or relative frequency) $= \dfrac{\text{number of successful trials}}{\text{total number of trials}}$

You can estimate the number of times an outcome will occur using the formula

expected number $=$ P (a successful outcome) $\times$ number of trials

How to do it

Using a sample

It is believed that a rare bird species has more males than females. After a long search, a team of scientists find a colony of these birds. 16 of them are male and 4 are female.

a Estimate the probability that a bird of this type is male.

b Comment on the sampling method.

Solution

a The total number of birds is $16 + 4 = 20$.

The estimated probability that a bird in the population is male is $\frac{16}{20} = 0.8$.

b The sample is not random. It is just one group of the birds.

This is an example of an **opportunity sample**.

The sample is small so the estimate is unlikely to be accurate. If the scientists can find more birds, and so have a larger sample, the estimate should be more reliable.

Relative frequency

An insurance company investigates a number of speeding convictions in a country. It classifies them according to the age and gender of the offender. The data are summarised in this table.

		Age	
		Under 25	**25 and over**
Gender	**Male**	455	284
	Female	120	141

a Calculate the probability that a speeding conviction belongs to

i a male person

ii someone under 25.

b Do the data support the claim that males drive faster than females?

Solution

Start by adding an extra row and an extra column to the table.

a

		Age		Total
		Under 25	**25 and over**	
Gender	**Male**	455	284	739
	Female	120	141	261
	Total	575	425	1000

i P(male) $= \frac{739}{1000}$

There are 739 convictions for males.

There are 1000 convictions in total.

ii P(under 25) $= \frac{575}{1000}$

There are 575 convictions for under 25s.

b No. The data do not tell you how many male and how many female drivers there are.
It is possible that in that country there are only a few female drivers but they drive fast and get convictions.
It is also possible that there are about equal numbers of male and female drivers and the males do drive faster.
You do not have the information to draw a conclusion.

Learning exercise

① A shop sells newspapers.

Here are the sales for a few days.

Newspaper	*Daily Post*	*Gazette*	*The News*	*The Tribune*
Frequency	186	84	216	105

a Estimate the probability that the next newspaper sold will be *The News*.

b The owners open a new shop in the same area. They expect to sell 1000 newspapers each day. How many of each one should they stock?

② A computer repair company keeps a record of the faults in the computers it repairs.

Component	Motherboard	Power supply	Hard drive
Frequency	67	215	103

a Millie is called out to do a repair. Estimate the probability that she will find the fault is in the

i motherboard

ii power supply

iii hard drive.

b The engineers say that it is more likely than not that the fault will be in the power supply. Are they correct?

c The computer company normally repairs 5000 computers each year. Estimate how many of each component it will need.

③ Ben has a gold coin. It is going to be used for the toss in the World Cup Final.

Ben wants to be sure his coin is unbiased.

He throws his coin and records how many heads he gets.

Number of throws	20	50	100	400	1000	2000	5000
Number of heads	12	27	46	211	486	982	2516
Relative frequency	0.6						

a Copy and complete the table.

b Do you think his coin is unbiased? Give a reason for your conclusion.

④ The police force in a large city selects vehicles at random for road-worthiness checks.

The table shows the results of a traffic survey at a particular road junction.

Vehicle	Lorry	Van	Bus or coach	Car
Frequency	15 305	2411	1064	24 792

a Estimate the probability of each type of vehicle being selected on one occasion.

b In one week the police carry out checks on 190 vehicles at the same road junction.

Estimate how many of these vehicles are buses or coaches.

c On one occasion, across the whole city, the police select 2400 vehicles for checks.

Comment on the accuracy of using the data from the table to estimate the total number of lorries checked.

Give a reason for your answer.

⑤ A bag contains some coloured balls. They are red, blue or green.

Eleanor takes a ball out at random, notes its colour and replaces it.

She does this 40 times.

Here are her results.

Colour	Red	Blue	Green
Frequency	12	18	10

a Estimate the probability that the next ball she draws is

i red **ii** blue **iii** green.

b There are 12 balls in the bag.

i Estimate how many there are of each colour.

ii What could Eleanor do to make her estimate more reliable?

⑥ Kwame and Kofi are investigating a type of moth.

There are two varieties of this moth: light and dark.

They collect moths at different sites, A and B, and record which variety each moth is that they collect.

Kwame (Site A)			
Light	Dark	Dark	Light
Dark	Light	Dark	Dark
Light	Dark	Dark	Dark
Light	Light	Dark	Dark
Dark	Dark	Dark	Dark

Kofi (Site B)			
Dark	Light	Light	Light
Light	Dark	Dark	Light
Light	Light	Light	Light
Dark	Light	Light	Dark
Light	Dark	Light	Light

a Estimate the probability that a moth is dark

i using all the data

ii using only Kwame's data

iii using only Kofi's data.

b Kwame says he thinks the moths are different at the two sites.

Explain why he says this.

7 **a** Sara, Pete and Ian each have an unbiased six-sided die.

Sara says that she normally throws a 6 every three throws.

Pete says he takes at least 20 throws to get a 6.

Ian says that he never gets a 6.

Who is correct?

b Tim also has a six-sided die. He throws the die 20 times and gets five 6s.

i Is the die biased?

ii Tim throws the die a further 100 times and gets 15 sixes.
Is it more or less likely that the die is biased? Explain your answer.

iii How can Tim check whether the die is unbiased?

8 Jane, Katie and Len want to know the probability that a letter used in written English is an E. (It can be upper case E or lower case e.) They each take a sentence as a sample to estimate this probability.

a In each case, use the sample sentence to estimate the probability that a letter in written English is an E.

i Jane: *The quick brown fox jumps over the lazy dog.*

ii Katie: *To be or not to be, that is the question.*

iii Len: *Never in the field of human conflict was so much owed by so many to so few.*

b The probability of a letter being E is actually 0.12.

i Which of the sentences gave the closest estimate?

ii Explain why Jane's sentence cannot be regarded as random.

c Suggest how they can obtain a more accurate estimate.

d There are 100 tiles in a Scrabble set. Two of them are blank and the other 98 have letters on them.

i How many tiles would you expect to have the letter E in a set designed for players using English?

ii Find out how many Es there are in a standard Scrabble set.

9 Here is the result of a traffic survey a council did over one week on a busy road. It shows the numbers of lorries, buses and cars that used the road.

Vehicle type	Lorries	Buses	Cars	Motorbikes
Frequency	16 000	10 000	240 000	9000

a Keith stands at the side of this road.

Estimate the probability that the next vehicle to go past Keith is

i a car

ii a motorbike

iii not a lorry.

b In one day, 15 000 vehicles go past a particular point.

Estimate the number of buses to pass that point.

⑩ A biased tetrahedral die has four sides numbered 1, 2, 3 and 4.

The die is thrown 20 times and the number it lands on is recorded.

1	3	4	2	4	4	3	2	2	1
1	2	2	4	3	2	1	1	1	2

a Copy and complete the relative frequency table.

Lands on	1	2	3	4
Relative frequency				0.2

These are the relative frequencies after the die is thrown 100 times.

Lands on	1	2	3	4
Relative frequency	0.3	0.4	0.15	0.15

b Which of the two relative frequencies for 'lands on 4' is the better estimate of the probability of the die landing on 4?

Give a reason for your answer.

The die continues to be thrown.

c Estimate the number of times it will land on '1' in 1000 throws.

⑪ A four-sided spinner has the numbers 1, 3, 5 and 7 on it.

The table shows the probability that it will land on each number.

Number	1	3	5	7
Probability	0.15	0.4	0.25	0.2

Harry spins the spinner 150 times.

a How many times should he expect to score

i 3

ii 5?

b How many times does he expect not to score 3?

c What is the probability that he does not score 3?

⑫ At a charity fete there is a stall selling tickets.

The probability of winning £1 with one ticket is $\frac{1}{8}$.

Tickets cost 20p each.

The charity would like to make a profit of £120.

How many tickets does it need to sell?

Do I know it now?

① A doctors' surgery keeps a record of how early or late patients are seen.
The results are recorded in the table below.

Time patient seen	Frequency
$\frac{1}{4}$ hour early or more	4
Up to $\frac{1}{4}$ hour early	20
On time	8
Up to 5 minutes late	4
5 minutes to 10 minutes late	3
More than 10 minutes late	1

a Use the table to find the probability that a patient selected at random will be seen

i up to 5 minutes late

ii 5 minutes to 10 minutes late

iii more than 10 minutes late.

b The next day the surgery sees 124 patients. Estimate how many of them will be seen more than 10 minutes late.

There is a larger doctors' surgery in the same town. In one day, the larger surgery sees 438 patients.

c Comment on the accuracy of using the data from the table to estimate the probability that a patient is seen late at the larger surgery.

Give a reason for your answer.

22.3 The multiplication rule

What you need to know

Events are **independent** if the outcome of one does not affect the outcome of the other.

For two independent events, A and B, then

P(A and B) = P(A) × P(B)

When a coin is thrown and a die is rolled, the score on the die does not change the likelihood of getting heads on the coin. So 'rolling a 6' and 'heads' are **independent**.

P('6' and 'heads') $= \frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$.

Events are **dependent** if the outcome of one event affects the outcome of the other.

The events 'first marble is red' and 'second marble is red' without replacement are dependent events.

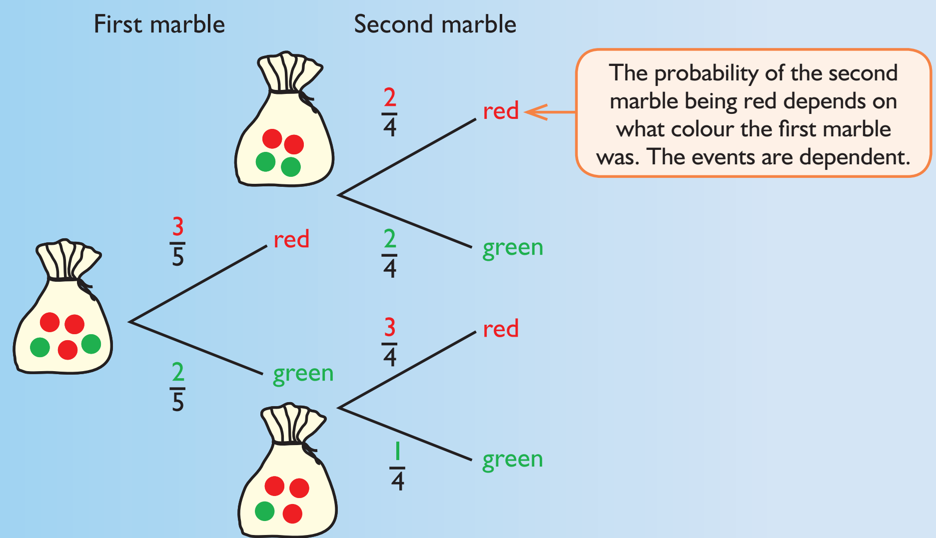
Did you know?

Game shows are almost always based on chance. Whatever the presenter may say, your chance of winning is determined by the laws of probability.

The first event has had an impact on the probability of the second event.

A **tree diagram** can be used to show all the possible outcomes of an event or sequence of events.

This tree diagram shows the possible outcomes of taking marbles (without replacement) from a bag containing three red marbles and two green marbles.



You can multiply the probabilities as you move along the branches to work out the probability of combined events.

P(red marble followed by a green marble) $= \frac{3}{5} \times \frac{2}{4} = \frac{6}{20} = \frac{3}{10}$

How to do it

Independent events

A coin is thrown and a die is rolled.

a The coin is thrown 10 times and each time shows a tail.

What is the probability that the 11th time it will also show a tail?

b A die is rolled 100 times and a 6 has not yet been rolled.

What is the probability that the next roll will give a 6?

c What is the probability of getting a 'tail' and a '6' together?

Solution

a Each throw of the coin is an independent event. What has happened in previous throws does not affect it, so the probability is $\frac{1}{2}$.

b Each roll of a die is an independent event so the probability of scoring a 6 on each roll is exactly the same. The probability of scoring a 6 on the 100th roll is $\frac{1}{6}$.

c The events are independent, so P(tail and 6) = P(tail) × P(6)

$$= \frac{1}{2} \times \frac{1}{6}$$

$$= \frac{1}{12}$$

Wrongly believing that a head is more likely to turn up as it is 'overdue' is known as the 'Gamblers' fallacy'.

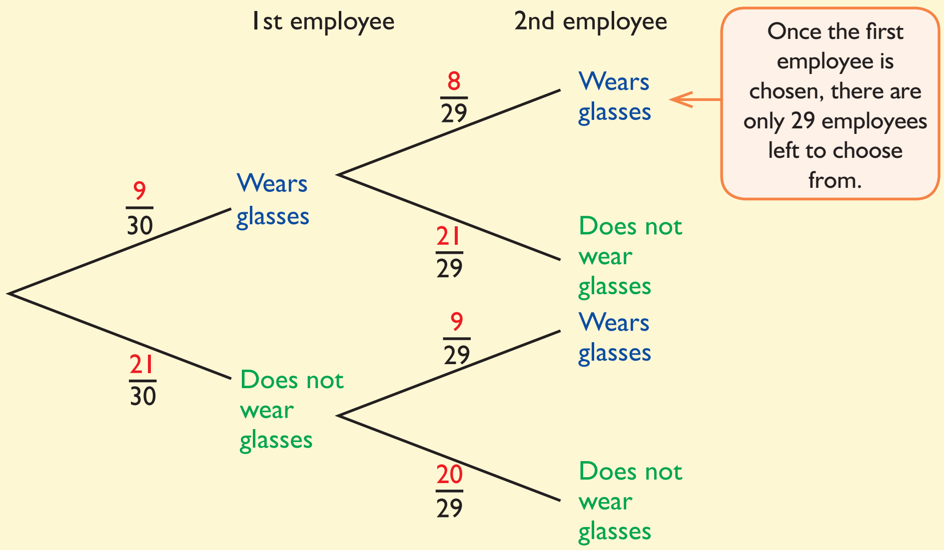
Without replacement

There are thirty employees in an office. Nine of them wear glasses.
Two employees are chosen at random from the group.

a What is the probability that the second employee chosen wears glasses?

b What is the probability that exactly one of the employees wears glasses?

Solution



a The second employee wears glasses in two of the paths on the tree diagram.

P(1st wears glasses and 2nd wears glasses)

$$= \frac{9}{30} \times \frac{8}{29} = \frac{72}{870}$$

The probability that the second employee chosen wears glasses is found by adding these two probabilities together.

P(1st does not wear glasses and 2nd wears glasses) $= \frac{21}{30} \times \frac{9}{29} = \frac{189}{870}$

P(2nd employee wears glasses) $= \frac{72}{870} + \frac{189}{870} = \frac{261}{870}$

$$= \frac{87}{290}$$

Simplify to $\frac{87}{290}$ by dividing the 'top' and 'bottom' by 3.

b There are two paths through the tree diagram that lead to exactly one employee wearing glasses.

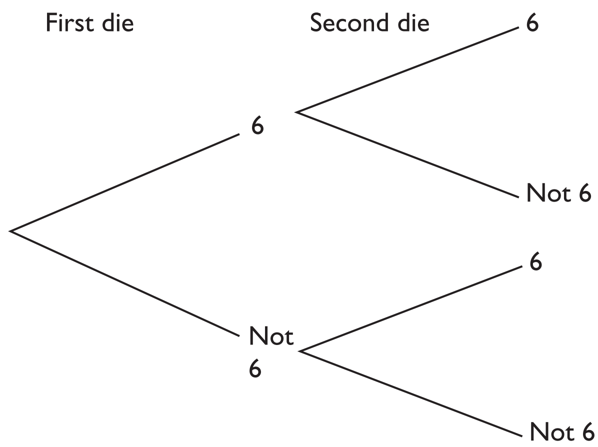
P(1st wears glasses and 2nd does not wear glasses) $= \frac{9}{30} \times \frac{21}{29} = \frac{189}{870}$

P(1st does not wear glasses and 2nd does wear glasses) $= \frac{21}{30} \times \frac{9}{29} = \frac{189}{870}$

So, P(exactly one wears glasses) $= \frac{189}{870} + \frac{189}{870} = \frac{378}{870} = \frac{63}{145}$

Learning exercise

① Two dice are rolled.



a Copy and complete the tree diagram.

b Use this tree diagram to calculate the probability that

i both dice show a 6

ii neither die shows a 6

iii at least one die shows a 6

iv exactly one die shows a 6.

② Arlo and Bekah are playing cards. There are 52 cards in the pack; 4 of them are aces. They deal the cards and they are not replaced.

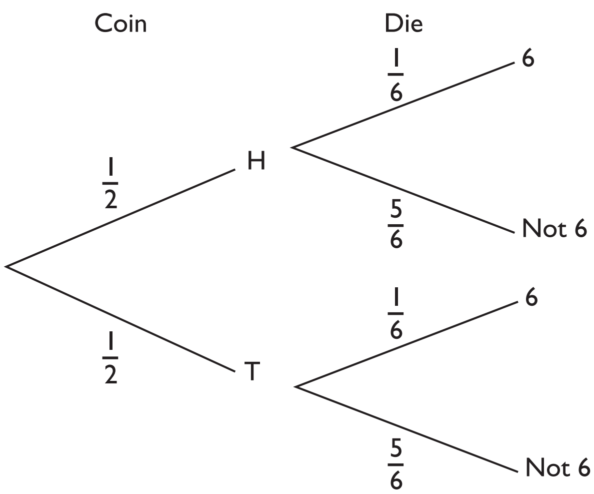
a Bekah deals the first card. What is the probability that it is an ace?

b Bekah's card is not an ace. Arlo gets the next card. What is the probability that it is an ace?

c Arlo's card is not an ace. Bekah deals the next card. What is the probability that it is an ace?

d Bekah's card is an ace. Arlo deals the next card. What is the probability that it is an ace?

③ A coin is thrown and a die is rolled.



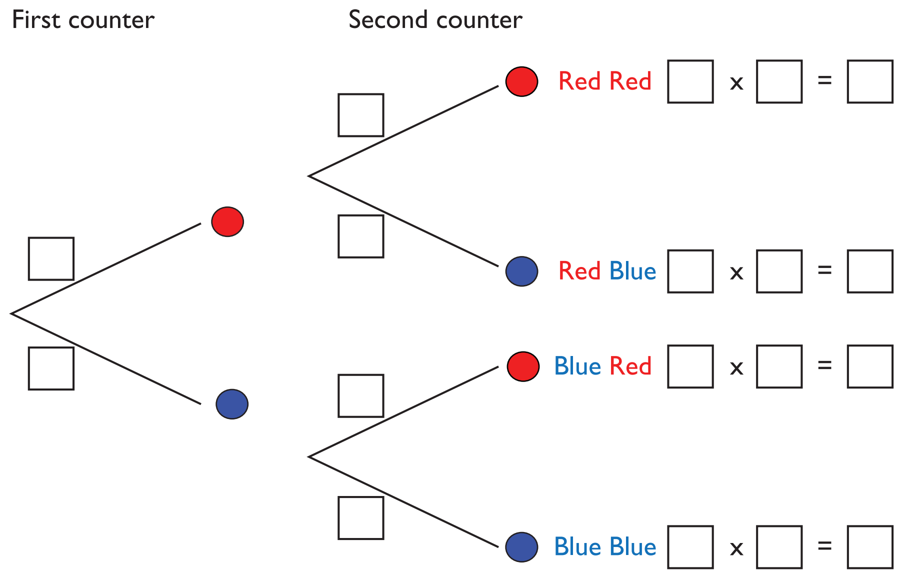
Use this tree diagram to calculate the probability of scoring

a a head and a 6 **b** a head and not a 6 **c** a tail and a 6 **d** a tail and not a 6.

④ 12 coloured counters are placed into a bag. 7 are red and 5 are blue.

A counter is drawn out of the bag. The counter is not replaced. A second counter is then drawn out of the bag.

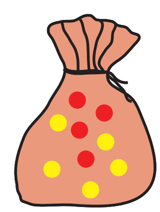
a Copy and complete this tree diagram.



b Another time, the first counter is replaced.

Draw the tree diagram for this situation.

⑤ Della has these coloured counters in a bag.



a She takes out a counter without looking.

What is the probability that it is red?

b The first counter was red. Della does not put it back.

How many red counters and how many yellow counters are left in the bag?

c Della shakes the bag and takes out another counter.

i What is the probability that it is red?

The counter is red and again Della does not put it back.

She takes out another counter from the bag.

What is the probability that it is

ii red

iii yellow?

⑥ A class of students has 16 girls and 13 boys.

Two names are chosen at random from the register.

a Draw a tree diagram to show the possible outcomes.

b Using the information from your tree diagram, work out the probability that

i both students are girls

ii both students are boys

iii at least one of the students is a boy

iv exactly one of the students is a boy.

⑦ In each case, state whether P and Q are independent or dependent.

a P: I throw a coin and record how it lands.

Q: I roll a die and record its score.

b P: It rains some time today.

Q: It rains some time tomorrow.

c P: A person lives to be 100 years old.

Q: A person smokes.

⑧ Lucie has two dice. One is blue and has eight sides numbered from 1 to 8. The other is red and is a normal, six-sided die.

a Lucie rolls both dice.

i What is the probability that she rolls a 6 on the blue die?

ii What is the probability that she rolls a 6 on the red die?

b Lucie rolls both dice again.

i What is the probability that she rolls a 6 on the blue die?

ii What is the probability that she rolls a 6 on the red die?

c Lucie rolls both dice again. She rolls a 6 on the red die.

What is the probability that she rolls a 6 on the blue die?

d Are the outcomes of the two dice independent or dependent?

Problem solving exercise

① The probability that Seb has toast for breakfast is 0.4.

The probability that Seb has toast for lunch is 0.05.

The events are independent.

Select the right calculation for each of the following.

a The probability that Seb doesn't have toast for lunch.

b The probability that Seb doesn't have toast for breakfast.

c The probability that Seb has toast for breakfast and lunch.

d The probability that Seb has toast at least once a day.

	$1 - 0.05$	0.95×0.4
	$1 - 0.95 \times 0.6$	$1 - 0.4$
	0.05×0.4	$1 - 0.05 \times 0.4$

② James says that he threw a coin 20 times and got tails every time. Joe says that this is impossible and that James is not telling the truth.

Explain why Joe is wrong.

③ A box contains two white balls and one black ball.

A bag contains yellow and green balls in the ratio 2 : 3.

Mary takes a ball at random from the box.

She then takes a ball from the bag.

What is the probability that she will take

a a white ball and a yellow ball

b a black ball and a yellow ball

c a black ball and a yellow or green ball?

④ A manager wants to select two people to look after a stand at a conference. They are chosen from eight female staff, including Mrs Bell, and six male staff, including her husband Mr Bell.

a His first plan is to select the two people at random from all fourteen staff.

What is the probability that Mr and Mrs Bell are both chosen?

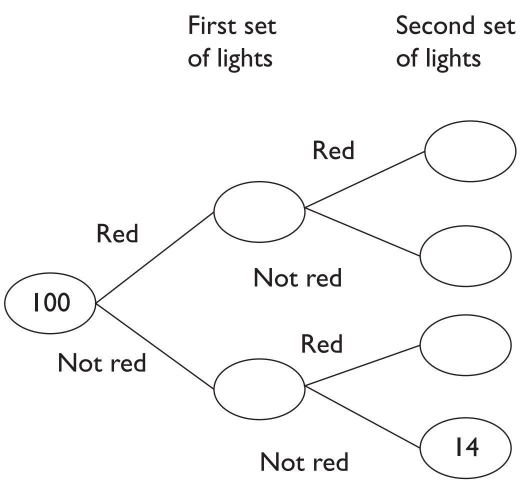
b He then decides he should select one woman and one man.

What is the probability that Mr and Mrs Bell are both chosen now?

⑤ On the way to work, Millie has to pass through two sets of traffic lights.

The probability that the first set of lights is red is 0.8 and the probability that the second set of lights is red is 0.3.

a Copy and complete this frequency tree to show the results for 100 journeys to work.



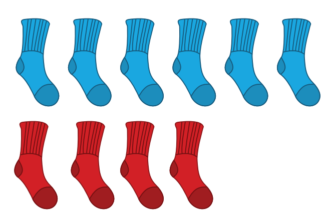
Over a 25-day period Millie has to stop at both sets of lights six times on her way to work.

b Millie thinks the lights work independently.

Do you agree? Give a reason for your answer.

c If Millie is right, how often would you expect her to have to stop at just one set of lights in a 25-day period?

⑥ There are six blue socks and four red socks in a drawer.



Freddie takes out two socks at random.

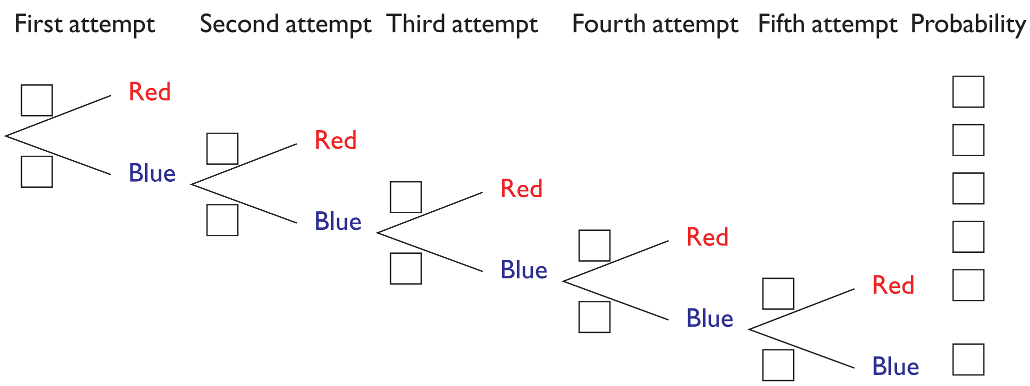
a Work out the probability that Freddie takes out two socks of the same colour.

b What is the maximum number of socks he would need to take out at random to ensure he had a red pair?

⑦ Jamil has a bag containing five counters. Four are blue and one is red.

Jamil wants the red counter. He takes the counters out one at a time until he comes to the red one. If he picks a blue counter, he does not replace it.

a Copy and complete this tree diagram.



b Jamil can take between one and five attempts to get the red counter.

- **i** Before he starts, what is the probability that he will take each of these numbers of attempts?
- **ii** What does the total of these five probabilities tell you?

c Investigate the situation if Jamil replaces any blue counters he takes out.

- **i** How does this affect the tree diagram?
- **ii** What is the probability that he gets the red counter in his first five attempts?

Do I know it now?

① Fifteen counters are placed in a bag. Five are red and the rest are green. Two counters are drawn from the bag in succession, without replacement. What is the probability of drawing out

a two red counters

b two green counters

c one of each colour

d at least one green counter?

② The probability that it will rain today is 0.2.
If it rains today, the probability that it will rain tomorrow is 0.15.
If it is fine today, the probability that it will be fine tomorrow is 0.9.

a Draw a tree diagram and calculate the missing probabilities.

b What is the probability that at least one of the two days will be fine?

Can I apply it now?

① Naela takes the train to work.
When it is raining, the probability the train is late is 0.3.
When it is not raining, the probability the train is late is 0.05.
In January, she worked on 22 days. It rained as she went to work on 8 of those days.

a Work out the probability that the train was late on a day in January.

b The railway company advertised that fewer than 10% of its trains were late during January. Could this claim be true for Naela's train? Explain your answer.

22.4 The addition rule

What you need to know

Two events or outcomes are **mutually exclusive** if they cannot both happen.

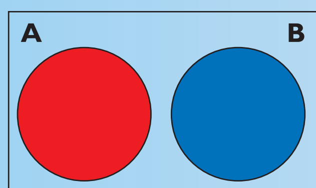
(When you toss a coin, the outcomes ‘heads’ and ‘tails’ are mutually exclusive – you cannot get both heads and tails!)

When two events or outcomes are mutually exclusive, the probability that one or the other of them happens can be found by **adding** their individual probabilities.

The events must be mutually exclusive for the **Addition Rule** to work.

You can use **Venn diagrams** to show possible outcomes.

This Venn diagram shows two mutually exclusive events.

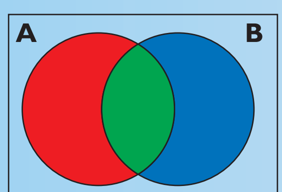


There is no intersection as A and B can never both happen!

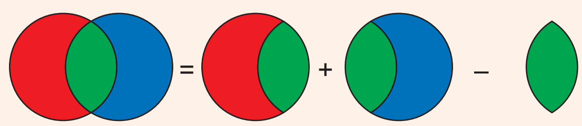
P(A or B) = P(A) + P(B)

For example, event A is ‘throwing a die and getting an odd number’ and event B is ‘throwing a die and getting 2’.

This Venn diagram shows events that are not mutually exclusive.



The intersection shows that A and B can happen at the same time:



P(A or B) = P(A) + P(B) – P(A and B)

For example, event A is ‘someone owning a cat’ and event B is ‘someone owning a dog’, the intersection is ‘someone owning a cat and a dog’.

Set notation: E = $\{x : x \text{ is a factor of } 24\}$ means E is the set of all the numbers that are factors of 24, i.e. 1, 2, 3, 4, 6, 8, 12, 24.

Did you know?

Games such as Snakes and Ladders involve no skill at all as they are based entirely on chance. So young children are just as likely to win as the adults or older children they play against.

How to do it

Mutually exclusive events

A bag contains 20 counters. Five of them are blue, two are green and the rest are other colours.

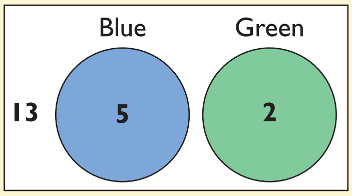
a Draw a Venn diagram showing this information.

b A counter is selected at random. What is the probability that it is

i blue **ii** green **iii** blue or green **iv** neither blue nor green?

Solution

a



b **i** P(blue) $= \frac{5}{20} = 0.25$

ii P(green) $= \frac{2}{20} = 0.1$

iii Choosing a blue counter and choosing a green counter are mutually exclusive events, so you add their probabilities.

P(blue or green) = P(blue) + P(green)

$= \frac{5}{20} + \frac{2}{20}$

$= \frac{7}{20}$ or 0.35

iv Choosing a counter that is neither blue nor green is the opposite of choosing one that is either blue or green.

So P(neither blue nor green) = 1 − 0.35 = 0.65

The total probability is 1.

Another way of doing this is to look at the Venn diagram.

There are 20 − 5 − 2 = 13 other colours.

So the required probability is $\frac{13}{20} = 0.65$.

Events that are not mutually exclusive

A fair die is rolled.

a Draw a Venn diagram to show the events A and B, where A = {prime numbers on a die} and B = {even numbers on a die}.

b Use your Venn diagram to find

i P(score is prime)

ii P(score is even)

iii P(score is prime and even)

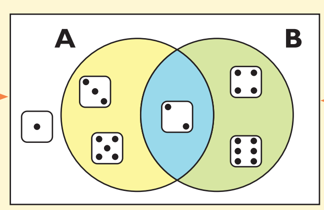
iv P(score is prime or even).

Solution

a The prime numbers on a die are 2, 3 and 5. So A is {2, 3, 5}.

The even numbers on a die are 2, 4 and 6. So B is {2, 4, 6}.

1 is neither prime nor even so it goes outside the circles.



2 is prime and even, and goes in the intersection.

b **i** P(score is prime) $= \frac{3}{6} = \frac{1}{2}$

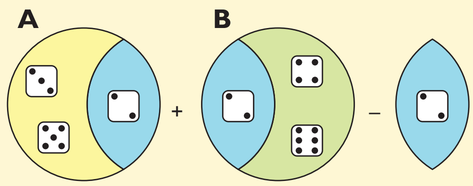
3 out of the 6 possible scores are prime.

ii P(score is even) $= \frac{3}{6} = \frac{1}{2}$

3 out of the 6 possible scores are even.

iii P(score is prime and even) $= \frac{1}{6}$ ← 2 is the only even prime.

iv P(score is prime or even) $= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$ ← 2, 3, 4, 5 and 6 are prime or even.



Learning exercise

① A bag contains 15 counters. Three are red, two are blue and the rest are yellow. A counter is chosen at random. What is the probability that the counter is

a red **b** blue **c** red or blue?

② A raffle has tickets numbered from 1 to 300. Richard has five tickets and Sue has seven tickets. One ticket is drawn to find the winner of the star prize.

What is the probability that

a Richard wins the star prize **b** Sue wins the star prize

c either Richard or Sue win the star prize?

③ Pete is listening to his music on shuffle.

He has 600 songs which are all rated from 0 to 5. He has rated 57 songs as 5 star songs and 72 songs as 4 star songs. What is the probability that the next song he plays is

a a 5 star song **b** a 4 star song

c either a 4 star or 5 star song **d** rated with 0, 1, 2 or 3 stars?

④ A normal six-sided die is rolled. What is the probability that the number that comes up is

a 2 or less **b** 5 or more **c** either 2 or less, or 5 or more?

⑤ In a survey 50 students were asked whether they liked soccer or rugby.

21 liked only soccer.

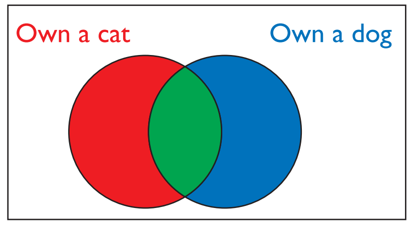
10 liked both.

8 liked neither soccer nor rugby.

a Draw a Venn diagram showing this information.

b What is the probability that a student chosen at random from this group liked only rugby?

⑥ In a class of 30 students, 8 own a dog, 11 own a cat and 4 own both a cat and a dog.

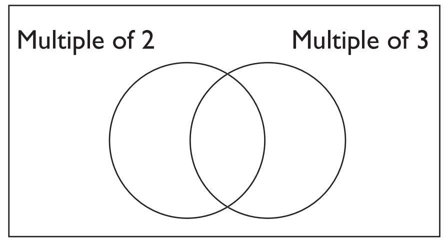


a Copy and complete the Venn diagram.

b What is the probability that a student picked at random has

i a dog **ii** a dog and a cat **iii** a dog or a cat or both **iv** no pets?

⑦ Emily has cards numbered from 1 to 15.



a Write each number from 1 to 15 in the correct place in a copy of the Venn diagram.

b Find the probability that a card Emily selects at random is

i a multiple of 2 **ii** a multiple of 3

iii a multiple of 2 and a multiple of 3 **iv** a multiple of 2 or a multiple of 3

v neither a multiple of 2 nor a multiple of 3.

⑧ There are 52 cards in a pack. Each card is equally likely to be chosen.

What is the probability that a card chosen at random is

a a club **b** not a club **c** a spade, heart or diamond

d a 2 or a 3 **e** neither a 2 nor a 3?

⑨ This two way table shows how many members of an amateur dramatics club fit certain descriptions.

	Blonde hair	Brown hair	Black hair	Total
Wears glasses	3	4	0	
Does not wear glasses	9	10	3	

a Copy and complete the table.

b What is the probability that a member chosen at random has

i blonde hair **ii** blonde hair or black hair

iii blonde hair or wears glasses, but not both **iv** black hair or wears glasses, but not both

v black hair and wears glasses?

⑩ Ryan writes down each of the numbers from 1 to 20 on a separate card.

He then decides which cards belong in the following sets:

A = $\{x: x$ is a factor of $18\}$

B = $\{x: x$ is an odd number$\}$

a Draw a Venn diagram to show the two sets A and B.

b Ryan shuffles his cards and selects a card at random.

Work out the probability that a number on a card selected at random is

i odd **ii** a factor of 18 **iii** an odd number and a factor of 18

iv an odd number or a factor of 18 **v** neither an odd number nor a factor of 18.

⑪ A vet is investigating the possible effects of inbreeding in a breed of pedigree dog.

Two possible effects are bad eyesight and weak hips.

The vet looks at a sample of 80 dogs and finds that 44 have neither condition, 24 have bad eyesight and 32 have weak hips.

a Show this information in a Venn diagram.

b **i** Estimate the probability that a dog of this breed suffers from both defects.

ii Say why your answer is an estimate and not necessarily accurate.

c Find the probability that a dog of this breed suffers from at least one of the defects, by using

i the formula P(A or B) = P(A) + P(B) − P(A and B)

ii the numbers on the Venn diagram.

Problem solving exercise

① 100 patients take part in a trial of new medicines.
36 patients receive medicine X only.
25 patients receive medicine Y only.
15 patients receive both medicine X and medicine Y.
The remaining patients receive a placebo.

a Draw a Venn diagram showing this information.

b What is the probability that a patient chosen at random receives a placebo?

② 100 people belong to a health spa club. It has a swimming pool and a gym.
70 use the swimming pool only.
10 use both the swimming pool and the gym.
1 uses neither the swimming pool nor the gym.

a Draw a Venn diagram showing this information.

Members are chosen at random to take part in a survey.

b What is the probability that someone chosen for the survey uses the gym?

c What is the probability that someone chosen for the survey uses one (but not both) of the club's facilities?

③ At a supermarket, 80% of customers bring their own carrier bags and 25% of customers have loyalty cards.

15% of customers have both their own bags and a loyalty card.

a Draw a Venn diagram to show this information.

b Find the probability that a randomly chosen customer has

i a loyalty card but no carrier bag

ii a carrier bag but no loyalty card

iii a carrier bag or a loyalty card or both

iv neither a carrier bag nor a loyalty card.

④ Ben always cycles, walks or drives to work.
On any day the probability that Ben cycles to work is $\frac{1}{3}$ and the probability that he drives is $\frac{1}{4}$.
Calculate the probability that

a Ben walks to work

b Ben either walks to work or he cycles.

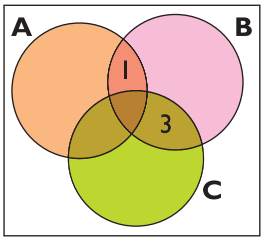
⑤ There are 80 A-level students at a college. 40 of them study at least one science.
28 students study physics.
28 students study chemistry.
3 students study physics and biology only.
14 students study physics and chemistry only.
1 student studies chemistry and biology only.
5 students study all three sciences.
One of the 80 students is chosen at random to meet a visiting television science presenter.

a Identify the sets in this Venn diagram.

b Copy the diagram and fill in all the missing numbers.

c Find the probability that the student chosen studies two or three science subjects.

d Which is more likely: the student chosen studies some science or the student studies no science?



Do I know it now?

① **a** A die is rolled. What is the probability that the score is

i even **ii** 3 or less **iii** either even or 3 or less?

b Are the outcomes 'even' and '3 or less' mutually exclusive?

Explain your answer.

Can I apply it now?

① In a group of 30 students:

- 8 study history only
- 5 study both history and geography
- 2 study neither history nor geography.

a Draw a Venn diagram showing this information.

b A student is chosen at random from this group. Find the probability that the student studies one out of history and geography but not both.

MINI TEST PAPERS

Next Steps Test Paper 1

Total number of marks: 50

Suggested time allowed: 1 hour

1 Here are five different numbers.

−8	3	−4	9	0

a Copy the statement below and write one number in each box to make the largest product. The numbers must be different.

□ × □ [1]

b Work out the difference in value between the largest product and the smallest product. [2]

2 The table shows some information about the English grades for some students.

Grade	Frequency
A*	6
A	15
B	12
C	5
D	2

Draw a pie chart to show this information. [3]

3 I think of a number and call it n. I add 10 to my number. I then multiply the answer by 5. This gives me the same answer as when I subtract n from 80.

What number did I think of? [3]

4 The mean mass of five young people is 30 kg.

Another young person joins the group and the mean mass is now 29 kg.

What is the mass of the young person who joins the group?

[2]

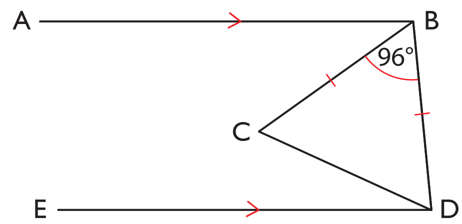
⑤ AB and DE are two parallel lines.

BC = BD

Angle CBD = 96°

Angle ABC = 2 × angle EDC

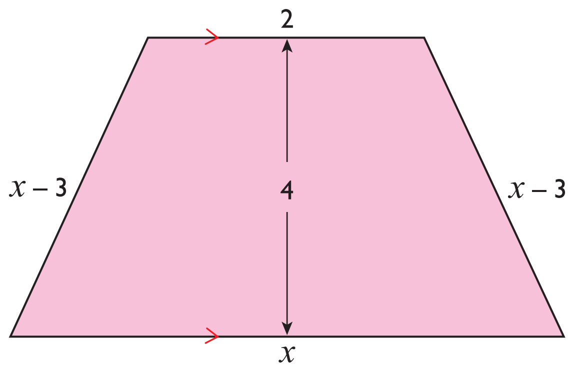
Work out the size of angle ABC. [3]



⑥ Here is a trapezium. All the measurements are in centimetres.

The area of the trapezium is $20\,cm^2$.

Work out the perimeter of the trapezium.



[4]

⑦ The diagram shows the cross section PRQ of a large tower.

RT is a radio mast.

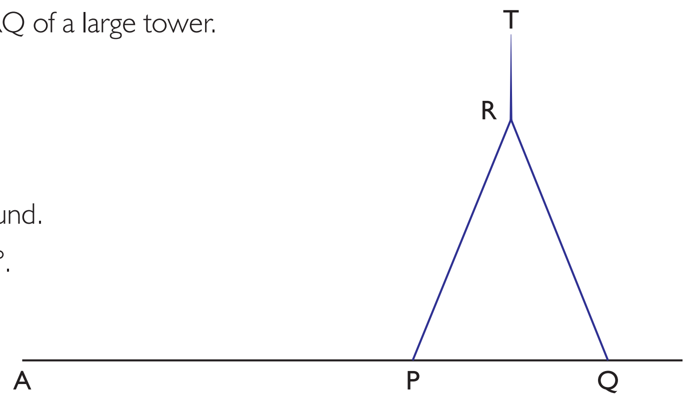
PQ = 30 m

PR = QR = 45 m

A, P and Q are points on horizontal ground.

The angle of elevation of T from A is 33°.

AP = 80 m



Use a scale drawing to find an estimate for the height of the top of the mast T above the ground. [3]

⑧ ABCD is a rectangle.

AB = 10 cm

The perimeter of the rectangle is 36 cm.

Work out the angle between the diagonal DB and the side BC. [4]

⑨ Brendon estimates that one fifth of his earnings are deducted from his pay each month.
He spends one third of the remainder on the rent and utilities for his apartment.

He estimates that he splits the rest of his monthly pay equally between food, clothes and savings.

What fraction of Brendon's monthly pay does he save? [3]

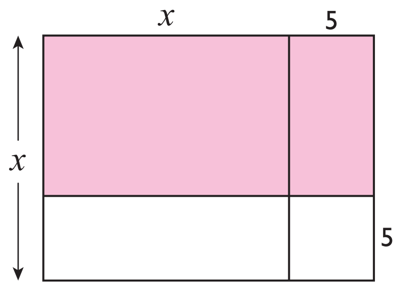
⑩ Darren buys some melons and some pineapples.

He buys three times as many melons as he buys pineapples. The cost of a pineapple is £1.45.

Darren pays a total of £16.60.

If the cost of a melon is under £1, work out the number of melons and the number of pineapples Darren buys. [3]

⑪ Show that the region shaded in this rectangle has an area of $(x^2 - 25)$ square units.



[2]

⑫ A square has a perimeter of $(40x + 60)$ cm. A regular pentagon has the same perimeter as the square.

Show that the difference between the length of the sides of the two shapes is $(2x + 3)$ cm. [3]

⑬ In 2010 the population of an endangered species of spider in Australia was 2.5 million. It is estimated that this population is decreasing at an annual rate of 8.5%.

a What was the population in 2009? Write your answer to the nearest thousand. [3]

b In what year was the population greater than 5 million? [3]

⑭ Robbi's monthly bill, B pounds, for using a mobile phone when he makes c calls and sends t texts is given by the formula

$$B = \frac{8c + 2t}{100} + 20.$$

Last month Robbi made 80 calls and sent 40 texts.

a How much was his bill? [2]

This month Robbi's bill was £27. He made 60 calls.

b How many texts did he send? [3]

⑮ Find the value of n to make these statements true.

a $a^5 \times a^n = a^{12}$ [1]

b $\frac{12p^9}{3p^n} = 4p^3$ [1]

c $(y^n)^2 = y^{12}$ [1]

Next Steps Test Paper 2

Total number of marks: 50

Suggested time allowed: 1 hour

1 **a** Anwar worked out the value of $8 - 4 \times 2 + 5$. He got an answer of 13. This answer is wrong.

Explain the mistake that Anwar may have made and give the correct answer. [2]

b Bronwin worked out the value of $3y^2 + 7$ when $y = 2$. She got an answer of 43. This answer is wrong.

Explain the mistake that Bronwin may have made and give the correct answer. [2]

2 Sandra recorded her science test marks over seven weeks. The table shows this information.

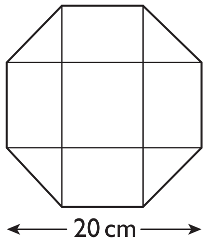
Week	1	2	3	4	5	6	7
Score	6	8	6	20	12	11	14

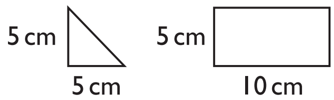
a Plot a vertical line graph for this information. [3]

b Why do you think Sandra scored higher in week 4 than any other week? [1]

c Can you draw a trend line from week 1 to week 7 to predict future scores? Explain your answer. [1]

3 Here is a piece of card in the shape of an octagon.





The octagon is made of 4 triangles, 4 rectangles and a square.

Harry has a large piece of card 60 cm by 80 cm. He cuts out as many of these octagons as possible from the large piece of card. Work out the area of card left over. [4]

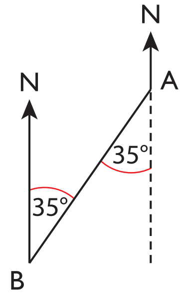
4 **a** The bearing of town A from town B is 035°.

What is the bearing of town B from town A? [1]

b The bearing of town C from town B is 125°.

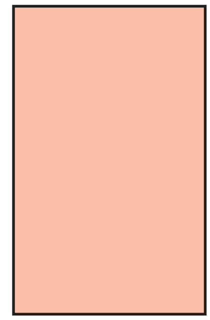
Town B is 40 km from town A. Town B is 40 km from town C.

Work out the bearing of town C from town A. [3]



⑤ Here is a rectangle.

Use a ruler and a pair of compasses to construct an isosceles triangle with the same area as the rectangle. Do not use a ruler to measure any lengths. [3]



⑥ Boris says 'I think of a number and subtract 4. I then square the result. My final answer is 4.'

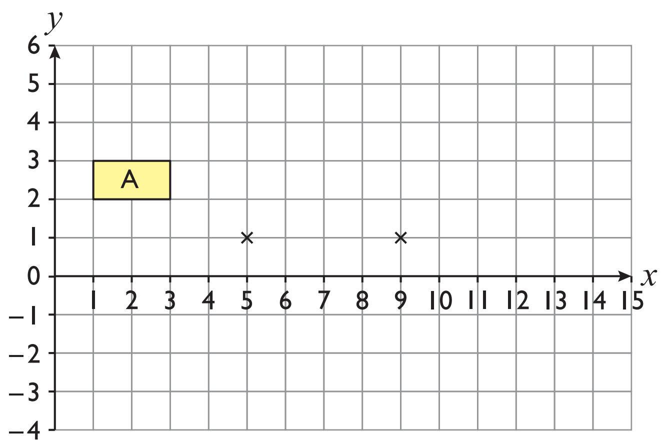
What numbers could Boris be thinking of? [3]

⑦ The angles of a hexagon, in clockwise order, are in the ratio 3 : 2 : 3 : 3 : 2 : 3.

The sides of the hexagon are equal in length. The perimeter of the hexagon is 24 cm.

Draw an accurate diagram of this hexagon. [3]

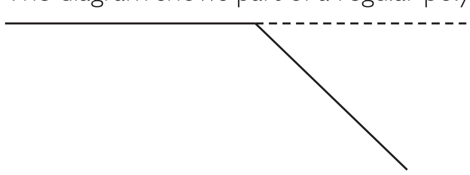
⑧ Shape A is a rectangle.



Shape A is rotated 90° clockwise about the point (5, 1) to give shape B. Shape B is rotated 90° anticlockwise about the point (9, 1) to give shape C.

Describe the single transformation that will map shape A onto shape C. [4]

⑨ The diagram shows part of a regular polygon.



Sam says 'the interior angle is 40° more than the exterior angle'. Explain why Sam is wrong. [3]

⑩ Jean Pierre lives on a very busy road in France.

He constantly complains to the local authority that vehicles go too quickly past his house.

Jean Pierre decides to measure the speed of vehicles passing his house. He buys a meter that can measure the speed of a passing vehicle. The meter gives the speed in metres per second.

There is a 50 km/h speed limit on the road where Jean Pierre lives.

The table shows the speeds, in metres per second, of a number of vehicles passing his house.

Vehicle	car	motorcycle	van	sports car	bicycle
Speed (m/s)	13	15	12	14	10

Which of these vehicles were exceeding the speed limit? [3]

⑪ Dennis collects old cigarette cards. He has 24 580 cards in his collection and it is valued at £7200.

Alison also collects old cigarette cards. She has just 6850 cards in her collection.

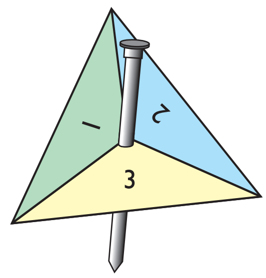
Alison sells her collection to Dennis. Dennis says that this will increase the value of his collection by 15%.

Work out the mean value of each of Dennis's cigarette cards after buying Alison's collection. [4]

⑫ A racing car is travelling at 25 m/s. It then accelerates at 2.5 m/s^2 for 8 seconds.

At what speed will the car be travelling now? Give your answer in km/h. [3]

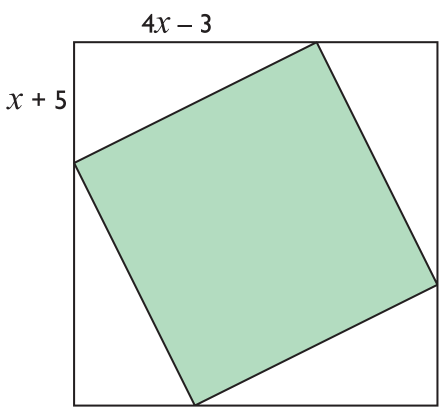
⑬ George plays a game with two spinners with sides numbered 1, 2 and 3.



He spins both spinners and records the results.

He wins the game if he gets an odd number and an odd number or an even number and an even number. Work out the probability he wins the game. [3]

⑭ The shaded square in the diagram is formed by cutting the four right-angled triangles each with base length $(x + 5)$ and height $(4x - 3)$, from each corner of the large square, as shown.



Show that the area of the shaded square is $17x^2 - 14x + 34$. [4]

Next Steps Test Paper 3

Total number of marks: 50

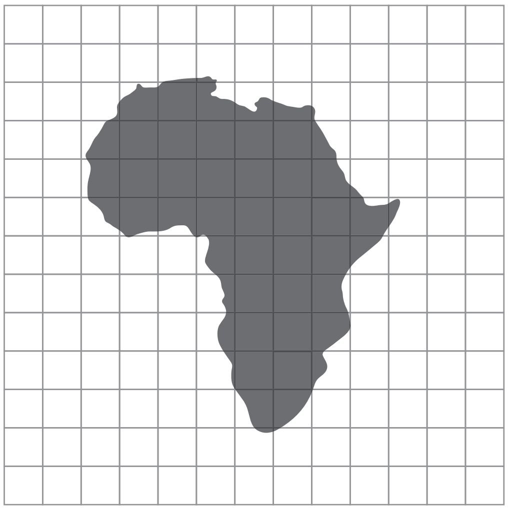
Suggested time allowed: 1 hour

1. Peter and Tim go into a fish and chip shop.

   Peter buys six portions of chips. It costs him £5.10. Tim buys four portions of chips and three steak pies. It costs Tim £8.20.

   Work out the cost of one steak pie. [3]

2. The diagram shows a map of Africa on a grid. The area of each square is about 8×10^5 square kilometres.



   Estimate the area of Africa, in square kilometres. Give your answer in standard form. [3]

3. Rolacola produces cans of fruit juice.

   Each can holds 330 ml of fruit juice and costs £1.80 per can.

   In an effort to improve sales, Rolacola produces a new size of can offering a 30% increase in the amount of fruit juice it contains. The new size of can costs £2.30.

   a Is the percentage increase in cost greater or less than 30%? [2]

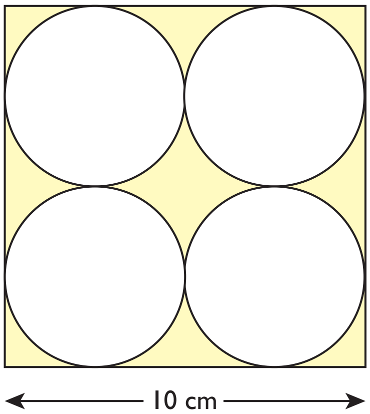
   b Which size of can is the better buy? [1]

4. A train is travelling at an average speed of 108 mph.

   a Work out the time, in seconds, it takes for the train to travel 1 mile. [2]

   b Work out the distance the train travels in 35 minutes. [2]

⑤ The diagram shows a square plate with four identical circles cut out. The length of the square is 10 cm.



Work out the area of the shaded region. [4]

⑥ Shape A is a rectangle. Shape A is reflected in the y axis and its image is then reflected in the line $y = x$.

Show that this gives a different answer to when shape A is reflected in the line $y = x$ and its image is then reflected in the y axis. [4]

⑦ Tom wants to invest £12 000 in a bank account for four years.

His local bank pays compound interest at an annual rate of 3.5% for the first year and 1.4% for each of the other three years.

The bank in the city pays compound interest at an annual rate of 3.1% for the first two years and 0.9% for each of the other two years.

In which bank should Tom invest his money? [4]

⑧ Luca uses this formula to change between degrees Fahrenheit (F) and degrees Celsius (C):

$$F = \frac{9C + 160}{5}$$

The boiling point of water is 100 °C.

a What is 100 °C in degrees Fahrenheit? [2]

b What temperature is the same in degrees Celsius as it is in degrees Fahrenheit? [2]

⑨ Seb and Nikki are racing cars around a track.

Each lap takes Seb a time of 1 minute 30 seconds. Each lap takes Nikki a time of 1 minute 40 seconds. Seb and Nikki start the race at the same time.

a How many laps have they each raced when Seb overtakes Nikki at the start line? [2]

Seb wins the race having covered 70 laps.

b How many times did Seb overtake Nikki during his race? [2]

⑩ The density of copper is 8.94 g/cm^3.

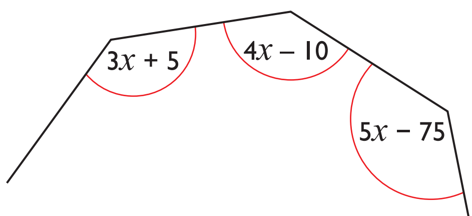
a Work out the mass of 250 cm^3 of copper. [1]

A piece of brass is made from combining 250 cm^3 of copper and 100 cm^3 of zinc. The density of zinc is 7.13 g/cm^3.

b Work out the density of the piece of brass. [3]

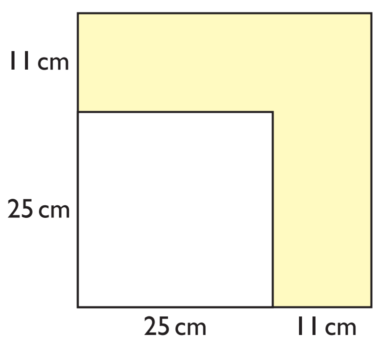
c State one assumption you have made to get your answer to part **b**. [1]

⑪ Here is part of a polygon.



Explain why the polygon cannot be regular. [3]

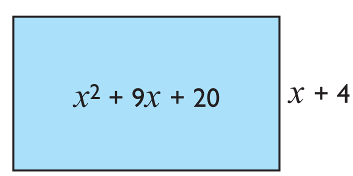
⑫ Work out the shaded area in this shape. Do not use a calculator.



[3]

⑬ The area of a rectangle in cm^2 is $x^2 + 9x + 20$.
The width of the rectangle is $(x + 4)$ cm.

Write an expression, in terms of x, for the length of the rectangle. [2]



⑭ A box contains white and black balls in the ratio 3 : 1. A bag contains yellow and green balls in the ratio 4 : 3.

Mary takes a ball at random from the box. She then takes a ball at random from the bag. What is the probability that she will take

a a white ball and a yellow ball [1]

b a black ball and a yellow ball [1]

c a black ball and a yellow or green ball? [2]